Int. J. Engng Ed. Vol. 18, No. 6, pp. 652–666, 2002
Printed in Great Britain.

0949-149X/91 $3.00+0.00

Formulation of Tutoring Policy for Maximising Student Learning using a Decision-Theoretical Approach*

PENG-KIAT PEK
School of Mechanical & Manufacturing Engineering, Singapore Polytechnic, 500 Dover Road, Singapore 139651

KIM-LENG POH
Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260. E-mail: isepohkl@nus.edu.sg

This paper examines the nature of decision-theory approach and its application to education. In particular, a case study on selection of tutoring policy for maximising students' learning at Singapore Polytechnic is discussed. On the one hand, students' ability, availability of resources, and lecturers' preparation time are uncertainties. On the other hand, decision is influenced by individual preference for tutorial formats such as student-centred, chalk-and-talk, or computer-based tutoring. Moreover, the course of action is dependent on trade-off in values. Using decision analysis, uncertainties, preferences, alternatives and values are modelled and a defensible claim on maximising student learning can be made.

INTRODUCTION

A LEARNER is a cognitive system that develops by his own information and knowledge-processing activities. To maximise the learner's cognitive development, knowledge-intensive environments are essential to help him explore a situation, construct his own concepts, and discover general laws by his own problem-solving activity [1]. The lecturer, as knowledge facilitator, has an extremely complex problem on his hands. Before deciding exactly what course of action to take, he needs to consider many issues, including suitable environments and the uncertainties involving students' abilities and school resources.

Decision analysis provides effective methods for organising a complex problem into a structure that can be analysed [2]. In particular, elements of a decision structure include the possible courses of action, the possible outcomes that could result, the likelihood of those outcomes, and eventual consequences to be derived from the different outcomes. Figure 1 shows a flowchart for the decision analysis process. For illustration, assume a lecturer needs to make a decision on which tutoring method to apply for motivating a class of underachievers in engineering mechanics. A few alternatives may be considered: drill and practice, peer tutoring, hands-on activity, and on-line tutoring. Thereafter, variables associated with the alternatives are identified. The variables may be uncertainties such as students' interest, their abilities, and availability of computer resources. Utility functions are assessed in order to model the way the lecturer values different outcomes and trade-off competing objectives.

Decision analysis tools such as influence diagrams [2, 3] and decision trees [4, 5] are then used to model the problem for determining a preferred alternative. For complex models, computer software such as DPL [6] is available to automate the computation. Additional analysis such as sensitivity study [7] may be performed to answer 'what if' questions such as: 'If a computer resource is available, does it imply that on-line tutoring leads to a better student motivation?' If the answer is positive, then the lecturer may want to consider obtaining more information on that variable *prior* to making the decision.

Figure 1 also shows that the lecturer may return to the previous steps of the decision analysis process. It may be necessary to refine the objectives or to include objectives that were not previously considered. When new alternatives are identified, the model structure, the uncertainties and preferences may also need to be modified. This process may go through several iterations before a satisfactory solution is found. The final solution that contains the essential components is known as the *requisite model* [8]. The approach allows inclusion of personal judgements about uncertainties and values [9] for making good decisions.

Examples of education policy formulation where decision-theoretical techniques can be applied are listed in Table 1. For each area of application

* Accepted 27 February 2002.

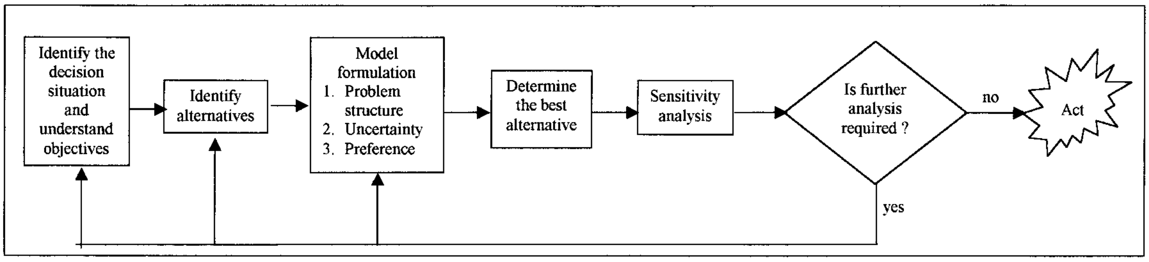

Fig. 1. Decision analysis cycle.

mentioned in the table, some examples of decisions and possible variables that may affect the alternative outcomes are provided in columns two and three respectively. Consider a team of academic staff who are to formulate a school promotion strategy, where some decisions they may be making include type of advertisement, duration of advertisement, and extensiveness of staff involvement. If the decision is on type of advertisement, then possible alternatives may be newspapers, magazines, Internet, radio, television, road show, and open house exhibition. Variables that may affect the alternatives are budget, links with outside organisations, staff interest, accessibility of each media to the public, and interest areas of potential applicants. How these variables (deterministic or stochastic) may affect the alternatives has to be identified. The team has to agree on what value they consider important before a most satisfying alternative can be determined. For example, attracting applicants with the desired academic qualifications could be the most important objective.

The issues of uncertainties, subjective judgements, and trade-offs in values are further discussed in the following sections to provide the readers with essential decision-theoretical foundation before they walk through a case study. We have selected the case study to illustrate how a module team may apply the decision-theoretical approach to determine policy that maximises student learning in tutorials.

PROBABILITY ASSESSMENT

Uncertainty in decision problems is represented by probability. Besides interpreting probability in terms of *long-run frequency*, one can consider it to represent an individual's *degree of belief* that a particular outcome will occur. There is no correct answer when it comes to subjective judgement: different people have different beliefs and hence will assess different probabilities. However, as long as the probability axioms [10] are not violated, decision-theoretical approach being normative rather than descriptive it is able to explain the course of action.

One of the methods to assess probabilities adopts a thought-experiment strategy [11] in which the decision-maker compares two lottery-like games, each of which can result in a prize (A or B). Consider the situation to assess the student's probability distribution for the number of hours (uncertain variable X) he spent in extra-curricular activities. The probability wheel (see Fig. 2) is used to determine the size of the unshaded sector in which the lecturer is just undecided between the two options:

1. Spins the wheel and wins \$100 or nothing.
2. Checks the real value of x (assuming it can be done) such that if $x \leq 2$, he wins \$100, otherwise he gets nothing.

If p is the value indicated by the wheel, then $P(x \leq 2 \mid \xi) = p$ where 'ξ' refers to background knowledge that the decision-maker brought to

Table 1. Possible areas for the application of decision-theoretic approach

Applications	Decisions@	Variables@
School promotion strategy	type of advertisement, duration of advertisement, extensiveness of staff involvement	budget, industrial links, students' participation, staff interest, accessibility of media to the public, interest areas of potential applicants
Manpower planning and career advancement	effectiveness of teaching, research involvement, promotion criteria	enrolment, domestic economic condition, staff turn-over rate, staff interest
Determination of tuition fee	stages of increments, fee structures for different disciplines	attrition rate, enrolment, staff strength, cost of utility, national policy
Diagnoses on faulty students' learning	learning outcomes, measurement of learning, types and duration of remedial lessons	student ability, effort, item difficulty, item suitability, availability of resources
Allocation of elective modules	modules combination, module pre-requisites, benefits to post-graduate studies, usefulness to job prospects	staff specialities, module requirements, students' choices, demand of job markets

@ *Note:* The above decisions and variables may not be exhaustive. In addition, the variables may not be mutually independent

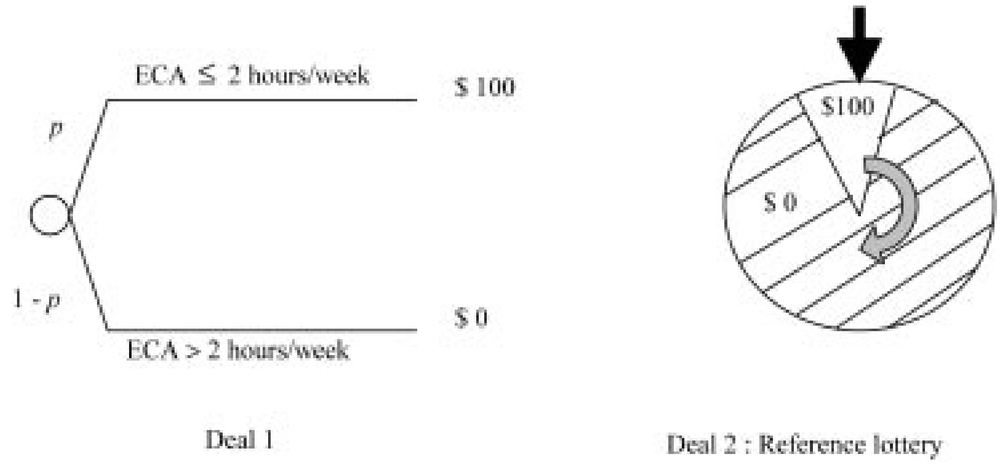

Fig. 2. Probability assessment with equivalent-lottery method.

bear on assessing the uncertainty. This process is repeated for different values of x. The cumulative probability distribution can be plotted as shown in Figure 3a.

