

Engineering Applications in Differential and Integral Calculus*

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The authors describe a two-year collaborative project between the Mathematics and the Engineering Departments. The collaboration effort involved enhancing the first year calculus courses with applied engineering and science projects. Two enhanced sections of the differential (first semester) and integral (second semester) calculus courses were offered during the duration of the project. The application projects involved both teamwork and individual work, and we required use of both programmable calculators and Matlab for these projects. Some projects involved use of real data often collected by the involved faculty. The paper lists all the projects, including where they fit within the course topics. Some selected projects are described in detail for both the differential and the integral calculus courses. The paper also summarizes the results of the survey questions given to the students in two of the courses followed by the authors own critique of the enhancement project.

INTRODUCTION

AT PENN STATE, most of Math 140 covers differential calculus, while about 30% of the course is devoted to integral calculus. Among the topics covered are: limits and rates of change, continuous functions, derivatives of polynomials, rational functions, trigonometric functions, curve sketching and optimization, applied word problems, the Riemann integral and the Fundamental Theorem of Calculus, areas between curves, and volumes of solids of revolution. Almost all of the topics covered in Math 141 involve the integral calculus including: inverse functions, derivatives and integrals of exponential and logarithmic functions, techniques of integration, infinite sequences and series, parametric equations, and polar coordinates.

From Fall 1997 to Spring 1999, we offered enhanced sections of the Math 140 and Math 141. The objectives were:

- to introduce team-based projects in engineering and science,
- to convey to the students the importance of mathematics in engineering and science,
- to use Matlab and graphics calculators to analyze experimental data and perform mathematical operations.

We offered one enhanced section of Math 140 in Fall 1997 and also in Fall 1998. We also offered one section of enhanced Math 141 in Spring 1998 and in Spring 1999. In this paper, we will describe

the projects used, the grading system, and survey results of the students' experiences.

BRIEF DESCRIPTIONS OF THE PROJECTS

The following are the projects (with pertinent mathematical topics in parentheses) we used in the enhanced section. In the next section, selected projects are described in detail.

Math 140 Projects, Fall 1997

1. Data on strength of basswood samples (fitting data with linear and polynomial functions)
2. Temperature data from State College, Pennsylvania (fitting data with sine functions, comparing average and instantaneous rates of change)
3. Beam analysis in mechanics (piecewise linear, quadratic, and cubic functions, and differentiability of a function)
4. Electrical circuit analysis (exponential functions and derivatives)
5. Fitting a pipeline with minimal cost (optimization of a function on a closed interval)
6. Crankshaft design (optimization of a function on a closed interval)

Math 141 Projects, Spring 1998

1. Analysis of beams in mechanics (polynomial integration and optimization of a function on a closed interval)
2. Tuning a radio (integration of sine and cosine functions)
3. Application of parametric curves (Cubic Bezier Curves)
4. Wrecking ball (approximating an integrand

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using a Taylor polynomial and finding roots of the resulting polynomial equation)

Math 140 Projects, Fall 1998

1. Hydraulic engineering (fitting data with power functions and properties of log and exponential functions)
2. Bridge project (piecewise functions and differentiability of a function)
3. Automobile velocity data (fitting polynomial functions to velocity data and testing models)
4. Optimization of an irrigation channel (plane geometry, trigonometry, and minimization of functions).

Math 141 Projects, Spring 1999

1. Exponential growth and decay (exponential functions, the definite integral and average value of a function)
2. Drag force effect on a skydiver free fall (integration of rational functions and use of integration tables)
3. Automobile velocity data (fitting polynomial functions to velocity data and numerical integration)
4. Application of parametric curves (Cubic Bezier Curves).

SELECTED PROJECTS FROM FIRST SEMESTER CALCULUS

Hydraulic Engineering (Torricelli's Principle)

Let f denote the volume flow rate of a liquid through a restriction, such as an opening or a valve, out of a tank. Torricelli's principle states that f is proportional to the square root of the volume V of liquid in the tank. Thus

$$f = r\sqrt{V}$$

where r is a constant. The idea is to use regression to fit a curve of the form

$$f = rV^m$$

to the given data. To use linear regression, transform the variables using logarithms to base 10 first, giving a linear equation between the variables $x = \log_{10} V$ and $b = \log_{10} r$.

The following problem, taken from [6], was covered in class:

'A 15-cup coffee pot was placed under a water faucet and filled to the 15-cup line. With the outlet valve open, the faucet's flow rate was adjusted until the water level remained constant at 15 cups, and the time for one cup to flow out of the pot was measured. This experiment was repeated with the pot filled to the various levels shown in Table 1.'

If t equals the time for one cup to flow out of the pot, then the flow rate equals $1/t$. During class we first found the linear regression line $f = c_1 V + c_2$ to the given data, without transforming the variables. Later we compared this fit with the one obtained by transforming the variables. Matlab

Table 1.

