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On Teaching the Simplification of Block Diagrams*

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Due to their simplicity and versatility, block diagrams are widely used by control engineers to model all types of dynamic systems. The complexity of a block diagram is in general caused by the existence of summing/pickoff points within a loop. A novel concept, the main/branch stream concept and the corresponding shifting rule for the relocation of summing/pickoff points are introduced in this paper. It is found that the new concept and the corresponding shifting rule greatly help the simplification of block diagrams and make teaching the simplification of block diagrams much easier.

SUMMARY OF EDUCATIONAL ASPECTS OF THE PAPER

1. The paper discusses materials/software for a course in Control Engineering.
2. Students from the Mechanical Engineering and Electrical Engineering departments are taught in this course.
3. Level of the course (year) is Senior (4th year) as a regular course presented as lectures and labs.
4. The course is normally a 4-credit-hour course (that is, 4 lecture-hours weekly)
5. Student homework or revision hours required for the materials average 4 hours weekly.
6. A novel concept, the *main/branch stream* concept and the corresponding *shifting rule* for the relocation of summing/pickoff points are introduced in this paper. The introduction of the shifting rule eliminates the necessity of the existence of dozens of block diagram algebra-related rules, hence it greatly helps the simplification of block diagrams and makes teaching the simplification of block diagrams much easier.
7. The standard text recommended in the course, in addition to author's notes:

   K. Ogata, *Modern Control Engineering*, 3rd edition, Prentice-Hall, New Jersey, (1997).
   B. C. Kuo, *Automatic Control Systems*, 7th edition, Prentice-Hall, New Jersey, (1995).

1. INTRODUCTION

MOST PHYSICAL systems can be represented by block diagrams—a group of properly interconnected blocks, with each interconnected block representing and describing a portion of the system [1–4]. In control engineering, the block diagram is a primary tool that together with transfer functions can be used to describe cause-and-effect relationships throughout a dynamic system. The manipulation of block diagrams adheres to a mathematical system of rules often known as block diagram algebra.

In general, the interrelationships of causes and

Fig. 1. The components of a block diagram.

Fig. 2. Basic block diagram algebra with regard to series/cascaded blocks.

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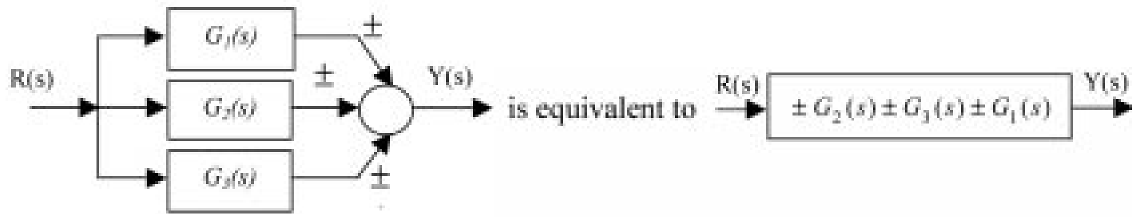


Fig. 3. Basic block diagram algebra with regard to parallel blocks.

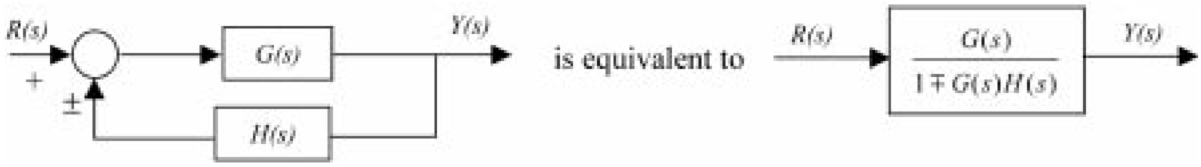


Fig. 4. Basic block diagram algebra with regard to a general feedback loop.