Another method for probability assessment is to use *theoretical probability models* [12, 13] and their associated distributions. For example, if we believe that the cognitive abilities of students follow the familiar bell-shaped curve, which is the *normal distribution*, then we may use the distribution to generate probabilities. Such probability modelling is just as subjective as a directly assessed probability distribution because judgement is being made that students' abilities can be adequately represented using the theoretical model. When *historical data* is available it is possible to use it to construct probability distributions [14]. We can use the data to understand and model relationships among variables.

The way to use a continuous distribution in a decision tree is to approximate it with a discrete distribution. A few representative points in the distribution are selected and assigned specific probability values. A simple approach (see Fig. 3) known as the Pearson-Tukey method [15] uses the 5, 50, and 95 percentiles as the representative points. In assigning probabilities, the 50 percentile gets a probability of 0.63, and the 5 and 95 percentiles each has a probability of 0.185.

PREFERENCE AND RISK ATTITUDES

In this section, we examine the representation of the decision maker's preference or their attitude towards risk. Examples of risk taking are willingness to try new or unproven tutoring methods such as on-line assessment, video conferencing, and peer tutoring. Modelling a person's preference by assessing their *utility function* is a subjective procedure much like assessing subjective probabilities [16].

A utility function can be specified in terms of a graph or in a mathematical expression. Traditionally, the utility function has been used to translate dollars into a utility (or satisfaction) unit. Individuals who are sensitive to risk are called *risk-averse* [17]. Some examples of mathematical expressions that have the general concave shape (opening downward) are:

$$U(x) = a \log(x) \tag{1}$$

$$U(x) = a - b\,\mathrm{e}^{-x/\rho} \tag{2}$$

$$U(x) = x^{a} \tag{3}$$

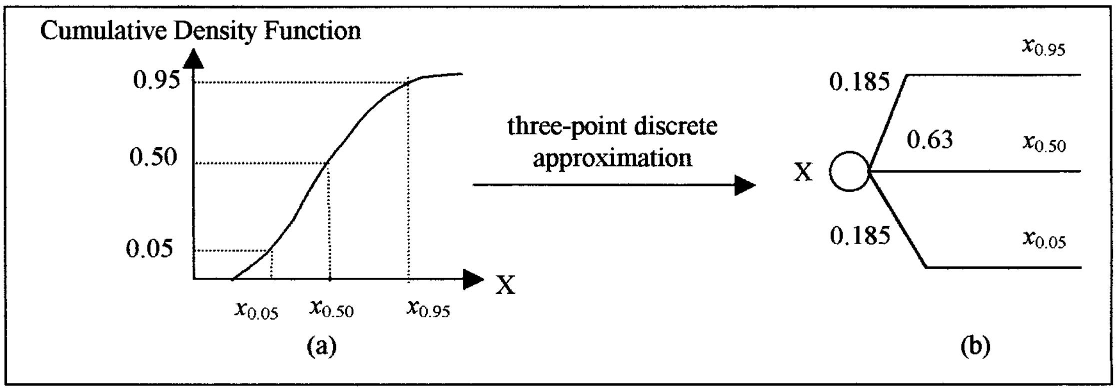

Fig. 3. Pearson-Tukey method.

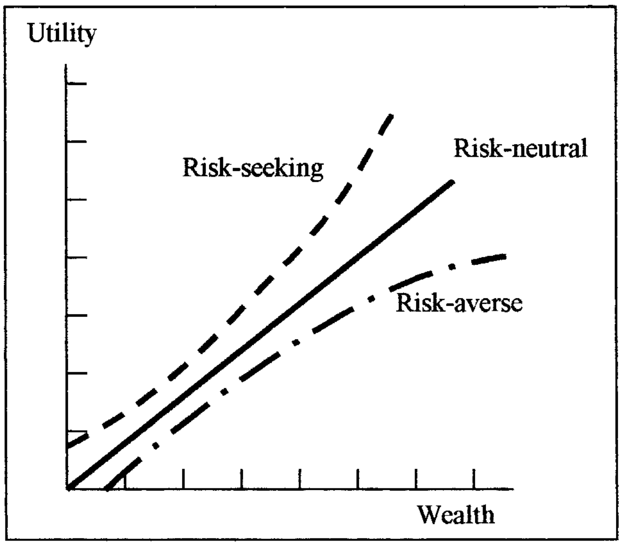

Fig. 4. Utility functions for three risk attitudes.

where a and b are constants that can be determined using boundary conditions, while ρ is the risk tolerance [5, 18] value. x is any quantity where the satisfaction of possessing it is expressed in the utility function.

Not everyone displays risk-averse behaviour all the time. A convex (opening upward) utility curve indicates *risk-seeking* behaviour (see Fig. 4). Alternatively, an individual can be *risk-neutral.* Risk neutrality is reflected by a utility curve that is a straight line. A person who is risk-neutral does not care about risk of the alternatives that he or she faces.

Utility scales can also be used for measuring how satisfaction varies with non-monetary objectives, including:

- quality of classroom facility;
- students' motivation;
- lecturers' preparation time;
- students' pass rates;
- classroom air quality;
- lecturers' morale;
- recreational opportunities;
- students' travelling time to school, etc.

An approach to elicit a decision-maker's utility scale for non-monetary objectives is known as the *probability-equivalent* assessment technique [11]. The best and worst possible outcomes for a prospect are first identified. A utility score of 0 is assigned to the worst and 1.0 to the best outcome. Next, the intermediate values (x_i, p_i) are determined using a reference gamble (see Fig. 5). The (x_i, p_i) values are plotted on a graph as a continuous curve.

While most non-monetary objectives have the natural order of more being better, some require that fewer are better. Consider the case where the objective is to maximise students' learning through the use of information technology, the utility function for the number of students to a computer is a decreasing curve. Because a utility function incorporates a decision-maker's attitude towards risk, he or she may choose the alternative that maximises their *expected utility* [19, 20]:

$$\max_j \sum_{i=1}^{n} u_i p_i^j \tag{4}$$

where p_i^j is the preferred probability of j^{th} decision

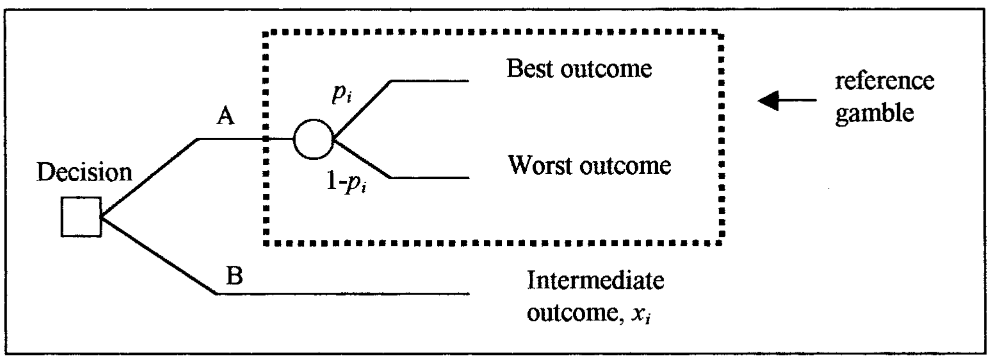

Fig. 5. Probability-equivalent assessment.