V (cups)	15	12	9	6
t (s)	6	7	8	9

was used to find and plot the regression curves, along with the data, and then to extrapolate from the data. It was emphasized to the class that it is important not to use software blindly, and to understand some of the underlying mathematical principles. In this case, one must understand the properties of logarithms and exponential functions. We also emphasized the importance of knowing some theory as well as the given data. In this case, the theory is Torricelli's principle, which says that $f = r\sqrt{V}$. Of course, for *experimental data*, the relationship is approximately $f = rV^{1/2}$. Here is where theory meets practice. The fit obtained without transforming the variables is $f = .0062V + .0718$, as shown by the linear regression of Fig. 1. If one transforms the variables first, the Matlab commands are: L2 = polyfit(log10(cups),log10(flow),1), which gives L2 = 0.4331, -1.3019; So $m \approx .4331$, $b \approx -1.3019$. To obtain f , $10^{\log f} = 10^{b+mx} = 10^b 10^{mx} = 10^b (10^{\log V})^m = 10^b V^m = rV^m$, where $r = 10^b$. For this example, $f = '10.^{-1.3019}.*v.^{.4331}'$, or $f = .0499V^{.4331}$. A plot of both fits together (see Fig. 1) shows that the line diverges from the power function as V becomes larger. Hence the power function is the better estimate as V grows large.

The faculty involved in the project obtained the data given to the student teams for their project. We drilled several holes in a plastic container (Fig. 2). For a given opening size, the time to fill a cup, t , was recorded for constant water levels ($V = 6, 9, 12, 15$ cups) in the container. The constant water level was maintained by adjusting the faucet water entering the container. Each team received a set of four data values for t . The values remained the same as in the sample done in class. The teams were asked to do what was done for the sample in class, and were encouraged to use Matlab.

Designing a pipeline with minimum cost

This project involved derivatives and global extrema of functions. It required determining the most cost effective pipeline route in connecting various wells in an oil fertile area [7]. Figure 3 shows a schematic view of the wetland and the corresponding simplified rectangular model. In this problem we are connecting a pipeline from well at point A to another well at point B. Costs are associated with material (cost of pipe of \$1.50/foot) and terrain type (normal terrain installation cost of \$1.20 per foot). Installation in the wetland requires an additional Track Hoe at a cost of \$60/hour. In a 10 hour day, the Track Hoe can dig approximately 300 feet of trench, and thus there is an additional cost of

$$\frac{\$60/hr.}{30ft./hr.} = \$2/ft.$$

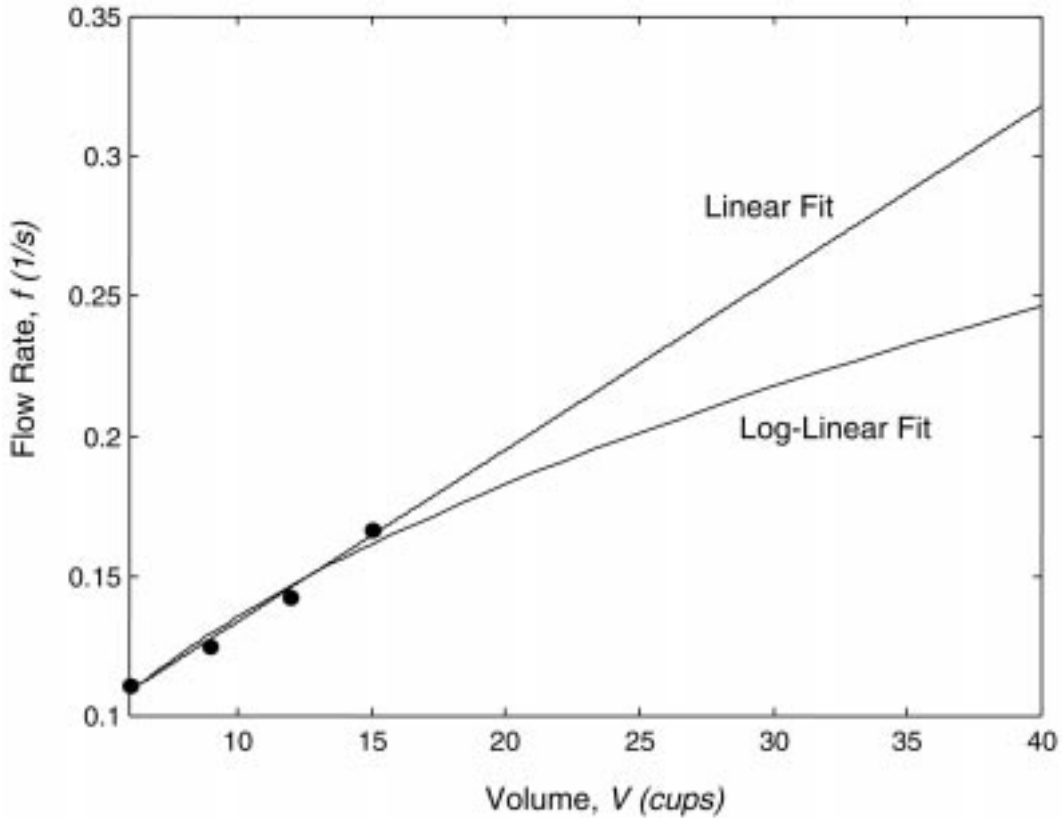


Fig. 1. Flow rate vrs. volume for the Hydraulic Engineering Project.



Fig. 2. Hydraulic engineering experimental setup.

This gives a wetland installation cost of \$4.70 per foot. Three routes were considered in class (labeled 1, 2, and 3 in Fig. 3):

$$\text{Route 1: Cost} = 2.7(d_1 + d_2 + d_3)$$

$$\text{Route 2: Cost} = 4.7\sqrt{(d_1 - d_3)^2 + d_2^2}$$

$$\text{Route 3: Cost} = 2.7x + 4.7\sqrt{(d_1 - x)^2 + d_2^2}, \quad 0 \leq x \leq d_1 - d_3$$

The solution for the third route was explained in detail in class using geometry, differential calculus, and Matlab. Here is a brief summary of the commands:

Define variable, $x = \text{sym}('x')$

Define cost function, $C(x)$

Find the derivative of the cost function, $dC = \text{diff}(C)$

Set the derivative equal to zero, solve for x, and find the global minimum cost.