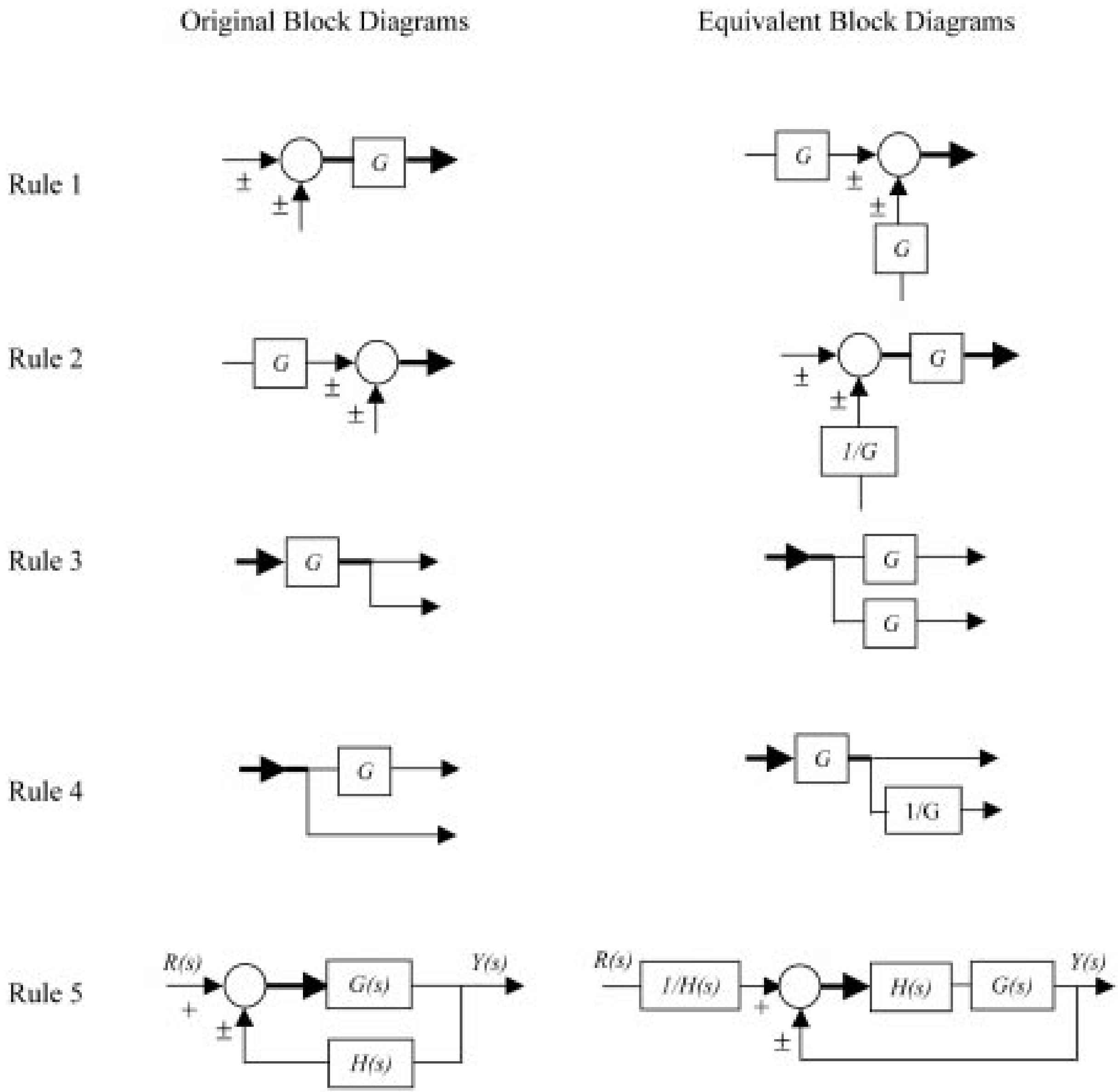


Fig. 5. Example rules related to the simplification of block diagrams using block diagram algebra.

effects of a system are not straightforward and the simplification of block diagrams is required. The signal flow graph (SFG) developed by Mason [5–6], which is based on a representation of a system by line segments, is used to simplify block diagrams. When using the SFG method, the cause-and-effect relationships are normally obtained by applying 'Mason's gain formula'. Applying the formula however involves a relatively complex process such as identifying the nodes, the various paths (forward/loop, touching/non-touching) and calculating the corresponding path gains. Block diagrams can also be simplified using the rules of 'block diagram algebra'. This too is a relatively complex process where up to 33 rules were introduced [7] that may need to be applied.

In this paper, a novel concept, the concept of *main/branch stream* and the corresponding *shifting rule* are introduced to simplify the complexity of the rules in the simplification of block diagrams using the block diagram algebra.