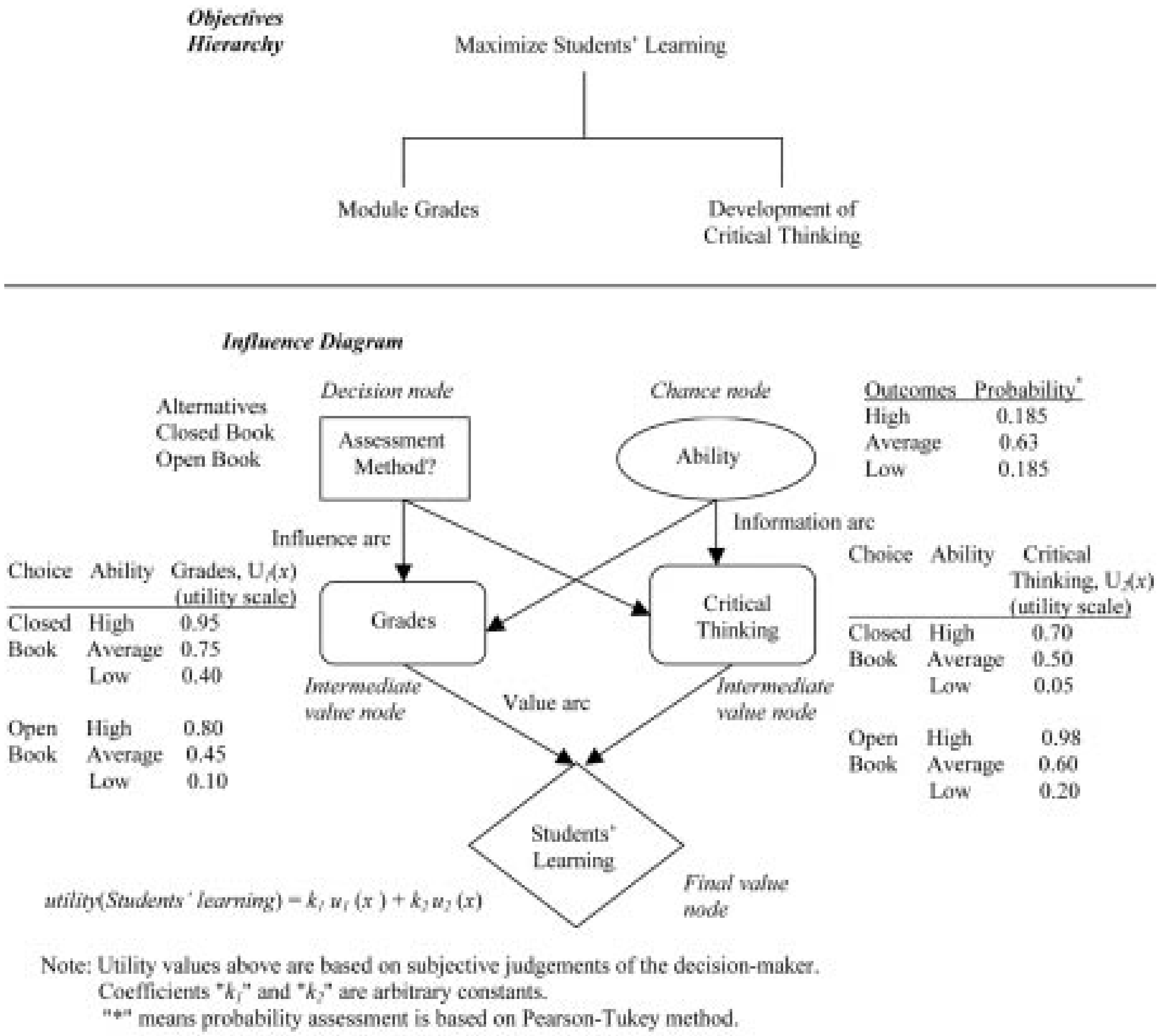

Fig. 6. Lecturer's decision with two objectives.

which deals with outcomes $\underline{A}_i$ (worse prospect) and $\overline{A}_i$ (best prospect); and u_i is the preference probability (or utility) of outcome A_i.

STRUCTURING VALUES

An *Influence diagram* is a graphical structure for modelling uncertain variables and decisions, and *explicitly revealing probabilistic dependence and flow of information*. Figure 6 shows a simple two-level objectives hierarchy and the corresponding influence diagram. The figure illustrates how the objectives hierarchy is reflected in the pattern of value nodes in the influence diagram; two value nodes labelled 'Grades' and 'Critical thinking' represent the lower-level objectives which in turn are connected to the 'Students' learning' value node. The aim is to determine the most preferred alternative that maximise the utility of the decision-maker.

The chance node represents uncertainty associated with student's ability and it has three possible outcomes. The table besides 'Critical thinking' node shows that if the decision for assessment is open book, the satisfaction (utility) depends on the outcome of student's ability. Open book assessment places greater emphasis on higher order cognitive skills (such as application and evaluation) than does closed book assessment [21]. Students who are trained for open book assessment are more aware of critical thinking techniques and will be likely to use it. However, it requires greater efforts and training for students to master higher order cognitive skills, which may not be currently available. Consequently, students' grades are likely to be better for closed book assessment than open book assessment as reflected by the higher utility values for the former option. The mathematical expression besides the final value node indicates the trade-off between the two intermediate objectives as represented by the constants k_1 and k_2. The next section illustrates the method to estimate these constants and to determine the preferred alternative in this decision problem.

MULTIPLE ATTRIBUTE UTILITY AND PREFERRED DECISION

A method to assess the constants (k_1 and k_2) is known as Saaty's Eigenvector Method [22]. Consider the example illustrated in Fig. 6, assuming the decision maker decides that attribute 1 (Grades) is half as important as attribute 2 (Critical thinking), then the **A** matrix is:

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}.$$

Through solving the matrix for eigenvectors, $k_1 = \frac{1}{3}$ and $k_2 = \frac{2}{3}$.

In general, for an outcome that has m objectives, the multiple attribute utility is given as:

$$\mathrm{U}(x_1, \ldots, x_m) = k_1\mathrm{U}_1(x_1) + \ldots + k_m\mathrm{U}_m(x_m) = \sum_{i=1}^{m} k_i\mathrm{U}_i(x_i) \qquad (5)$$

where $k_i \geq 0$, $\sum_{i=1}^{m} k_i = 1$, $0 \leq \mathrm{U}_i \leq 1$.

A necessary and sufficient condition for Equation (5) to hold is that the m attributes (also known as stochastic variables) are *mutually utility independent* [23].

Figure 7 shows an equivalent decision tree representation of the previous influence diagram (see Fig. 6). It is created to understand how the preferred alternative is determined. First, the options represented by branches from a decision (square) node must be such that the decision-maker can choose only one option. Second, each chance (circle) node must have branches that correspond to a set of *mutually exclusive* and *collectively exhaustive* outcomes. Third, the decision tree represents all of the possible paths that the decision-maker may follow through time. The preferred alternative is found by selecting the maximum utility at the decision node, after the expected utility has been computed at each chance node.

A CASE STUDY

Decision context

For most engineering schools in Singapore Polytechnic, the lecture-tutorial-practical system is a dominant pedagogical method. Each academic year consists of two semesters, where a semester consists of 15 weeks. Table 2 lists the modules taken by first-year students in the School of Mechanical and Manufacturing Engineering. For each semester one group of students takes stage A modules while the other group takes stage B modules. In the next semester, those in stage A will switch over to stage B, and vice versa. Within each stage, the students are further divided into tutorial classes, each class typically has about 15 to 22 students.

Prior to the commencement of an academic session, module teams usually meet to decide among other matters policy on the conduct of tutorials. In tutorials, lecturers have the flexibility to adopt the *traditional chalk-and-talk* method or the *student-centred* method. Concerning the chalk-and talk method the lecturer is the key person who orchestrates the learning process, while students are passive learners who follow the curriculum closely. Lesson plan, course content, and tutorial problems are closely followed and discussed. Concerning the student-centred method, lecturers present the outline of essential course content while