For team assignments, we gave the teams different values for the rectangle representing the wetland and the coordinates for wells at points A and B. Also, given the costs involved, we required the students to find the total cost of the routes considered.

Student comments were generally very favorable about the project. We feel that this project seems very applicable to something in real life, and the mathematics involved is not too difficult.

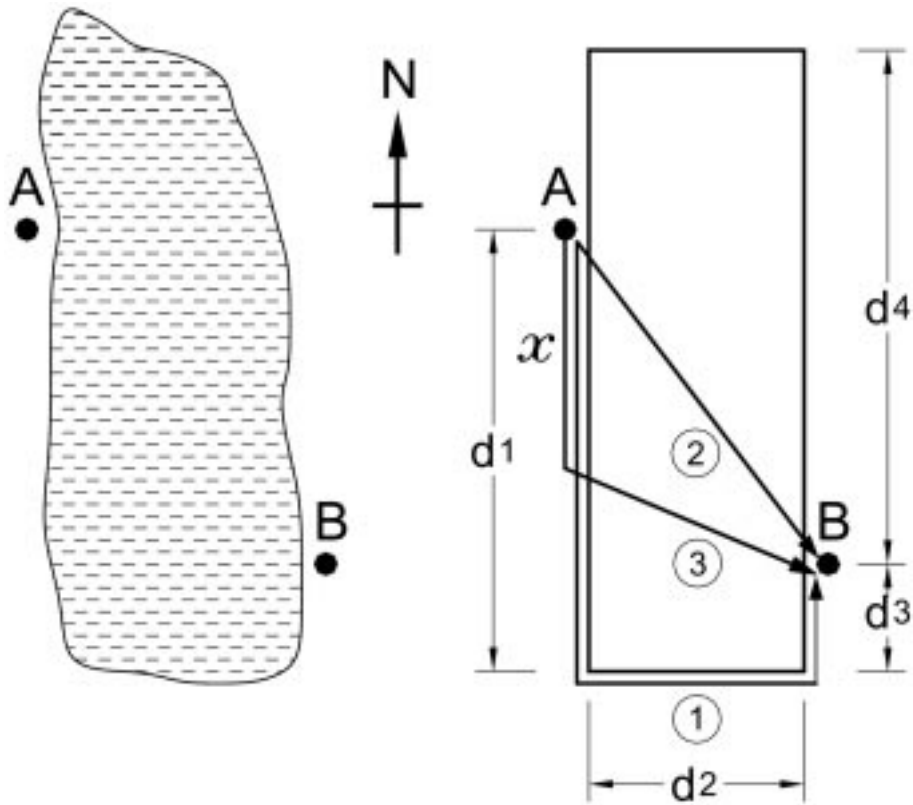


Fig. 3. Plan view of the wetland for the Pipeline Project.

Optimization of an irrigation channel

The following sample problem was covered in class [6]. Figure 4 shows the cross-section of an irrigation channel, which is a trapezoidal region. To maintain a certain volume flow rate, we want the cross-sectional area, A , to be fixed, to say $A = 100$ sq. ft. The objective is to minimize the amount of concrete that must be used to line the channel. To do this one minimizes the length, L , of the channel's perimeter, excluding the top. For the classroom example, we assumed that $\theta_1 = \theta_2 = \theta$, $h_1 = h_2 = h$, and $e_1 = e_2 = e$. Using geometry and trigonometry, one can express L as a function of θ and d alone.

$$L = \frac{100}{d} - \frac{d}{\tan \theta} + \frac{2d}{\sin \theta}$$

Minimizing L as a function of both θ and d requires multivariable calculus, something that the students had not yet learned. We first reduced the problem to single variable calculus by specifying either θ or d .

Fix a value of θ , and minimize with respect to d

Example: $\theta = \theta/3 \Rightarrow L = f(d) = 100/d + \sqrt{3}d$. We want the global minimum of L for $0 < d < \infty$. Setting $f'(d) = 0$ yields $d = 10/\sqrt[4]{3} \cong 7.5984$. Since $f''(d) = (200/d^3) > 0$ on $(0, \infty)$, $f(7.5984) \cong 26.3215$ is the global minimum.

Fix a value of d , and minimize with respect to θ

Example: $d = 1 \Rightarrow L = g(\theta) = 100 - (1/\tan \theta) + (2/\sin \theta) = 100 - \cot \theta + 2\csc \theta$. We want the global minimum for $0 < \theta < \pi/2$. Setting yields $\theta = \cos^{-1}(1/2) = \pi/3$. The First Derivative Test for local extrema shows that $g(\pi/3) \cong 101.73$ is the global minimum.

The students were asked to work individually for the assignment, though they were encouraged to consult with other members of their team. Each student, however, was given a different problem to answer. The faculty wanted the students to use some of the geometry and trigonometry covered in class to derive certain formulas on their own. Hence, the students could *not* assume that $\theta_1 = \theta_2 = \theta$, $h_1 = h_2 = h$, and $e_1 = e_2 = e$. Each student was given a value for A , θ_1 , and θ_2 and asked to:

- obtain a formula for the perimeter L (excluding the fourth side);
- minimize L with respect to d on $(0, \infty)$, and explain why it's the global minimum;
- use their given value of θ_1 , along with $d = 1$. Then minimize L with respect to $\theta = \theta_2$ on $(0, \pi/2)$, and again explain why it's the global minimum.

