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Use of a Physical Linear Cascade to Teach Systems Modelling*

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Entire undergraduate programs have been known to adopt PBL. This might be considered a 'macro-application' of this pedagogical approach. Sometimes micro-scale applications can be useful, i.e. introduction of a single challenging and rather open-ended problem. The problem selected should be generic in nature and rather difficult because to promote learning by the mere mimicking of specific examples is poor pedagogy. Micro-to-meso-scale PBL initiatives have a number of advantages: they familiarise faculty with the PBL approach, they have lower cost and risk, the tutors start at an appropriate position on the PBL learning curve, and they are administratively simple. A comparison between the activities of Engineering Design and Process Modelling is presented. The PBL exercise described herein was developed around a set of linear reservoirs that were specially constructed to provide the students with a hands-on verifiable experience with mathematical modelling. This physical cascade system is unique in that an exact analytical solution exists for the n^{th} reservoir. The degree of success with this modelling exercise is discussed.

INTRODUCTION

THERE ARE MANY WAYS to try to model naturally occurring and/or engineering phenomenon. Three broad categories or methods are:

1. The building of physical models, followed by laboratory measurements during model operation.
2. Via deterministic modelling using analytical solutions of, or numerical approximations to, the governing differential equations.
3. Via non-deterministic modelling using 'best-fit' equations to mimic the observed processes (without seeking a deeper understanding of the underlying mechanics).

Numerical solutions themselves represent a large number of possible approaches and much is known about the magnitude of the errors that may be expected for a given method. All of these approaches are interesting and useful in their own ways, but it is still a challenge to make this material seem interesting to engineering students. Today's engineering professor is also confronted with a large array of software that purports to be able to engage in process modelling in new and better ways. This paper describes how the phenomenon of level-pool hydrologic routing was used in a civil engineering course as a vehicle to introduce students to all of these approaches, including a powerful simulation software package that emphasises the development and use of the user's intuitive understanding of the processes being observed. Our on-going desire is to increase the amount of environmental modelling in the course, thereby increasing both its usefulness and the level of interest experienced by students.

ACADEMIC CONTEXT

Dalhousie University course CIVL4720 'Civil Engineering Computations' was originally conceived as one in which various numerical methods would be taught using examples specific to civil engineering. Except for this selectivity it was, prior to the fall of 2000, a fairly typical numerical methods course. It had no laboratory component, which is quite typical of such courses. This paper describes an experience with teaching modelling using the well-known hydrologic phenomenon of level-pool routing as the pedagogic vehicle. In the fall of 2000 a simpler version of the experiment described herein was incorporated. It was based on

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the more common case of non-linear reservoirs (Hansen and George [1]); specifically, a cascade of small cylinders with outlets controlled by orifices—and for which no analytic solution exists for the n^{th} outflow hydrograph. The experimental set-up was both inexpensive and easy to build. The experiment described herein was executed using a more elaborate system: a cascade of linear reservoirs, for which an analytic solution does exist for the n^{th} outflow hydrograph. The level-pool routing phenomenon was modelled in CIVL4720 in five ways:

- physically;
- numerically, using traditional numerical schemes;
- analytically;
- statistically, using non-linear transformations and ordinary least squares regression (OLS);
- using a systems-simulation software package known as Stella® (HPS 2000).

It is believed that this last approach represents quite a departure from what civil engineering students normally encounter in their undergraduate programs.

PROBLEM-BASED, DESIGN, AND MODELLING

The underlying idea of PBL is already so familiar to engineers that they may be wondering what all the excitement is about. We would argue that engineering design is the quintessential PBL activity. Efforts by engineering professors to incorporate more design in their academic programs could be considered to be efforts to make their curricula more PBL in orientation. Students generally love design, but are not so sure about modelling. It is therefore interesting to compare and contrast the intellectual activities of 'design' and of 'modelling'. Table 1 presents such a comparison.