A brief review of the block diagram basics is presented in part A of section II. The current practice in the simplification of block diagrams and the proposed alternative approach in the simplification of block diagrams using the novel main/branch stream concepts and the corresponding shifting rule are described in part B and C of section II respectively. Two illustrative examples are given in section III, which is followed by the conclusion.

II. THE BLOCK DIAGRAMS

A. Block diagram basics

In general, the block diagram of a linear time invariant system consists of four components, namely signal, block (with transfer function), summing point and pickoff point as shown below in Fig. 1.

The basic block diagram algebra involves algebra with regard to series/cascaded blocks, parallel blocks and a general feedback loop, as shown in Fig. 2 to 4. Series blocks combine with each other by multiplication and parallel blocks combine with each other by algebraic addition. The combined block is interchangeable in sequence in both cases. The simplification of the general feedback loop described in Fig. 4 can be obtained by performing simple algebraic operation around the loop, which can be found in most textbooks on control engineering [e.g. 1–4].

Note that the understanding of the properties of summing junction in series is crucial in the simplification of some block diagrams: summing points in series can be interchanged, combined and separated algebraically. The equivalency of summing junction in series to mathematical summer is self-explanatory.

B. The current practice in the simplification of block diagrams using block diagram algebra

The block diagram is in general complicated by the existence of the summing/pickoff point(s) within a loop. However the simplification of the block diagram can always be achieved through the relocation of such summing/pickoff point(s) appropriately. In the current practice of simplifying block diagrams using block diagram algebra, in addition to the three basic rules described in part A (Figs 2 to 4), various numbers of other rules are introduced in various textbooks with regard to the relocation of the summing/pickoff point(s). Each rule involves a pair of equivalent block diagram. For example, Fig. 5 lists some of the rules introduced in [1]. The equivalency is not hard to verify by tracing the signals at the input through to the output and recognizing the output signal is identical in the corresponding cases.

However, it is hard enough for the students to memorize such rules, not to mention to let them decide when to use which rule while facing a complex block diagram.

C. The proposed alternative approach in the simplification of block diagrams by using the novel main/branch stream concepts and the corresponding shifting rule

The main/branch stream is defined as following: At a summing point or pickoff point, there are signals flowing in and out. The signal flow that goes in the unique direction, either into or out of the summing/ pickoff point, is defined as the main stream and the rest of the signal flows are defined as branch streams.

With the introduction of the main/branch stream concepts, summing/pickoff point(s) can be easily relocated for the purpose of simplification by simply following the shifting rule described below:

To remove a block G away from a main stream, a block G is to be added to each branch stream; to remove a block G away from a branch stream, a block G is to be added to the main stream and a block 1/G to be added to each of the rest branch streams.

Now let us go back to the equivalent block diagram pairs listed in Fig. 5. Firstly, we identify the main stream and the branch streams of each block with regard to its summing/pickoff point. It is not hard to see from the definition that the highlighted darker line represents the main stream in each case. The equivalency between the pair of block diagrams is then easily obtained by simply applying one single rule—the shifting rule.

It should also be pointed out that pickoff points are not summing points. However, their use follows the same main/branch stream related shifting rule; this again, greatly simplifies the rules with regard to the simplification of block diagrams.

From the above analysis, it is not hard to see that the new main/branch stream concepts and the corresponding shifting rule eliminate the necessity