Some students commented that this project was too difficult, though most of the students did it correctly. Part of the reason they thought it was difficult was the geometry and trigonometry involved with deriving the correct formulas, given

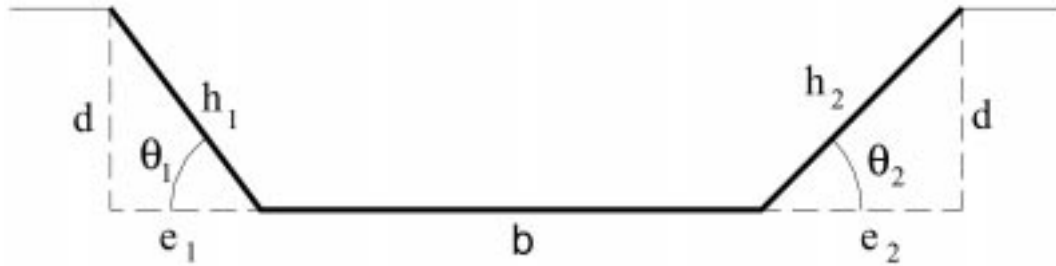


Fig. 4. Channel cross section for the Irrigation Project.

that the values of θ , h , and e were not assumed to be equal. The instructors of the course feel that it is good if the students are encouraged to obtain formulas on their own.

SELECTED PROJECTS FROM SECOND SEMESTER CALCULUS

Exponential decay (temperature data)

The definition and calculus of exponential functions are covered during second semester calculus at PSU. This project was designed to show where these kinds of functions might be useful. The faculty collected experimental data of the temperature, T , versus time, t , of a heated container of water. The water was allowed to cool down to room temperature. Students were required to

- find an exponential fit to the data;
- predict the temperature using interpolation and extrapolation;
- compare estimated average temperature using numerical integration and the given data.

Outside of class we collected data by heating water in a cup to an initial temperature of approximately 98°C , with the surrounding room temperature at approximately 24°C . The temperature was recorded at one minute intervals, from $t = 0$ to $t = 60$. In class we used data corresponding to $t = 0$ to $t = 15$ and discussed how to fit an exponential function to the data using linear regression and the natural logarithm. We fitted a curve of the form $T(t) = ce^{kt} + 24$ to the data, and plotted the data along with the graph of $T(t)$. Matlab was used along with properties of exponential functions. We also predicted the temperature at a time *not* given in the data. The linear regression we obtained was $L(t) = -.0358t + 4.2668$, which gives the temperature function $T(t) = 71.293 1e^{-.0358t} + 24$. The resulting graph is shown in Fig. 5.

Again, to involve the student with understanding properties of exponential functions, we set the temperature equal to a given value and solved for t . Given a continuous function $f(x)$ defined on a closed interval $[a, b]$, in integral calculus one defines the average value of f to be

$$\frac{1}{b-a} \int_a^b f(x) dx.$$

For this project, we wanted the average value of $T(t)$ on $[0, 15]$,

$$T_{ave} = \frac{1}{15} \int_0^{15} (71.293 1e^{-.0358t} + 24) dt \cong 79.1627$$

Finally, we compared T_{ave} with the average temperature over the time interval $[0, 15]$ using the data only, which gives 81°C .

Each team was given a proper subset of the temperature data to work with, and asked to do the same work as with the sample problem in class.

Skydiver free fall

This project involved integration of rational functions and use of integration tables, and the effect of drag force on free fall velocity of a skydiver [4] provided a good application (Fig. 6). A mini-lecture was given to explain the relationship between the position, velocity, and acceleration in rectilinear motion of a particle.

The mathematics of the following problem was covered in class. Given the drag force acting on a 60 kg skydiver, $F_D = cv^2$, where $c = 0.2088 \text{ N s}^2/\text{m}^2$, determine (a) the terminal velocity, v_t , (b) the velocity after 300m of free fall; and (c) time for the skydiver to reach a speed of 160 km/h.

For Part (a), it was necessary to explain briefly Newton's second law and that terminal velocity is obtained when acceleration is zero. That is,

$$\sum F = ma; \quad mg - F_D = ma;$$

$$a = 9.81 - 0.00348v^2; \quad a = 0 \Rightarrow v_t = 53.1 \text{ m/s}.$$

Part (b) involved the following relation among acceleration, a , velocity, v , and displacement, x : $adx = vdv$; $dx = dv/(9.81 - .00348v^2)$; $\int_{x=0}^x dx = \int_{v=0}^v (v/9.81 - .00348v^2) dv$. Using the substitution $\alpha = 9.81 - .00348v^2$ gives $v = 53.094\sqrt{1 - e^{-.0069x}}$; $v_{300} = 53.094\sqrt{1 - e^{-.00696(300)}} \cong 49.7 \text{ m/s}$. Part (c) of the problem used the following relation: $dt = dv/a$; since $160 \text{ km/h} = 44.44 \text{ m/s}$, we have $\int_{t=0}^{44.44} dt = \int_{v=0}^{44.44} dv/(9.81 - .00348v^2)$. Making a substitution and using a table of integrals yields $t_{44.44} = \int_{v=0}^{44.44} (dv/9.81 - .00348v^2) = 2.706 \ln[(v + 53.0939/53.0939 - v)_0^{44.44}] = 2.706 \ln 11.2705 \cong 6.55 \text{ s}$.

Team assignments involved completing the above three parts using different values for the mass and drag coefficients.