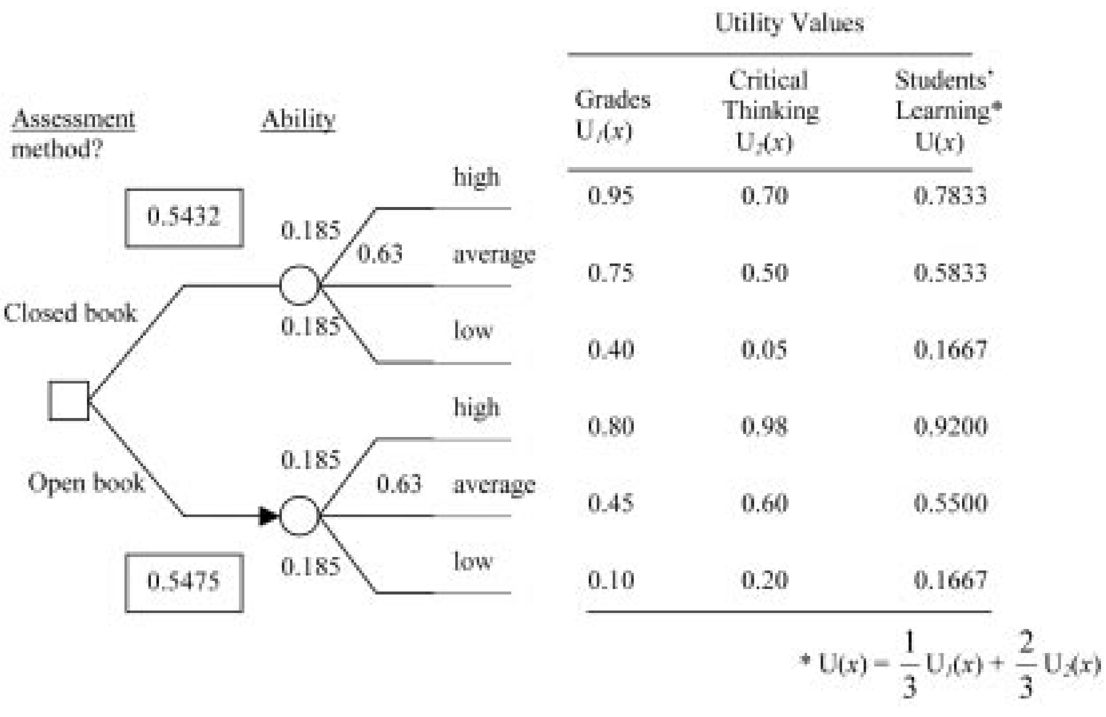

Fig. 7. Decision tree representation of two objectives.

Table 2. Full-time first year course modules in Diploma in Mechanical and Manufacturing Engineering

Stage 1A
Electrical Technology
Written Technical Communication
Computer Aided Drafting
Engineering Materials I
Fundamental Mathematics
Computer Programming
Character Education
Stage 1B
Electronics
Oral Communication
Workshop Practice
Mechanics I
Thermofluids I
Engineering Mathematics I

allowing students to search for additional information, pose questions and discuss solutions in class, and perform peer coaching [24] to weaker students.

With the advent of affordable high-speed personal computers and user-friendly software computer-based learning [25] is becoming a feasible alternative. Students are able to access information and to test their understanding of taught knowledge through computer administered quizzes. Such quizzes are currently limited to certain formats such as multiple-choice, fill-in-the-blanks, drag-and-drop, and matching of key words. Students are given feedback immediately after they complete the quizzes, mostly in pre-recorded messages. Depending on the amount of the lecturer's involvement, computer-based learning can be either supervised or unsupervised.

The motivation for this case study is to walk through one typical situation where a module team applies the decision-theoretical technique to formulate a satisfying policy for maximising students' learning in tutorials. **Mechanics I** module is used for illustration because the module team is aware of the benefits of computer-based learning, but is uncertain under which conditions the students will benefit most. The discussion is applicable to any module listed earlier (Table 2), and constraints unique to a particular module can be included as additional variables or decisions in the model. **Mechanics I** is taught using two 1-hour lectures, a 2-hour tutorial, and an hour of practical. The existing mechanics tutorials are generally chalk-and-talk method with the lecturers responding to students that require assistance. Currently, computer-based learning is voluntary and students access the materials outside the tutorials. The computer-based learning system is based on the Blackboard software platform [26] where the module materials, assessments, and web materials are accessed for self-paced learning.

Decision model

The influence diagram shown in Fig. 8 is a possible representation of the decision problem where the stochastic variables are mutually independent. The model consists of five stochastic variables, two sequential decisions, and an objective (or value) which the module team seeks to maximise. The five variables can be categorised into three groups: student, lecturer, and facility.

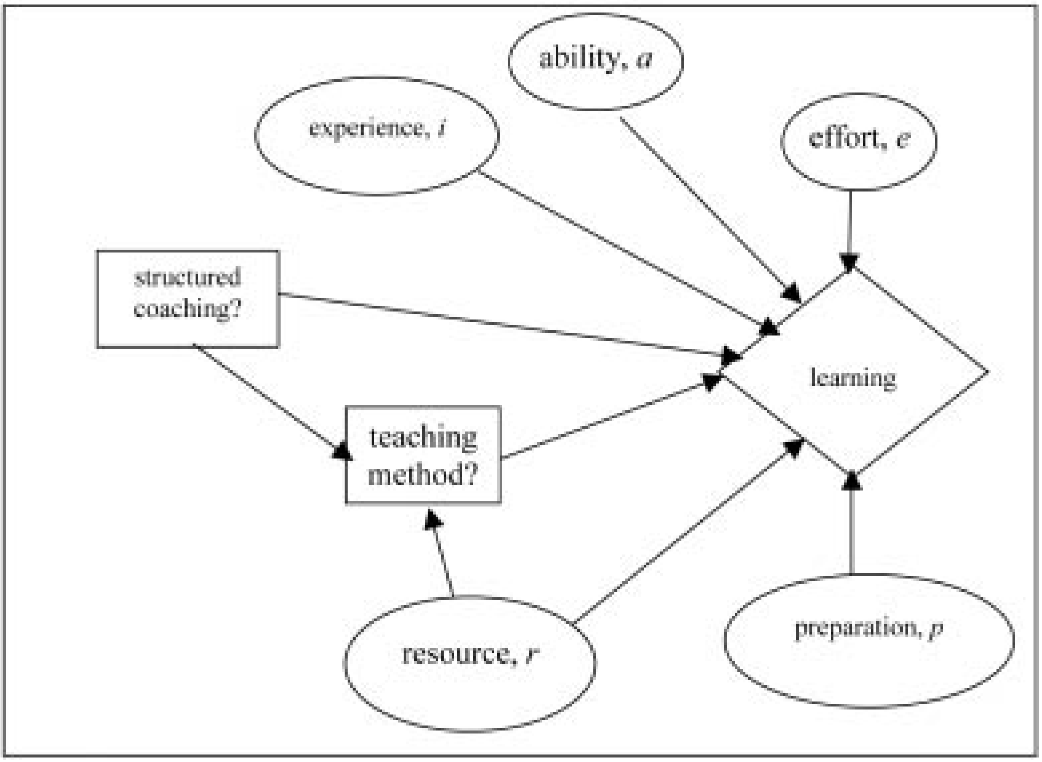

Fig. 8. Influence diagram of decision model.

Using data from the School of Mechanical and Manufacturing Engineering, Singapore Polytechnic, the range of values for the five variables are:

Students:
–3.0 < ability, $\boldsymbol{a} < 3.0$ (logit)
0 < effort, $\boldsymbol{e} < 20$ (hours/week)

Lecturers:
0 < preparation, $\boldsymbol{p} < 20$ (hours/week)
0 < experiences, $\boldsymbol{i} < 20$ (years)

Facility
2 < resource, $\boldsymbol{r} < 10$ (students/computer)

The variable *ability* refers to the scores students obtained from standardised tests and is measured on the logit scale [27]. *Effort* refers to the amount of time students may spend a week on learning the Mechanics I module. *Preparation* means the number of hours lecturers probably take for their lesson preparations and analysis of students' performance. *Experiences* denote lecturers' industrial experience (number of years). Finally, *resource* refers to the ratio of number of students to a personal computer available for computer-based learning in the school.

Two decisions are shown in the model:

1. *Should there be structured coaching for this class?* Structured coaching refers to the situation where activities conducted in the tutorials are pre-planned. The opposite of structured coaching is the situation where the lecturer plays the facilitator role during tutorials. The lecturers and students need not follow lesson plans strictly. Option 1: *Yes.* Option 2: *No*
2. *Which tutoring method is most suitable?* Although there are different tutoring methods, they can be broadly categorised into three major groups. Each group is different from others in terms of the extent of involvement of students, lecturers, and computer usage. Option 1: *Student-centred.* Option 2: *Computer-based tutoring.* Option 3: *Chalk-and-talk.*
   - *Student-centred.* Students play centre stage while lecturers facilitate the learning and ensure availability of facility. However, both students and lecturers followed a preset teaching plan. Computers are used only when essential to the lessons.
   - *Computer-based tutoring.* Computer-based learning attempts to tap the strength of information technology to enable students to learn at their own pace but within the curriculum time frame. This alternative requires ample availability of computer resources. When combined with unstructured coaching, it demands more contribution from student than the lecturer. Lecturers are indirectly involved in tasks such as answering students' queries not available through the computer help features, analysing class statistics, and refining the software package.
   - *Chalk-and-talk.* The chalk-and-talk method demands more contribution from the lecturer than students, and least on the availability of computer resources.