Therefore, although modelling is perhaps not as inherently PBL as the process of design, there appear to be many useful analogues between the thought processes involved. It is definitely true that sometimes the best way to advance a design effort is to build or test a model.

Savin-Baden [3] has presented and discussed a number of interesting anecdotes of student experiences with PBL under such categories as 'Fragmentation' and 'Self-validation'. 'Fragmentation' in the PBL context might be described as the student's intellectual and emotional discomfort caused by the expectation that he take greater responsibility for his own learning. The student is also expected to become facile with many disparate pieces of information and techniques that must be abstracted on a need-to-know basis from various fields of knowledge, often including (and perhaps especially) fields outside of the discipline in which the student is nominally enrolled. From the point of view of some students, particularly those already feeling besieged with work, this is worsened by the philosophy that they learn this rather foreign-looking material quite independently. Design, as an intellectual activity, is often affected by fragmentation, regardless of whether it is part of an undergraduate course in engineering or is being done for profit. One might also expect the activity of modelling to be affected by feelings of fragmentation, though perhaps to a lesser extent. In this PBL exercise the students were required to draw on the knowledge to which they

Table 1. Engineering design versus process modelling.

Design	Modelling
Nature: the quintessential PBL activity.	Nature: An attempt to replicate the behaviour of an observable phenomenon, often a naturally-occurring one.
Purpose: to create a product that can be sold at a profit.	Possible purposes: to understand nature, to predict an impact on a natural system, to optimise operation of an artificial system, to inexpensively pseudo-test a prototype.
Has a morphology (see Dieter [2] for typical flow charts).	Has a loose morphology, compared to design (see Figure 1).
Does not have one correct answer. Lateral thinking, criticism, cross-disciplinary investigations, and teamwork often needed.	Often quite obvious as to how good an answer is (closeness-of-agreement between observed and modelled outcomes).
Design has iterative components, esp. 'Needs Analysis', 'Problem Definition', and 'Generation of Alternatives'. These steps drive much of the information gathering and on-the-spot learning.	Best modelling approach might be arrived at iteratively; some are more powerful than others but may not be justifiable due to time constraints, or the need for a 'good enough' answer. Details of underlying physics and phenomenologies relatively important and may need to be re-visited and enhanced.
Requires intuitive leaps, so as to finally arrive at an outcome (prototype).	Nature of internal system connectivities requires the application of intuition.
New design cannot be called a success until production units have been subjected to long- term testing in the field.	Poorly designed and/or specified models (too few or too many parameters) can produce deceptively credible results when tested under a narrow range of conditions. Modelling tends to make modellers believe that their models are 'the truth'.
Outcomes (prototypes) cannot always be tested before being used (especially in civil engineering).	Outcomes can be tested, and at almost no cost in the case of mathematical modelling. Modelling permits repeatable vicarious experimentation.
Confidence in a given design may be improved *via* the testing of its systems and sub-systems.	Confidence in the final version of any model is undermined by the problem of lack of uniqueness: two different sets of parameter values can give virtually the same gross outcome.

had previously been exposed in courses in differential equations, fluid mechanics, hydraulics and hydrology. This material represented a sound basis from which students began their modelling efforts, but extension of this knowledge was also required. Students were also expected to use their own intuition to establish and test internal system connectivities and phenomenologies. Some students found this expectation disconcerting while others welcomed it.

With respect to self-validation, engineering students in conventional engineering courses in Canada are rather notorious for being reluctant to ask questions, being more accustomed to a learning style referred to as 'reproductive pedagogy' (Savin-Baden [3]). This attitude is very much the antithesis of one appropriate to PBL. Savin-Baden [3] found that PBL-based programs forced students to ask questions, which was interpreted as leading to greater student 'self-validation' and 'self-discovery'. That is, students began to realise that frequent questioning enhanced their own learning. Savin-Baden [3] describes this as 'students taking more personal responsibility for their learning, and in a manner or style unique to each individual.' Certainly, both design and modelling require that pride, passivity and fear be discarded; none are helpful in generating design alternatives or in improving a model. Neither do they assist in arriving at robust and innovative final outcomes. We would perceive these kinds of issues as depending largely on the basic maturity of the students and of the kind of classroom/laboratory culture that is fostered by the instructors. In this context it was repeatedly made clear that there was/is no such thing as a stupid question; all questions were treated in a welcoming manner and with the utmost respect and professionalism. A good PBL environment was further fostered by not immediately giving out all the information necessary to model the phenomenon in question.