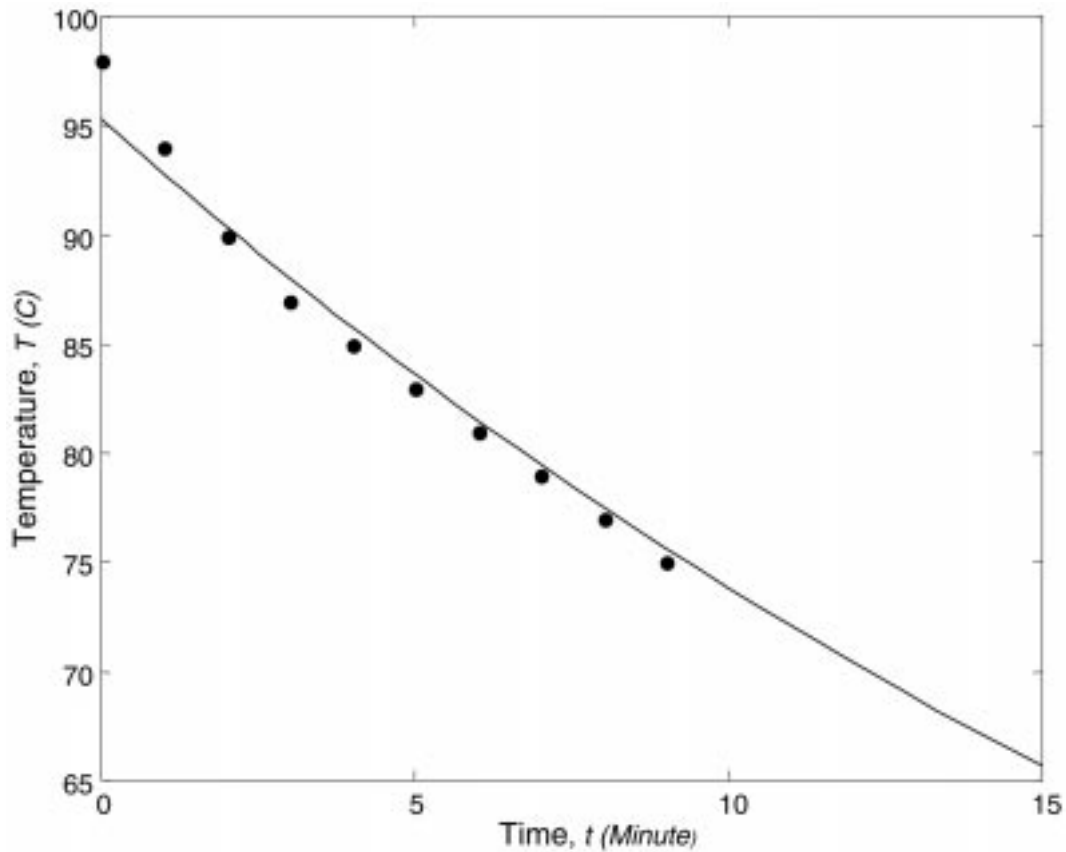


Fig. 5. Temperature vrs. time for the Exponential Decay Project.

Wrecking ball

This project combines three ideas from calculus in an interesting way: Taylor polynomials, approximate integration, and numerical solution of nonlinear equations are rarely used in conjunction with anything else.. The basic idea for this project was taken from [1]. We quote ‘A wrecking ball attached to a crane may be thought of as a pendulum. The period of such a pendulum is given (in seconds) by:

$$P = 4k \int_0^{\pi/2} \frac{1}{\sqrt{1 - a^2 \sin^2 x}} dx$$

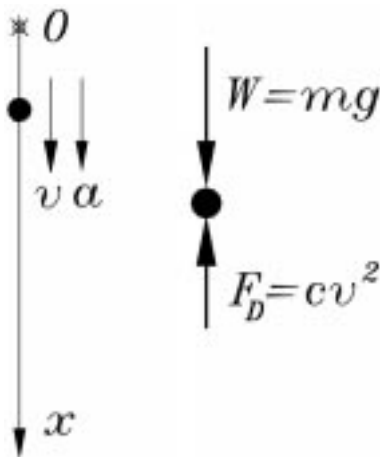


Fig. 6. The free body diagram for the Skydiver Free Fall Project.

where $a = \sin(\theta/2)$, θ is the angle of the swing (measured in radians from the rest position),

$$k = \sqrt{\frac{r}{g}}$$

r is the length of the cable holding the ball, and g is the acceleration due to gravity.’ The *derivation* of the formula for P above is beyond the scope of first semester calculus. Given the formula, the following sample problem was covered in class. Suppose that the construction engineer determines that a period of $P = 5$ seconds is required for the wrecking ball to have its maximum impact. Also, suppose that the cable is 18 ft. long (then $k = 3/4 \Rightarrow 4k = 3$). Then compute the angle θ required for the swing, rounded off to two decimal places. Note that, in the above calculation, the integral cannot be computed *exactly* and must somehow be estimated. While it is possible to use numerical integration for specific values of the parameter a , we require an answer in terms of a . This is an important point to emphasize with the students. Blindly using computer programs that execute numerical integration rules, such as the Trapezoidal or Simpson’s Rule, does not work here. We instead approximated the integrand using a Taylor polynomial, $T_5(x)$, of order five expanded about zero. While all the techniques necessary for computing T_5 , have been covered in class, the work is rather cumbersome. Hence we used Matlab for the computations, as well as for