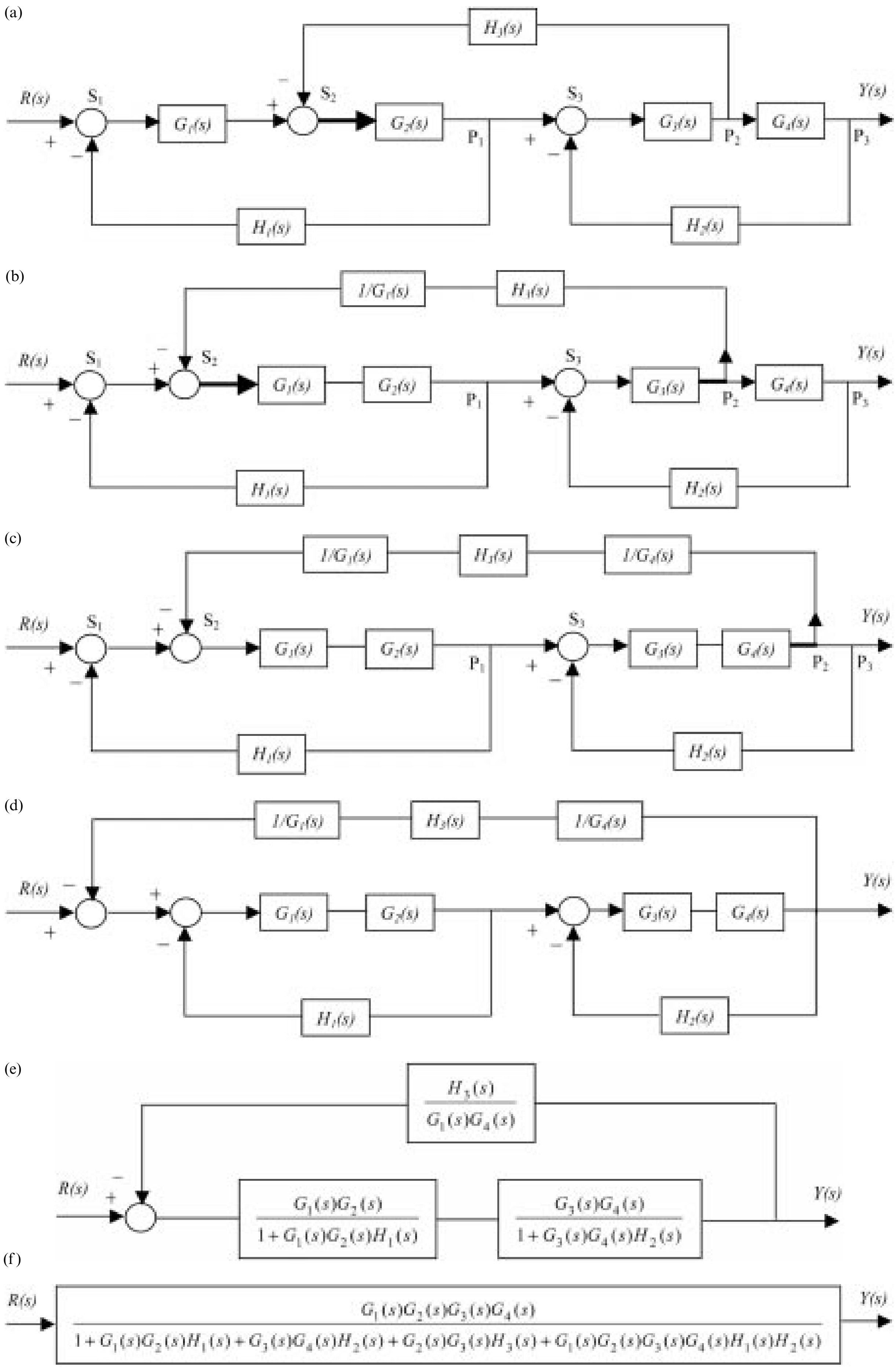
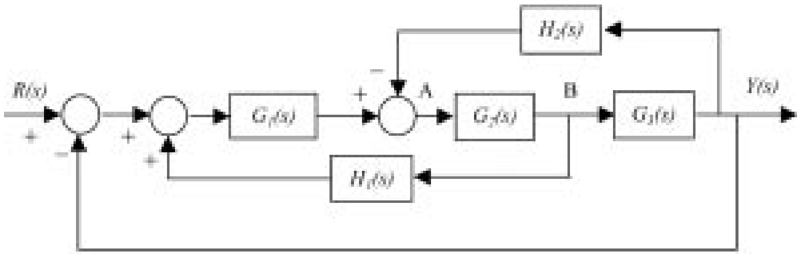


Fig. 6. (a) Block diagram of a dynamic system; (b)–(f) successive reductions of the block diagram.