Utility model for student learning

Utility values of the variables are elicited from the lecturers using the probability-equivalent assessment technique discussed earlier. The graphical representations of the utility values for various variables are shown in Figure 13. The equivalent mathematical functions of these graphs are given in Table 3. These functions are entered into the decision-theoretic software for analysis.

Assuming the decision-maker exhibits mutual utility independent attitudes as discussed in the previous section, the multiple attribute utility for students' learning can be represented by the *linear additive independence* expression:

$$\pi_j = k_a * \mathrm{U}(a) + k_\mathrm{e} * \mathrm{U}(e) + k_\mathrm{p} * \mathrm{U}(p) + k_\mathrm{r} * \mathrm{U}(r) + k_\mathrm{i} * \mathrm{U}(i) \quad (6)$$

where $k_a + k_e + k_p + k_r + k_i = 100\%$ and j is one of the alternative paths.

The weights can be obtained by using Saaty's Eigenvector Method. The coefficients in these matrices are obtained through pair-wise comparison of the attributes. Since the comparison involves many attributes, *consistency ratios* (CR) are computed to determine if the weights are consistent. Consistency ratios less than 0.1 are considered acceptable. The various $\boldsymbol{A}$ matrices, utility functions for various alternative paths, and CR are shown in Table 4.

Deterministic sensitivity analysis and reduced model

Sensitivity analysis is performed to check if all the variables are essential to the decision model.

Table 3. Utility functions of the variables for the decision model

Variable	Utility function
ability, $\boldsymbol{a}$	U(ability) = $1 - 0.25*\mathrm{EXP}(-0.46*a)$
effort, $\boldsymbol{e}$	U(effort) = $1 - \mathrm{EXP}(-0.2*e)$
preparation, $\boldsymbol{p}$	U(preparation) = $1 - \mathrm{EXP}(-0.15*p)$
industrial experience, $\boldsymbol{i}$	U(industrial experience) = $0.045*i + 0.1$
resource, $\boldsymbol{r}$	U(resource) = $1.5*\mathrm{EXP}(-0.2*r) - 0.02*r$

Table 4. Comparative judgement of the variables for each alternative

Alternative j	A_j	ρ_j	CR_j
Structured Coaching and Student centred	*not feasible alternative*		
Supervised Computer Based Tutoring	$\begin{array}{cc} \begin{matrix} a & e & p & r & i \end{matrix} & \\ \begin{pmatrix} 1 & 3/2 & 1/2 & 1/4 & 7 \\ 2/3 & 1 & 4/5 & 1/3 & 6 \\ 2 & 5/4 & 1 & 1/3 & 4 \\ 4 & 3 & 3 & 1 & 5 \\ 1/7 & 1/6 & 1/4 & 1/5 & 1 \end{pmatrix} & \begin{matrix} a \\ e \\ p \\ r \\ i \end{matrix} \end{array}$	$16.8^*U(a) + 15.3^*U(e) + 19.4^*U(p) + 44.2^*U(r) + 4.3^*U(i)$	0.0824
Chalk and Talk	$\begin{array}{cc} \begin{matrix} a & e & p & r & i \end{matrix} & \\ \begin{pmatrix} 1 & 2/3 & 1/3 & 3/2 & 7 \\ 3/2 & 1 & 2/5 & 2 & 6 \\ 3 & 3 & 1 & 3 & 4 \\ 2/3 & 1/2 & 1/3 & 1 & 5 \\ 1/7 & 1/6 & 1/4 & 1/5 & 1 \end{pmatrix} & \begin{matrix} a \\ e \\ p \\ r \\ i \end{matrix} \end{array}$	$18.3^*U(a) + 22.4^*U(e) + 41.3^*U(p) + 13.6^*U(r) + 4.4^*U(i)$	0.0928
Student Centred	$\begin{array}{cc} \begin{matrix} a & e & p & r & i \end{matrix} & \\ \begin{pmatrix} 1 & 2/3 & 3/2 & 2 & 7 \\ 3/2 & 1 & 2 & 5/2 & 6 \\ 2/3 & 1/2 & 1 & 3 & 4 \\ 1/2 & 2/5 & 1/3 & 1 & 5 \\ 1/7 & 1/6 & 1/4 & 1/5 & 1 \end{pmatrix} & \begin{matrix} a \\ e \\ p \\ r \\ i \end{matrix} \end{array}$	$26.7^*U(a) + 34.0^*U(e) + 21.9^*U(p) + 13.2^*U(r) + 4.2^*U(i)$	0.0444
Unsupervised Computer Based Tutoring	$\begin{array}{cc} \begin{matrix} a & e & p & r & i \end{matrix} & \\ \begin{pmatrix} 1 & 2/3 & 2 & 1/2 & 7 \\ 3/2 & 1 & 5/2 & 1 & 6 \\ 1/2 & 2/5 & 1 & 1/3 & 4 \\ 2 & 1 & 3 & 1 & 5 \\ 1/7 & 1/6 & 1/4 & 1/5 & 1 \end{pmatrix} & \begin{matrix} a \\ e \\ p \\ r \\ i \end{matrix} \end{array}$	$21.9^*U(a) + 29.5^*U(e) + 12.3^*U(p) + 32.0^*U(r) + 4.3^*U(i)$	0.0273

Note: A_j denotes matrix for j^{th} alternative, π_j denotes multiple attribute utility, and CR_j denotes consistency ratio

Table 5. Deterministic sensitivity analysis

				Alternative 2			Alternative 3			Alternative 4			Alternative 5			
Var	Lo	Nom	Hi	Low	High	Δ	Low	High	Δ	Low	High	Δ	Low	High	Δ	Sen?
r	2.0	4.0	10.0	37	80	43	56	69	13	61	74	13	49	78	29	Yes
p	0.0	5.0	20.0	53	72	19	42	81	39	58	78	20	61	73	12	Yes
e	0.0	8.0	20.0	51	66	15	46	68	22	42	75	33	44	73	29	Yes
a	−3.0	0.5	3.0	50	66	16	49	66	17	48	73	25	50	71	21	Yes
i	0.0	4.0	20.0	63	66	3	63	67	4	68	72	4	67	71	4	No
Nominal					63.41			63.64			69.10			67.80		

Note: Lo denotes low, Nom denotes nominal, Hi denotes high, Δ denotes difference between high and low utility values, and Sen denotes whether the alternative is sensitive to change in values of the variable.

Table 6. Utility functions for various policies

Decisions		
Structured Coaching	Tutoring Method	Multiple Attribute Utility
Yes	Student Centred	$\rho_1 = 0$
	Computer Based Tutoring	$\rho_2 = 16.8*U(a) + 15.3*U(e) + 19.4*U(p) + 44.2*U(r) + 1$
	Chalk and Talk	$\rho_3 = 18.3*U(a) + 22.4*U(e) + 41.3*U(p) + 13.6*U(r) + 1$
No	Student Centred	$\rho_4 = 26.7*U(a) + 34.0*U(e) + 21.9*U(p) + 13.2*U(r) + 1$
	Computer Based Tutoring	$\rho_5 = 21.9*U(a) + 29.5*U(e) + 12.3*U(p) + 32.0*U(r) + 1$
	Chalk and Talk	$\rho_6 = 0$