computing $\int_0^{\pi/2} T_5(x) dx$. The resulting polynomial equation in a was also solved using Matlab (giving approximate solutions). Finally, we wanted to *check* how accurate the computed value of a was, by comparing $3 \int_0^{\pi/2} dx / (\sqrt{1/a^2 \sin^2 x})$ to $P = 5$. Earlier in the semester, the students were provided with a program for the TI graphing calculator that executes the Midpoint Rule. We used that to estimate the integral, with 500 subdivisions. The Taylor polynomial we obtained using Matlab was $T_5(x) = 1 + 1/2a^2x^2 + (-1/6a^2 + 3/8a^4)x^4$. Setting $5 = 3 \int_0^{\pi/2} T_5(x) dx$ yields the polynomial equation $3q(a) - 5$, where $q(a) = 1/2\pi + (1/48\pi^3 - 1/960\pi^5)a^2 + 3/1280\pi^5a^4$. Approximate solutions were found using the roots command in Matlab. Only one of the solutions is a positive real number, $a \cong .4504$. To check the accuracy of the computed value for a , we estimated $3 \int_0^{\pi/2} dx / (\sqrt{1 - .4504^2 \sin^2 x})$ using the Midpoint Rule. This gave 4.9833, which is very close to $P = 5$. Finally, $\theta = 2 \sin^{-1} a \cong 2 \sin^{-1} .4504 \cong .9344 \text{ radians} \cong 53.54^\circ$.

Each team was given specific values for N , k , r , and P , and were asked to do work similar to the sample problem done in class using both Matlab and their graphing calculator.

Each team met with the mathematics professor who led them through the steps necessary to do the work for this project. One student commented that they should have been required to do more of the work on their own. We agree, although the students were not asked to cover any of the other projects in this way. Other students commented that they could do the work for this project, but were not very sure what they were doing. That was probably because Taylor polynomials are difficult for students to understand, as well as evaluating a definite integral with a parameter involved. Nonetheless, we feel that the wrecking ball project exposed the students to an interesting combination of ideas and methods.

Automobile velocity data (integration)

This project was first introduced in Math 140, where given velocity data and polynomial regression were used to predict velocity and acceleration at times not in the data. This also provided a link with the regular syllabus, where one differentiates a given formula for velocity to obtain acceleration. In Math 141, one learns that total distance traveled is the integral of the velocity function. When one is only given *data*, there are two ways to estimate the total distance—either use numerical integration and the given data, or find a polynomial regression curve to the data, and then integrate. One hopes that the two estimates are fairly close.

The faculty obtained and handed out automobile velocity data taken from [5]. The sample problem done in class used the data in Table 2 for the Mercedes-Benz SLK230, taken from the July, 1997 issue. The data were actually given with t as a function of v . We wanted to consider v as a

Table 2.

t (s)	0	2.3	3.6	5.0	6.9	9.0	11.4
v (mph)	0	30	40	50	60	70	80

function of t , and we also added the data point $(0, 0)$.

We estimated the total distance traveled, D , using the Trapezoidal Rule with six subdivisions. The result was $D = 564/3600 \cong .156667$ miles. We fitted the velocity data with a regression polynomial of degree three using Matlab. First we showed that polynomials of degree one or two did not fit the data well. We obtained the velocity function $v(t) = .0483t^3 - 1.2599t^2 + 15.0979t + .2883$, and we plotted the data along with $v(t)$ using Matlab as shown in Fig. 7. We estimated the total distance traveled using the regression polynomial. $D \cong (1/3600) \int_0^{11.4} v(t) dt \cong .1572$, which is very close to the other estimate. We also estimated when (in seconds) the total distance traveled equals a tenth of a mile. We used Matlab to approximate the roots of the corresponding polynomial equation. Setting

$$\frac{1}{3600} \int_0^t v(\pi) d\pi = .1 \Rightarrow \int_0^t (.0483\tau^3 - 1.2599\tau^2 + 15.0979\tau + .2883) d\tau = 360$$

$\Rightarrow .0121t^4 - .4100t^3 + 7.5490t + .2833t - 360 = 0$. The Matlab commands $p = [.0121 - .4100 7.5490 .2833 - 360]$, $\text{roots}(p)$ yielded only one *positive real* root, $t \cong 8.5116$ seconds.

Each team was given velocity data for the following cars, taken from [5].

- Team 1: Jaguar XK8 Convertible
- Team 2: Porsche Boxster
- Team 3: 1998 Ford Escort ZX2
- Team 4: 1998 Pontiac Trans AM
- Team 5: Acura Integra Type R
- Team 6: Honda Prelude Type SH
- Team 7: Nissan Skyline GTR
- Team 8: Nissan Altima
- Team 9: Ford Ranger XLT.

They were asked to do work similar to what was done in class.