Whiteman and Nygren [4] have presented a diagram that is relevant to modelling and PBL; it graphically illustrates the fact that nowadays software may come into the picture at any and all stages of the modelling process. For the modelling exercise described herein, the upper part of this paradigm did not apply—the physical reservoirs were not a representation of any real cascade system, and was therefore not refined. Apparently, this did not detract from its pedagogical value. However, the incorporation of a modern systems simulation software package, to be described later, was felt by the students to be critical to feeling successful about the overall experience.

METHOD

The phrase 'level-pool routing' in hydrology refers either to the manner in which water moves through a pond or reservoir, or to one of a number of algorithms that may be used to simulate this phenomenon. All such algorithms are based on the simple principle of the conservation of volume. A series of reservoirs is referred to as a cascade; a typical outflow sequence for such a cascade is shown in Fig. 2.

It was desired to make the students' modelling experience more than just successfully generating graph(s) that look like Fig. 2. It was hoped that by

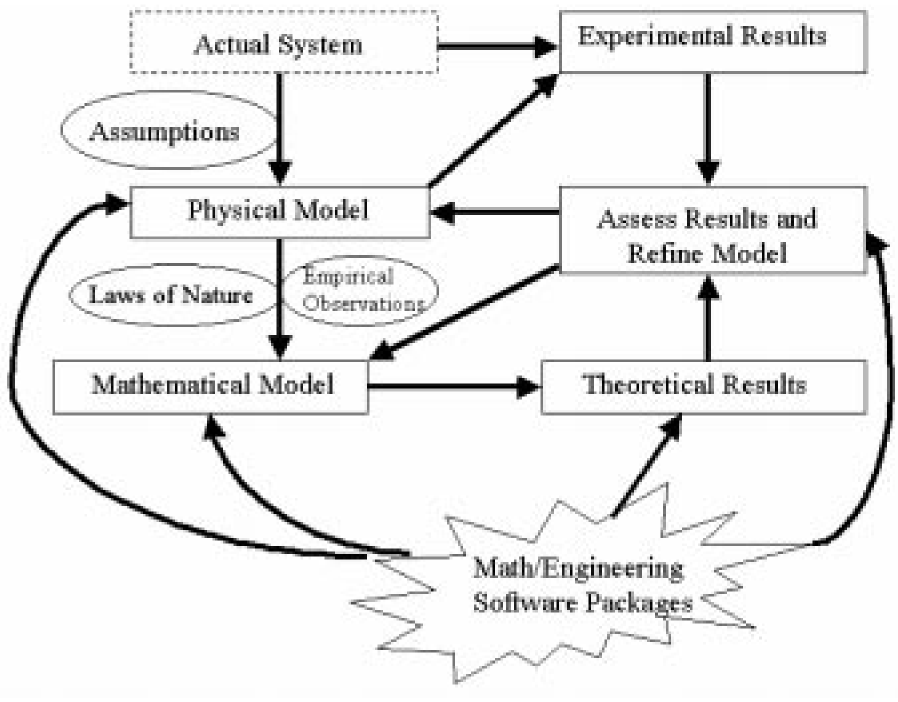

Fig. 1. A modelling paradigm (after Whiteman and Nygren, [4]).