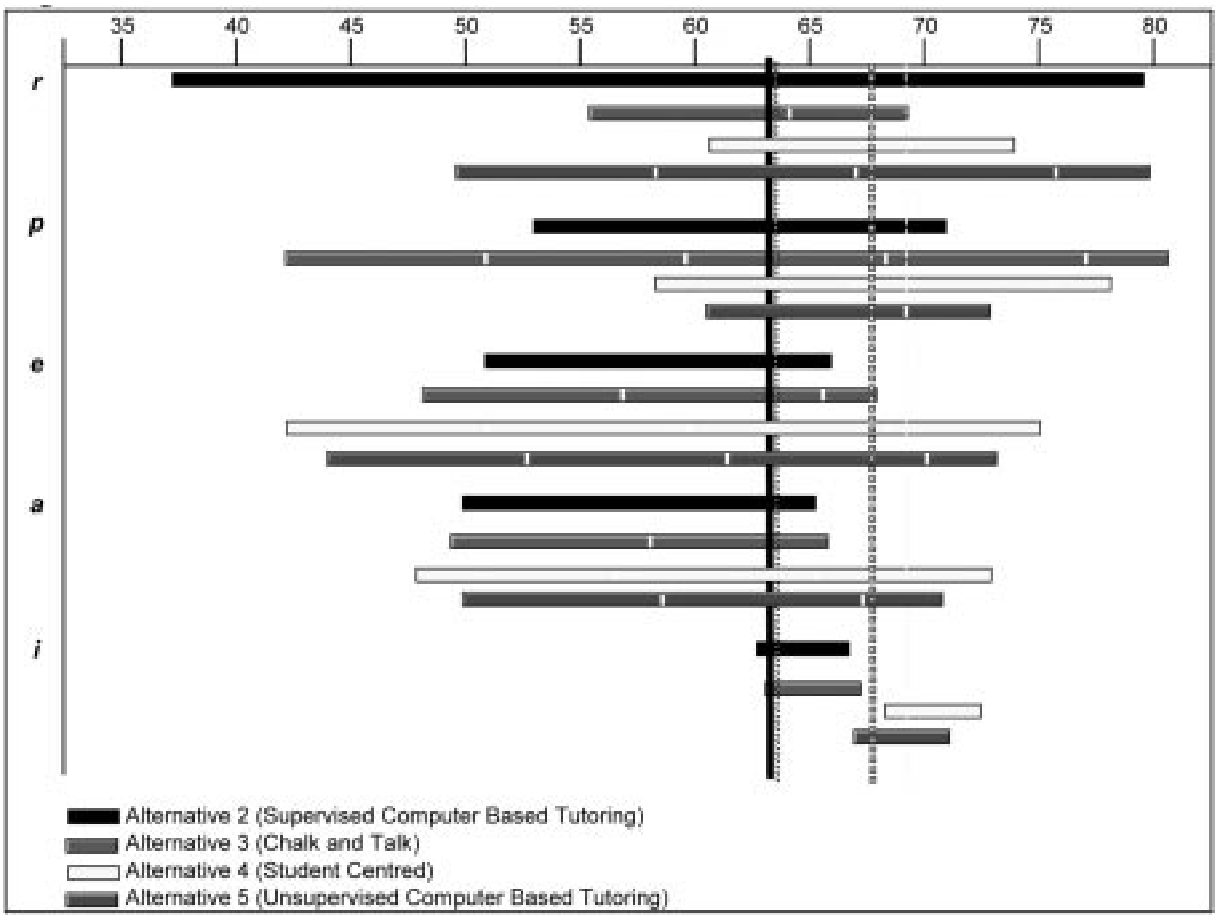

Fig. 9. Tornado diagram for four alternatives. (*Note:* The vertical lines indicate the means of the four alternatives.)

Table 5 shows the results of deterministic sensitivity analysis on the five variables by using DPL. It is noted that variable i (lecturer's industrial experiences) had the least influence on students' learning. The utility values are either 3% or 4% for all alternatives as variable i changes from 0 to 20 years. Hence, this stochastic variable is instantiated to $U(i) = 0.045 * 4 + 0.1 = 0.28$ by using its nominal value.

The updated utility function after the variable i has been removed is shown in Table 6. In addition, courses of action are checked to ensure there is no dominance from any one alternative over any other. Figure 9 shows the tornado diagram [28] illustrating the consequences of various alternatives. Since no alternative lies entirely to the right of others, it can be concluded that no alternative dominates the others.

Table 7. Distribution functions of the four variables

Variables	Theoretical distribution	Parameters: Mean	Parameters: Standard deviation
ability, a	Normal	0.5	2
effort, e	Normal	8	5
preparation, p	Normal	5	3
resource, r	Normal	4	2

Since ability (a), effort (e), preparation time (p), and resources (r) have significant changes in expected utility values over their possible ranges, probabilistic assessment of these variables is required. The variable on lecturer's industrial experiences in the inference diagram is removed and will not be considered in subsequent inferences. In this study, it is assumed that all the stochastic variables follow *normal distribution* with different parameters as shown in Table 7.

The preferred tutoring policy

Figure 10 illustrates the abridged policy tree of the most satisfying decision obtained using DPL. Expected utility value for this plan is 65.6468%. Since the cumulative distributions of the alternatives (see Fig. 11) overlap, there is no stochastic dominance.

The qualitative description of the preferred tutoring policy is:

use *unstructured coaching* for student and obtain the information on *resource*
if number of students sharing a computer is low (*resource* is available) then
perform *unsupervised computer-based tutoring*
else
select s*tudent-centred* tutoring method

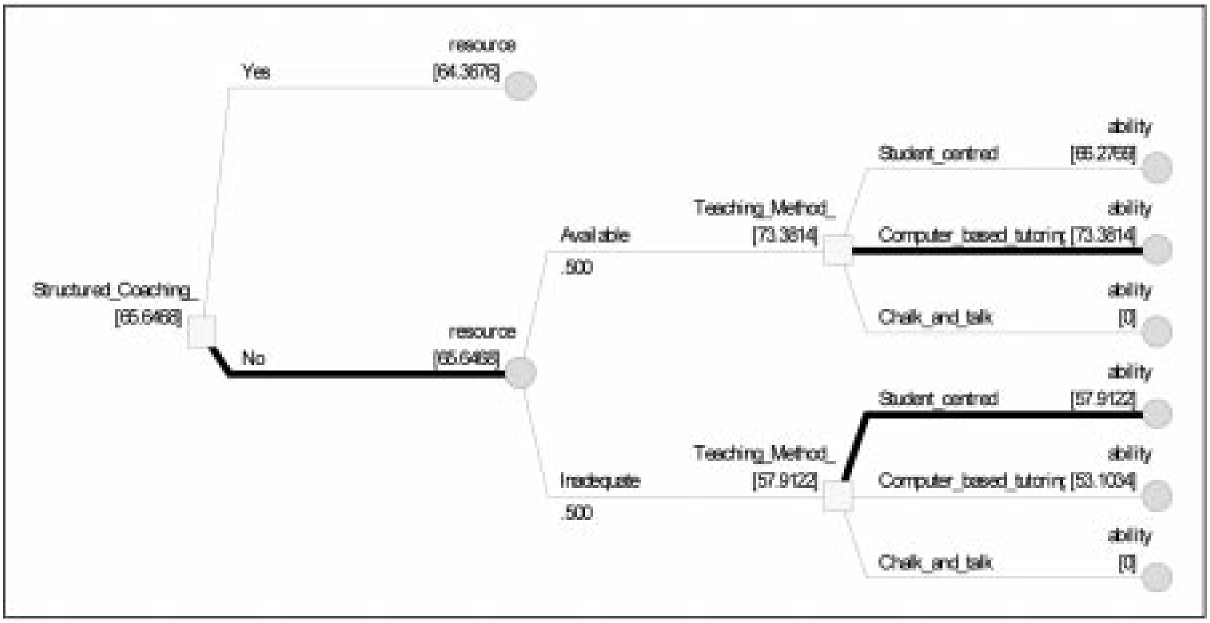

Fig. 10. Portion of *optimal* tutoring policy. (*Note:* The bold lines indicate preferred alternatives, and the circular nodes at the end of branches can be expanded to reveal sub-trees.)

DISCUSSION

Preference to collect information on stochastic variables

The decision model can be used to determine if there is any benefit in obtaining prior information on the variables. Figure 12 shows the abridged decision tree when the prior information on resource is known. The *current* expected utility is 66.5794% while the *original* expected utility is 65.6468%. Since there is an *increase* in expected utility, knowing the information on resource leads to higher satisfaction. When information on resource and effort is known, it can be also shown there is an increase of expected utility ($66.9525\% - 65.6468\% = 1.3057\%$). The increase in utility value is more than that based on resource only, indicating a stronger satisfaction for obtaining information on two variables.

Table 8 lists in ascending order the increase in utility values for information on all combinations of variables. Whether to proceed to gather information depends on the cost involved. In some situations, the cost of obtaining information on the variable(s) may not outweigh the benefits of the increase in utility.