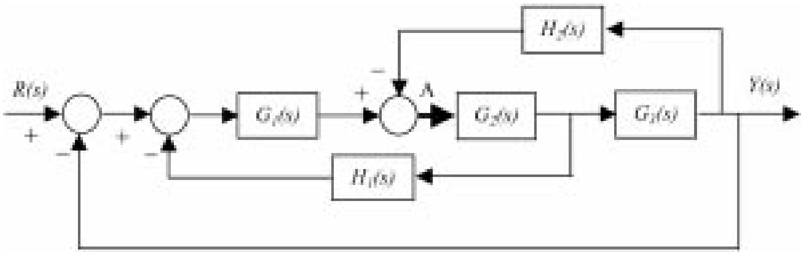
Table 1. Rules to be applied for simplifying the block diagram shown in Fig. 6(a)

	The current approach	The proposed alternative approach
	Rules to be applied	
From (a) to (b)	Rule 1	Shifting Rule
From (b) to (c)	Rule 4	Shifting Rule
From (c) to (f)	The basic rules described in Part A	

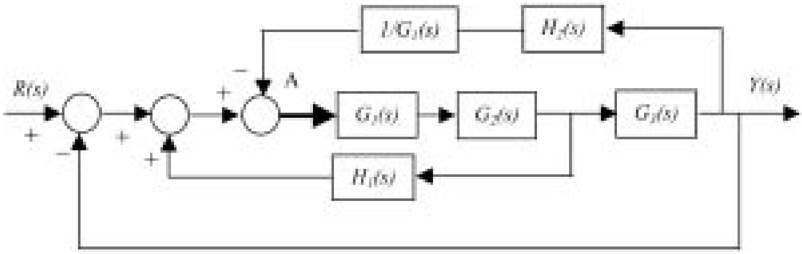
(a)



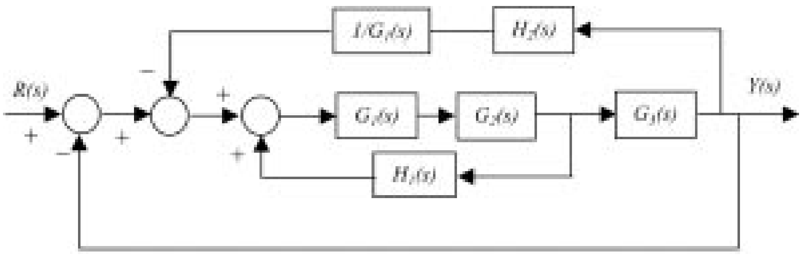
(b1)



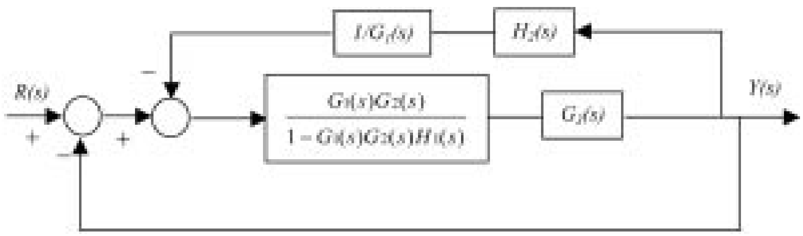
(c1)



(d1)



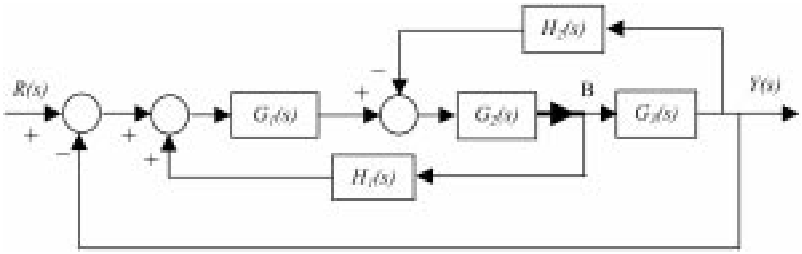
(e1)



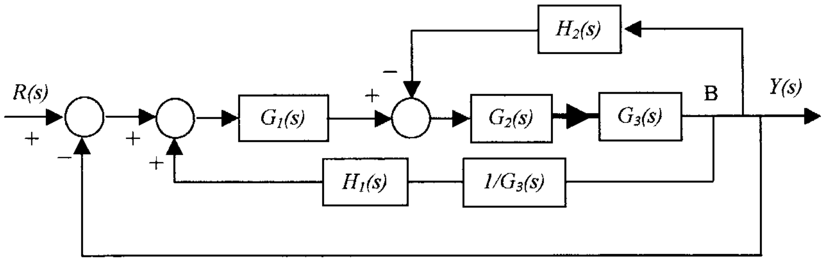
(f1)

(g1)