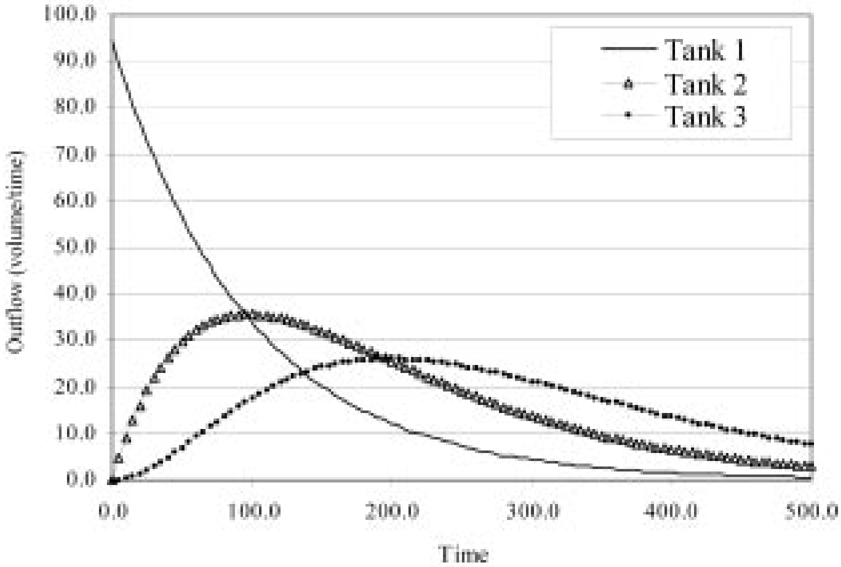

Fig. 2. Outflow hydrographs for a cascade of three reservoirs. (The first reservoir had no inflow; it drained an imposed initial volume).

making the modelling effort more experiential and by including an element of competition, that the level of student interest would be increased. Therefore, a linear cascade of three reservoirs was designed and assembled out of aluminum (see Fig. A1 in Appendix A), at a cost of about \$2000. Each tank was 1.2 m long by 0.25 m high by 0.5 m wide. The outflow from each of these reservoirs, placed in series, was controlled by a weir with a triangular opening having an angle of only 5°.

Students used a vernier caliper and a measuring tape to obtain 'all relevant physical dimensions' of the apparatus, such as weir angles and how the surface area of each tank varied with depth. Naturally, some students returned to obtain measurements that they later realised were needed in order to do the modelling. The procedure was as follows: water coloured with fluorescent green dye drained through a series of tanks and the students took pictures at regular intervals of the state of the system while the water was making its way though it. This was initiated by having one student remove a blockage from the top weir opening and start a clock at the same moment. Another student took a sequence of colour photographs with a digital camera, capturing the water levels in each tank via a bank of piezometers. Because there was a clock in the view of the camera, the students were able to obtain the time that had elapsed for any given set of water levels. These images were e-mailed to the students and they used the standard weir equation to convert the water levels, as measured from the images, into outflows. In this way they compiled all the data associated with the complete passage of the water that was initially only in the top tank. This system for physically modelling hydrologic routing was less expensive and complex than a data acquisition system with three water level pressure transducers connected to, say, a PC equipped with Lab-View.

THEORY AND EQUIPMENT

The phenomenon associated with how the water runs in, and out of, any given reservoir is governed by the following equation:

$$Q_{in} - Q_{out} = \frac{ds}{dt} \tag{1}$$

where 's' is the volume of water in any given reservoir; Q_{in} is the hydrograph supplied by the next-most upstream reservoir.