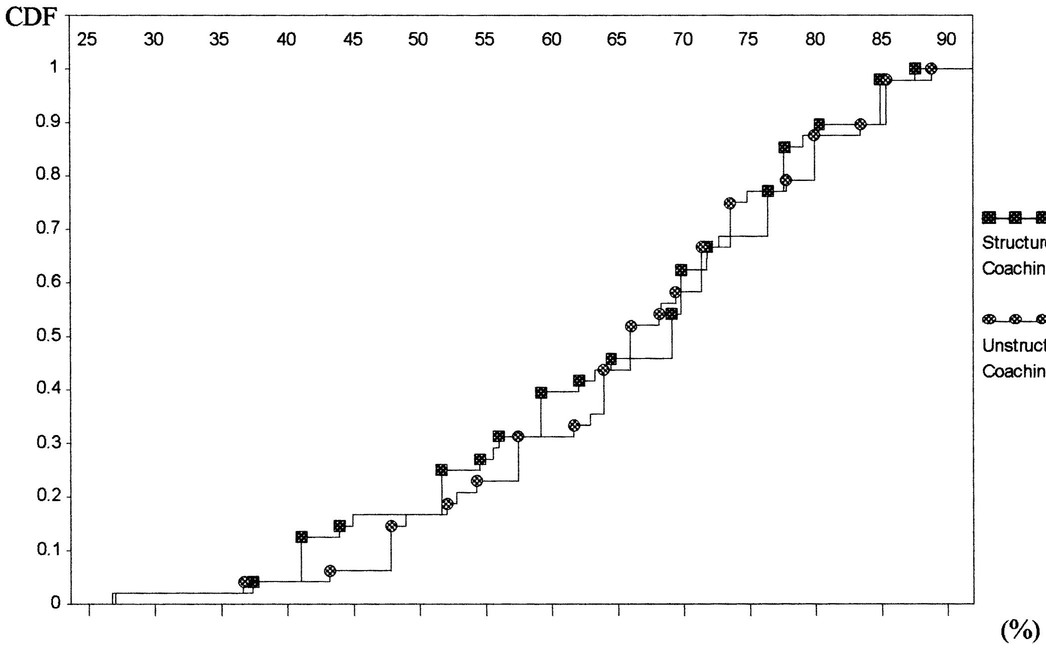

Fig. 11. Risk profiles for two coaching alternatives.

Table 8. Changes in *expected utilities* for variables *a, e, p*, and *r*

Variable	Current EU	Original EU	EU for perfect information
ability	66.1906	65.6468	0.5438
preparation	66.4624	65.6468	0.8156
ability & preparation	66.5025	65.6468	0.8557
effort	66.5346	65.6468	0.8878
ability & effort	66.552	65.6468	0.9052
resource	66.5794	65.6468	0.9326
preparation & resource	66.5794	65.6468	0.9326
ability & resource	66.6836	65.6468	1.0368
effort & preparation	66.8183	65.6468	1.1715
ability & preparation & resource	66.9096	65.6468	1.2628
effort & resource	66.9525	65.6468	1.3057
ability & effort & preparation	67.1161	65.6468	1.4693
ability & effort & resource	67.1991	65.6468	1.5523
effort & preparation & resource	67.2846	65.6468	1.6378
All variables	67.4476	65.6468	1.8008

Effects of probabilistic assessments

Selection of tutoring method has been based on the average **ability** level of students. One frequently asked question is 'Which method is suitable for tutoring a class where the majority are under-achievers?' To obtain some insights of this problem, the decision model is analysed with the probability assessment for ability amended to *normal* $(-1.5, 0.8)$. The values for these parameters are based on item calibration [25, 29, 30] from standardised tests. The new preferred tutoring policy is:

use *structured coaching* for students and obtain the information on *resource*
if *resource* is available then
conduct *supervised computer-based tutoring*
else
use *chalk-and-talk* method

The expected utility value is 60.3029%, which is lower than that for average students. An explanation for the lower value is the belief that effectiveness of tutoring methods reduces when student's ability is low. The utility curve for ability (see Fig. 13) illustrates these general preferences among lecturers.

As the trend towards incorporating information technology in teaching and learning gathers momentum, it is expected most modules are able to achieve low students-per-computer ratio in the near future. To analyse the impact of the increased accessibility of computer, another decision analysis is performed. The decision model is used with probability distribution for **resource** amended as