GRADING SYSTEM

From 20% to 30% of each student's grade in the enhanced versions of both Math 140 and Math 141, consisted of team-based projects in engineering and science. Students carried out a small number of projects individually. Students carried out the work for each project outside class, and this always involved the use of technology—either Matlab or programmable calculators. In addition, each project had a direct connection to some area of engineering and/or physics. Finally, the projects

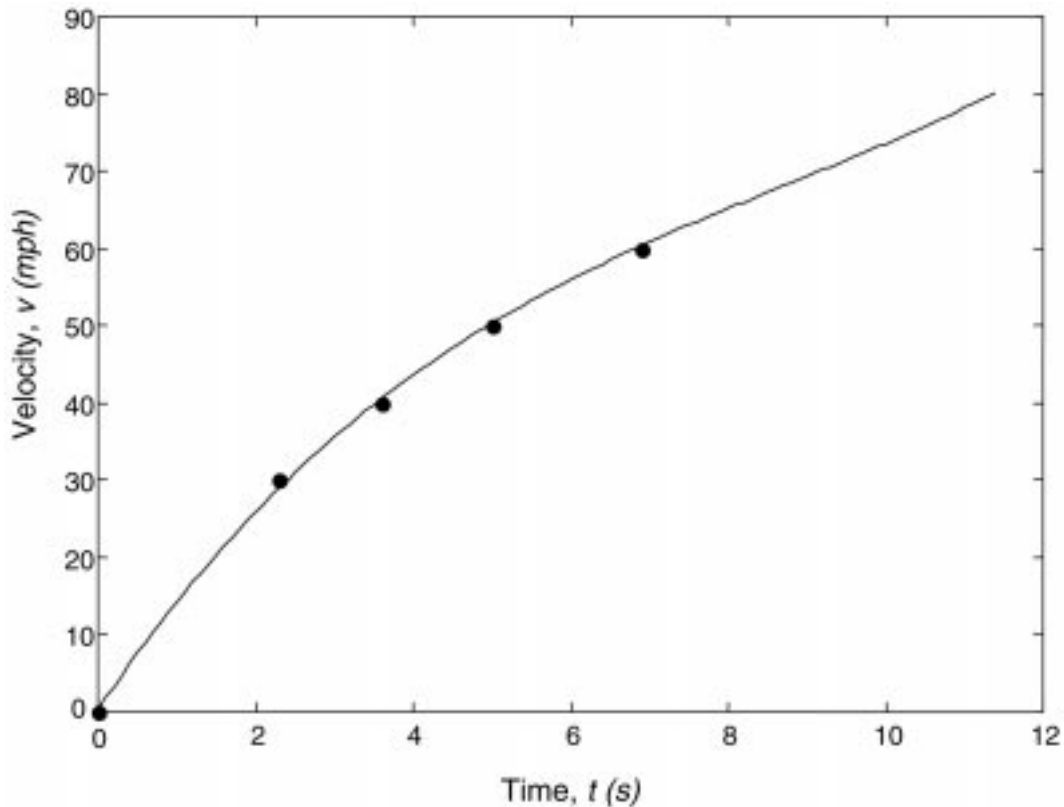


Fig. 7. Velocity vrs. time for the Automobile Velocity Project.

often involved data taken from real life situations. While individual and group take home problems were also assigned in the regular versions of Math 140 and 141, the key differences with the enhanced version were:

- A smaller percentage of a student's grade involved the take home problems. In addition, some group take home problems were assigned in the enhanced version in addition to the applied projects.
- The take home problems almost never had direct applications to science and engineering.
- The take home problems usually did not involve working with data.
- While the take home problems often did require the use of programmable calculators, they usually did not involve the use of computer software such as Matlab.

SURVEY QUESTIONS AND RESULTS

In the Fall 1997, Spring 1998, and Fall 1998 students were asked to complete informal evaluations of Math 140 and Math 141 during the last week of the class. The questions are listed below together with responses to questions for Fall 1997 and Spring 1998. Some of the students took both classes, while others only took one of the enhanced versions of Math 140 or Math 141.

Questions

1. On the scale of 1 to 5, rate the projects in MATH 140/141. You may rate them overall, or rate specific ones if you wish. Scale: 1 = worst to 5 = best. Also, make any comments you wish about the projects in MATH 140/141.
2. Did the projects help you understand and/or appreciate the practical applications of mathematics in science and engineering?
3. Were the explanations of your instructor and the engineering instructors helpful in understanding the projects?
4. What suggestions do you have for improving future projects?
5. Did members of your assigned group have any difficulty working as a team? Please explain.
6. Did your team manage to distribute the work equally among all members? Please explain.
7. On the scale of 1 to 5, rate MATLAB. Also, make any comments you wish about MATLAB.
8. Did you have any difficulty using MATLAB? Please explain.
9. Would you recommend this section of MATH 140/141 to other students?
10. *General Comments:* You are welcome to make any other comments which may not pertain to any of the questions asked.

Responses for Math 140, Fall 1997

Sixteen students filled out the evaluations out of a total of 18 in class.

Responses to questions

1. The average of the overall rating was 3.59. Some students rated individual projects. A few indicated that they had more difficulty with some projects than others.
2. Eleven students indicated yes; two answered no due to the level of difficulty of some of the projects; and the rest either did not answer or only addressed specific projects. Of those who answered yes, typical comments were:
 - ‘the projects made practical use of information that we learned in class and they helped me understand the subject matter fully;’
 - ‘I really didn’t understand what calculus was all about until these projects brought it together for me;’
 - ‘it showed us how math is used in other fields and gave us reason to understand and not just memorize for tests.’
3. The majority (12 out of 16) answered yes. Some indicated that some of the more challenging projects should have been explained in more detail.
4. The majority of answers had to do with more explanations about the projects. Some responded that projects should be less technical.
5. About half of the students answered no. The remaining students indicated some difficulty; reasons varied from having difficulty finding common meeting time to only one or two team members doing most of the work. Please note our campus is a commuter campus and many students have to work to support their college expenses.
6. Over half of the students answered yes. The remaining students indicated one or two people in the team did most of the work.
7. The average rating was 3.41. Typical comments included:
 - ‘sometimes it was hard to know what command to use; without the instructor, no one would have known what to do’;
 - ‘It was enjoyable to work with MATLAB; the program was relatively easy to work once you got used to it’;
 - ‘the program was easy to learn and the instructor helped explaining it.’
8. Over half of the students indicated some degree of difficulty using MATLAB. Most comments were very similar to responses to Question 7.
9. From those who responded to this question, almost everybody answered yes.
10. **Response to General Comments:** Only two students had comments; they were:
 - ‘a separate course should be available for people who are interested. I do not think it should be included in the MATH 140 course’; and
 - ‘the only negative comment is that it disturbed the flow of learning in the classroom, but overall very cool!’