Consider Fig. 3. If we integrate over H:

$$Q = C_d 2\sqrt{2g}\tan\frac{\theta}{2}\int_0^H (H-h)h^{1/2}dh$$

we obtain the Q_{out} governing a weir outlet:

$$Q_{out} = C_d \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}H^{5/2} \tag{2}$$

In this case it was found necessary to make θ only 5°. Computer simulations indicated that larger angles would drain the tanks so quickly that the students would probably not have time to collect a data set with enough points to nicely describe the gibbus (bump) of a hydrograph. This was also a concern because the digital camera used to record the state of the system at any given moment required a few seconds to write each image to its memory card. This did make the weirs more difficult to fabricate than would have been the case for larger openings. In the interests of future experimental flexibility, the tanks were designed to accommodate weirs with values of θ up to about 30° (see Fig. A1 in Appendix A). It is

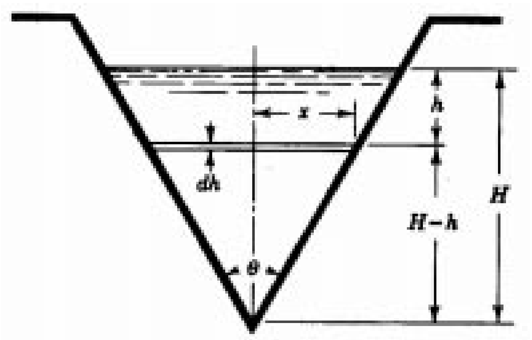

Fig. 3. Definition sketch used in derivation of expression for discharge from triangular weir.

noteworthy that the discharge coefficient C_d actually varies with the head.

Fortunately, the range of C_d is not large. The problem of how to account for C_d was representative of a very important question that we wanted the students to deal with: Is it always necessary to incorporate all the known physics in order to adequately replicate a phenomenon? The answer, of course, is that it is certainly not always necessary to do so. We deliberately refrained from providing the students with the above information until they demonstrated that they had successfully achieved a measure of success in their initial modelling efforts.

With respect to the reservoirs themselves, in order for an analytical solution to exist for the n^{th} reservoir, a linear relation between storage and outflow is needed:

$$s = KQ_{out} \tag{3}$$

It is a classical derivation in hydrology to combine Equations (1) and (3) through successive reservoirs in a cascade in order to obtain the n^{th} hydrograph. The result is founded on the gamma-function (see Appendix C). The side-walls of the tanks were designed so that the volume *s* in each tank increased with depth in such a way that the non-linearity of the weirs was offset, making Equation (3) a reality (see also the terms defined for Equation (4)).

MODELLING APPROACHES USED

Having researched the basic theory and having their data-set in hand, the students sought to computationally reproduce (model) what they had observed, in four ways:

- using the appropriate exact analytical solution for each tank;
- using numerical solutions to equations [1] and [2], executed in MS-Excel®;
- using a modern drag-and-drop icon-based simulation package known as Stella [6];
- statistically, via non-linear OLS curve-fitting.

The first three methods were presented in the lecture-component of the course as examples of 'deterministic modelling', the last as an example of 'non-deterministic modelling'.

Note on Physical modelling

This component has been described above. Low heads corrections to weir behaviour were not required of the students, although a couple of the more perspicacious ones asked if they should include this effect. A great deal is known about the physical modelling of hydraulic phenomena occurring in open channels (rivers) and over structures (such as spillways), which are usually based on Froude scaling laws. It does not appear that much work has been done on inferring the behaviour of real reservoirs using model reservoirs. This may be investigated at a future date, especially with regard to the trapping of model sediment. At this stage the students were not required to make any such inferences; the outcomes of the laboratory work were taken and used at face value.

Analytical solution

As previously mentioned, the surface area of these three reservoirs was designed to increase in such a way that the non-linearity of the outlet (a triangular weir) was negated. The closed-form

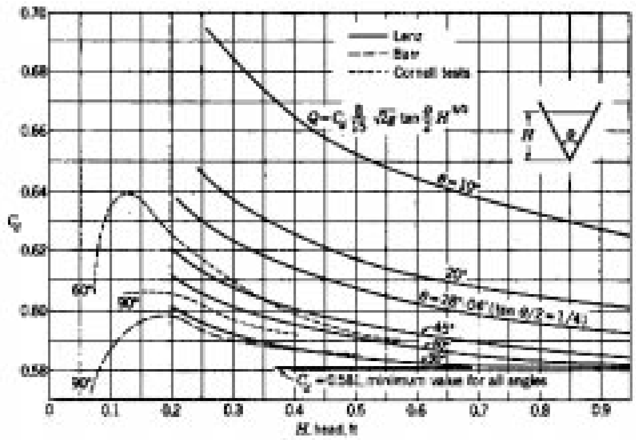

Fig. 4. Variation in weir coefficient with head (after Daugherty and Franzini [5]).