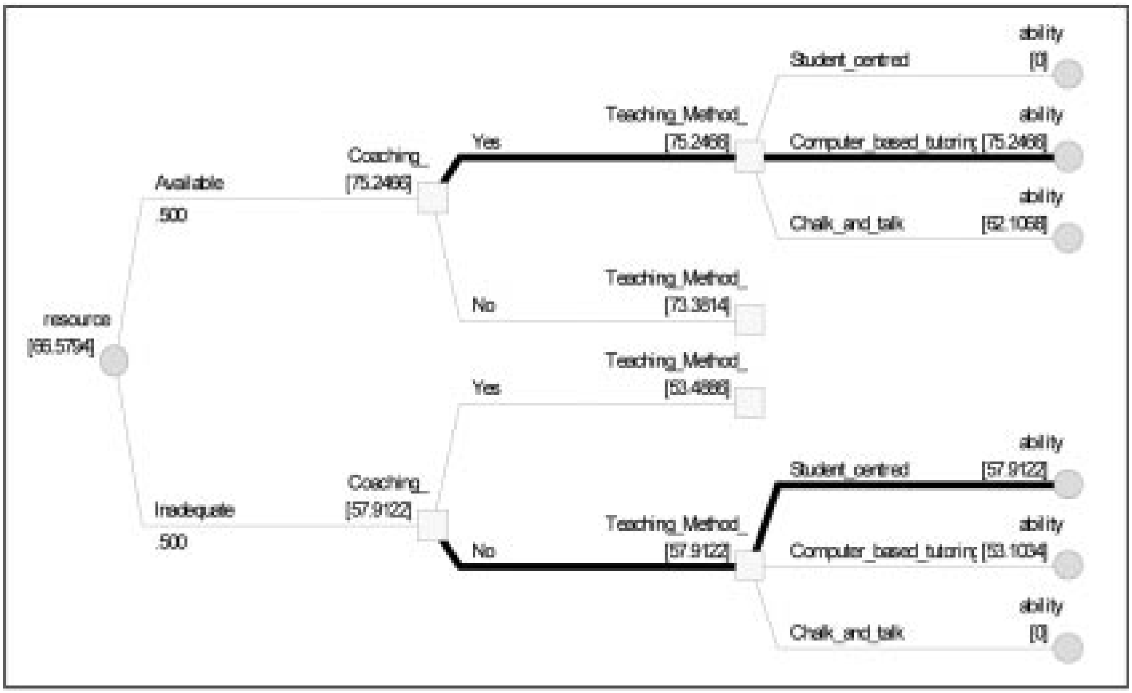

Fig. 12. The tutoring policy when the availability of resources is *known.*

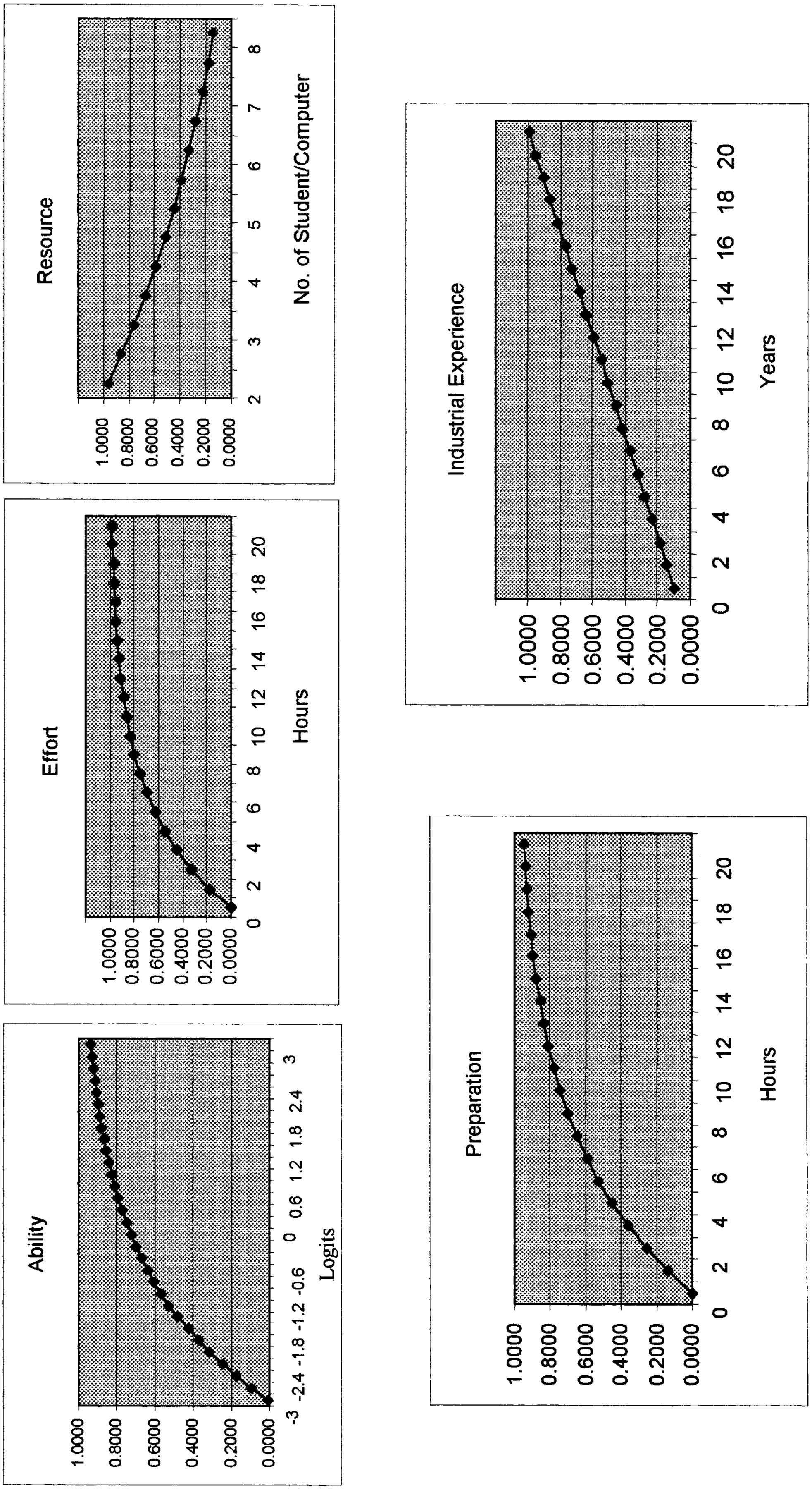

Fig. 13. Graphs of utility values for variables used in the model.

normal (2.5, 1). This reflects the situation where there is an average of 2.5 students per computer rather than the earlier projection of 4 students per computer. The preferred alternative from the analysis is to conduct *supervised computer based tutoring* regardless of the amount of contributions from other variables.

The expected utility value is 71.3858%, which is higher than that for resource of 4 students-per-computer (with standard deviation of 2). The higher value is in line with lecturers' belief that additional computer resource is beneficial to students' learning.

RESULTS

The mechanics module team adopted the policy for a semester and noticed a 5% improvement in pass rate. However, many factors may have influenced the improvement and additional data have to be taken over several semesters.

The module team is able to gain clearer understanding of the problems before implementation.

As a result of this study, the module team could focus their effort on other related issues such as:

- enhancement of *help* features and content materials to support computer-based learning;
- refinement of course materials to facilitate better activities for student-centred tutoring;
- creation of additional tutorial problems for lecturers to select during their chalk-and-talk coaching.

SUMMARY AND CONCLUSION

The modelling effort illustrated in the case study combines the pedagogical experience possessed by the lecturers and the decision-theoretical methodology. On the one hand, lecturer's experience is brought to bear on determining the decisions to make, available alternatives, nature of uncertain variables and their relationship (whether dependent or independent), probabilistic and preference assessments, and trade-off among values. On the other hand, it relies on a series of activities associated with the decision-theoretical approach to build a requisite model.

Sensitivity analysis is used to simplify the model by reducing stochastic variables to deterministic values. In the case study, using statistics from a school in Singapore Polytechnic, it was found that lecturers' industrial experiences have little influence on the expected utility values and would not affect the preferred policy. This variable was subsequently removed from the model. The requisite model consists of two sequential decisions and stochastic variables that influence decisions. In addition, conditional probabilities and utility values for different outcomes are included in the model. With the help of decision-theory software, the model is solved for the preferred policy to enhance students' learning. When information on computer resources, students' effort and lecturer's preparation time is available either singly or in combinations, it can lead to higher satisfaction so that the recommended policy improves students' learning.

This study has illustrated that decision analysis provides a normative rationale for achieving clarity of action under complex and uncertain decision situations. Although good decisions do not guarantee optimal outcomes all the time, a decision-theoretical approach ensures no unforeseen surprises. This paper has also shown how lecturers could construct graphical models for decision-making, in particular on selection of tutoring methods to maximise student learning. Subsequently, lecturers can take action for different situations with greater confidence that is gained through a clearer understanding of the problem.

REFERENCES

1. A. Bergeron and G. Paquette, Discovery environments and intelligent learning tools, in C. Frasson and G. Gauthier (eds), *Intelligent Tutoring Systems: At the Crossroads of Artificial Intelligence and Education*, Ablex Publishing Corporation, Norwood, NJ (1990).
2. R. A. Howard, Decision analysis: practice and promise, *Management Science*, **34**, 1988, pp. 679–695.
3. R. D. Shachter, Evaluating Influence Diagrams, *Operations Research*, **34**(6), 1986.
4. H. Raiffa, *Decision Analysis*, Addison-Wesley (1968).
5. C. A. Holloway, *Decision Making under Uncertainty: Models and Choices*, Prentice-Hall Englewood Cliffs, NJ (1979).
6. *ADA (Applied Decision Analysis)*, *DPL 4.0: Professional Decision Analysis Software*, Duxbury/Thomson Learning (1998).
7. D. Samson, *Managerial Decision Analysis*, Irwin, Homewood, IL (1988).
8. L. D. Phillips, A theory of requisite decision models, *Acta Psychologica*, **56**, 1984, pp. 29–48.
9. R. L. Keeney, *Value-Focused Thinking*, Harvard University Press, Cambridge, MA (1992).
10. S. Ross, *A First Course in Probability*, 5th edition, Prentice-Hall (1998).
11. C. S. Spetzler and C. A. Staël Jon Holstein, Probability Encoding in Decision Analysis, *Management Science*, **22**, 1975, pp. 340–352
12. R. T. Clemen, *Making Hard Decisions: An Introduction to Decision Analysis*, 2nd edition, Duxbury Press (1996).
13. W. Feller, *An Introduction to Probability Theory and Its Applications, Vol. 1*, 3rd edition, Wiley, New York (1968).

14. R. Peck, *Introduction to Statistics and Data Analysis*, Duxbury/Thomson Learning, (2001).
15. D. L. Keefer and S. E. Bodily, Three-point approximations for continuous random variables, *Management Science* 29, 1983, pp. 595–609.
16. A. L. Golub, *Decision Analysis: an integrated approach*, John Wiley & Sons (1997).
17. J. Pratt, Risk aversion in the small and in the large, *Econometrica*, **32**, 1964, pp. 122–136.
18. P. McNamee and J. Celona, *Decision Analysis for the Professional with Supertree*, Scientific Press, Redwood City, CA (1987).
19. F. P. Ramsey, Truth and probability, in R. B. Braithwaite, (ed.), *The Foundations of Mathematics and Other Logical Essays*, Harcourt, Brace, New York (1931).
20. D. Bunn, *Applied Decision Analysis*, McGraw-Hill, New York (1984).
21. T. Parry, and G. Gregory, *Designing Brain Compatible Learning*, SkyLight Training and Publishing Inc (1998).
22. T. L. Saaty, *The Analytic Hierarchy Process*, 2nd edition, RWS Publication (1990).
23. R. L. Keeney, and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, John Wiley (1976).
24. C. B. McCormick, and M. Pressley, *Educational Psychology: Learning, Instruction, Assessment*, Longman (1997).
25. B. R. Hergenhahn, and M. H. Olson, *An Introduction to Theories of Learning*, 5th edition, Prentice-Hall (1997).
26. Blackboard Inc., *http://www.blackboard.com* (2002).
27. R. K. Hambleton, H. Swaminathan and H. J. Rogers, *Fundamentals of Item Response Theory*, SAGE Publications (1991).
28. R. A. Howard, and J. E. Matheson, *Readings on the Principles and Applications of Decision Analysis*, Strategic Decisions Group, Menlo Park, California (1983).
29. F. M. Lord and M. R. Novick, *Statistical Theories of Mental Test Scores*, Reading: Addison-Wesley (1968).
30. B. D. Wright, Solving Measurement Problems with the Rasch Model, *J. Educational Measurement* **14**, 1977, pp. 97–116.

P. K. Pek is a lecturer in School of Mechanical & Manufacturing Engineering of Singapore Polytechnic since 1992. He obtained a M.Sc. degree with distinction in Systems, Information and Control Engineering from the University of Sheffield in 1997. He graduated with a M.Eng. and B.Eng. in Mechanical Engineering from National University of Singapore in 1992 and 1986 respectively. Mr Pek is a postgraduate research student working under the supervision of Dr K. L. Poh. His research area is on decision-theoretic intelligent tutoring systems.

K. L. Poh is an associate professor in Department of Industrial and Systems Engineering of National University of Singapore. He holds a doctorate in Engineering-Economic Systems and Operations Research from Stanford University. He is past president of the Operational Research Society of Singapore, and he serves on the Information Technology Committee of the Institution of Engineers, Singapore. Dr Poh's current research interests include decision systems, systems optimisation and meta-heuristics, and decision analysis in energy systems modelling and planning.