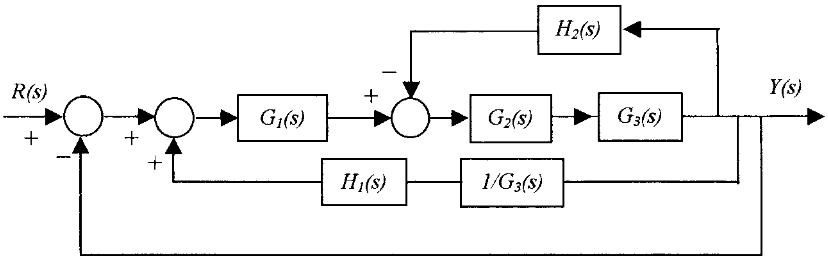
(b2)



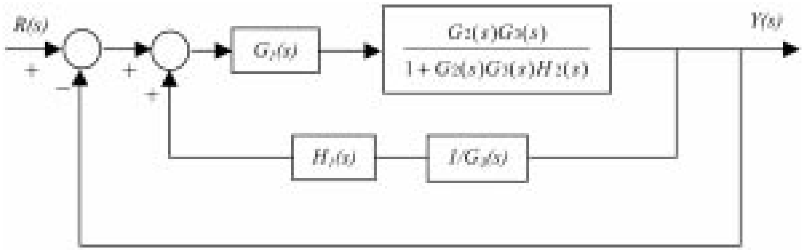
(c2)



(d2)



(e2)



(f2)

(g2)

Fig. 7. (a) Block diagram of a dynamic system; (b1)–(g1) successive reductions of the block diagram by relocating summing point A; (b2)–(g2) successive reductions by relocating pickoff point B.

Table 2. Rules to be applied for simplifying the block diagram shown in Fig. 7(a)

	The current approach	The proposed alternative approach
	Rules to be applied	
From (b1) to (c1)	Rule 2	Shifting rule
From (b2) to (c2)	Rule 4	Shifting rule
From (c) to (g)	The basic rules described in Part A	

of the introduction of dozens of block diagram algebra related rules (such as the ones listed in Fig. 5) in the simplification of block diagrams. As a result, it greatly simplifies the complexity of the rules of the block diagram algebra and makes the simplification of block diagrams a much easier task.

III. THE SIMPLIFICATION OF BLOCK DIAGRAMS—ILLUSTRATIVE EXAMPLES

Two example block diagrams are to be simplified. The same block diagram is simplified using both the current approach and the proposed alternative approach. Comparisons are made.

As a first example, let us have a look at the simplification of the block diagram as shown in Fig. 6. The block diagram is complicated due to the existence of summing points (S_2 and S_3) and pickoff points (P_1 and P_2) within the loops, as is normally the case. To simplify the block diagram, it is necessary to relocate such summing and pickoff points so as to move them out of the loops.

In Fig. 6, (a) shows the block diagram of an example dynamic system and figures (b) to (f) show the successive reductions of the block diagram. The rules that need to be applied for each successive reduction in the current practice and by using the proposed alternative approach are summarized in Table 1.

As a second example, we simplify the multiple-loop system as shown in Fig. 7 [Figs 3–9 in Reference 1]. The block diagram is complicated due to the existence of summing point A and pickoff points B within the loops. The block diagram can be simplified by either relocating summing A or pickoff B. The successive reductions are shown in Figs (b1) to (g1) and Figs (b2) to (g2) respectively.

The rules that need to be applied for each successive reduction in the current practice and by using the proposed alternative approach are summarized in Table 2.

The examples demonstrate that with the introduction of the shifting rule, it is no longer necessary to memorize dozens of rules related to block diagram algebra. This greatly helps the simplification of block diagrams.

IV. CONCLUSIONS

In this paper, the novel main/branch stream concepts and the corresponding shifting rule for the relocation of summing/pickoff points are introduced. The introduction of the shifting rule eliminates the necessity of the existence of dozens of block diagram algebra-related rules. Block diagrams can be simplified with the shifting rule and the three basic rules with regard to the simplification of series/cascaded blocks, parallel blocks and a general feedback loop without difficulty, as demonstrated by the illustrative examples. The introduction of the new concept and the corresponding shifting rule greatly helps the simplification of block diagrams and makes teaching the simplification of block diagrams a much easier task.

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