Responses for Math 141, Spring 1998

Twenty one students filled out the evaluations out of a total of 27 in class.

Responses to questions

1. The average of the overall rating was 4. Some students rated individual projects. From the ones that provided comments, most indicated that the projects addressed the practical applications of mathematics. Some commented that working as teams helped them better understand the projects.
2. Seventeen students indicated yes; two answered somewhat (one of the three preferred to see more practical applications in the regular course work); and one did not respond to this question. Typical comments of the students who answered yes were:
 - ‘I personally had more appreciation for the projects since they related to my scheduled courses in engineering;’
 - ‘Even though the projects were not applicable to me as an education major (Math option), I enjoyed seeing first hand some of the calculus’ theories used in real life situations;’
 - ‘I grew a better understanding of the material we were learning.’
3. The majority (19 out of 21) answered yes. Of those who answered yes, typical comments were:
 - ‘Absolutely, if it were not for the explanations, I would have been lost on certain parts;’
 - ‘They helped us through the material which could get complicated;’
 - ‘The handouts were helpful;’
 - ‘The explanations made the projects easier to understand.’
4. The majority of the students answered either none or did not answer this question. Of those who provided comments, typical comments were:
 - ‘I suggest having a couple more projects with practical applications;’
 - ‘More teaching of MATLAB, I was at a disadvantage sometimes due to lack of experience (presumably not having the enhanced section of Math 140 in the previous term);’
 - ‘Perhaps assign projects individually and not in groups.’
5. Fifteen students answered no; two answered yes; three had some level of difficulty; and one did not respond. Typical answers included:
 - ‘No, we got along well and working in groups helped us understand the material better;’
 - ‘No but it was hard to find times we could all meet;’
 - ‘Sometimes it was hard to get the whole team together, because of conflicting schedules.’

- The comment of one of the students who answered yes was: ‘Yes, motivation levels of members varied greatly.’
6. Over half of the students (11 out of 21) answered yes. The remaining students indicated one or two people in the team did most of the work. From those who answered yes, typical comments included:
 - ‘Yes, we each played equal roles in one way or another. Everything worked out great;’
 - ‘Yes, we all did even amounts of work.’
 From those who answered no, typical comments were:
 - ‘Not at all, when work was done it was done individually then compared to the group;’
 - ‘No, some were not at school at the time the project was being done.’
 7. The average rating was 3.71. Typical comments included:
 - ‘MATLAB is great and I really enjoyed using it;’
 - ‘I never really learned how to use it, so it was kind of confusing;’
 - ‘Good program, a little hard to use might be from a lack of prior use—would have liked to have done more with it.’
 8. Seven of the students responded no or minor difficulties; nine indicated yes; and the remaining students did not respond to this question.
 9. From those who responded to this question, everyone answered yes.
 10. *Response to General Comments:* Typical comments included:
 - ‘Thank you for a fun semester;’
 - ‘This was a great class and I hate to see it end so soon.’

SUMMARY AND CONCLUSIONS

Most of the students seemed pleased to see some applications of differential and integral calculus to other fields, such as engineering. While many of the students are engineering majors, a fair number are in other fields or are unsure of their major. These students were less enthusiastic about many of the projects, though several of the projects involved applications to science (usually physics). There was also some negative reaction to a few of the projects—most particularly the crankshaft design project and the optimization of an irrigation channel project. The major complaint was that the work was too difficult. Also, for some of the projects, the students could do the work

well, but did not fully understand what they were doing. Here is our critique of the enhancement project:

- While there was some enthusiasm for doing the projects, this did not spill over into the regular part of the calculus course. One of our objectives was to use the projects to motivate the students to want to do and understand calculus. We do not think that this was achieved. Part of the reason may be the nature of the assignments.
- The team and individual assignments were sometimes duplicated, and students could often do the work by following the steps done in the sample problem in class. While it was good that they saw some interesting applications, they were not really asked to do enough thinking on their own. This was partly due to the nature of the enhancement project.
- We were still required to cover the regular syllabus from Math 140 and Math 141. While there is some leeway in what and how certain topics are covered, time restrictions make it difficult to any project in depth. We also wanted to give a breadth of applications that used different areas of calculus and which applied to different areas of engineering, such as electrical, mechanical, and civil. We did offer fewer projects the second year, which allowed for [a little] more depth of coverage. However, the projects at times were grafted onto the regular course content, rather than interacting with that content in a vibrant way. We suspect, however, that many of the students will have a greater appreciation of the projects after they have taken other courses in engineering and the sciences.
- Perhaps a better way to offer an enhancement of calculus is to add an optional extra credit to first or second semester calculus. The extra time provided and the fact that the course is optional should improve the course.
- Finally, we should add that several students did express that they had fun doing the projects and/or learning that calculus can be useful in the real world. Many also seemed to enjoy using the software as well, though they expressed a desire to obtain a more thorough knowledge of Matlab.

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