solution for the discharge from the first tank is (see Appendix C):

$$Q_{out} = \frac{s_o}{K}\exp\left(-\frac{t}{K}\right) \quad \text{(C-1)}$$

The closed-form solution for the discharge leaving the second tank is (see Appendix C):

$$Q_{out} = \frac{s_o}{K^2}t\exp\left(-\frac{t}{K}\right) \quad \text{(C-11)}$$

The solution for n^{th} tank is (see Appendix C):

$$\frac{Q}{Q_{max}} = \left(\frac{t}{t_{Q\,max}}\right)^m \exp\left(\frac{t_{Q\,max} - t}{t_g - t_{Q\,max}}\right) \quad \text{(C-21)}$$

where: $m = t_{Q\,max}/(t_g - t_{Q\,max})$.

Discharge hydrographs computed using the above equations could therefore be compared with the hydrographs obtained from the laboratory measurements.

Numerical modelling

There are many well-known numerical schemes for solving both ordinary differential equations (Orvis [7]) and partial differential equations (Hansen [8], Hansen and Droste [9], Olsthoorn [10], Townsend *et al.* [11]) that can be executed efficiently in spreadsheets such as Excel® (see also Wolff [12]). In this case the method of Euler, as well as Heun's improvement upon it, (Chapra and Canale [13]) were applied to the differential equation in question (for the hydrologic theory see Bedient and Huber [14]):

$$\frac{dh}{dt} = \frac{Q_{in} - ph^q}{A_R} = f(h,t) \quad \text{(4a)}$$

$$A_R = mh^n \quad \text{(4b)}$$

where:

h = depth above the invert of the weir (L);
Q_{in} = inflow hydrograph to the tank in question (L^3/T);
p & q = empirical parameters governing outflow hydraulic (a triangular weir herein, so $q = 2.5$);
A_R = surface area of the reservoir at a given depth h;
m and n = empirical parameters relating A_R to h. In this case the flair of the side-walls was adjusted so that $m = 48$ and $n = 1.5$ (for A_R in cm^2).

The Euler-Heun algorithm can be efficiently executed in the tabular form for which spreadsheets are famous (see Tables 2a and 2b). Modern desktop computer CPUs are so fast that there now seems to be little interest in the relative efficiency of algorithms used to solve many civil engineering problems. In addition, we feel that it is more important in an educational setting that (i) students implement the relevant mathematics personally and pseudo-manually (not using black-box software), and that (ii) students be able to implement the mathematics efficiently. It seems that an excessive amount of time is often spent debugging conventional code, necessitating the impartation of fewer numerical methods and the assigning of fewer problems. This aspect is a very important consideration when teaching engineering students because their academic load is quite heavy.

The result of the first numerical solution is the outflow hydrograph from the first tank. This becomes the inflow hydrograph to the second tank, and so on.

Statistically

One generic aspect of CIVL4720 is the use of OLS curve fitting and nonlinear transformations to describe processes non-deterministically. The solution to the outflow from the first tank will be used to demonstrate what the students did in this regard. The analytic solution is:

$$Q_{out} = \frac{s_o}{K}\exp\left(-\frac{t}{K}\right) \quad \text{(5a)}$$

It is therefore appropriate to regress $\ln(Q_{out})$, as the dependent variable, against time as the

Table 2a. Tabular execution of level-pool reservoir routing, using the Euler method to solve Equation (4)

Time (sec)	Inflow Q_{in} (cm^3/s)	Head in tank 1 (m)	Outlow Q_{out} (cm^3/s)	Area A_R (cm^2)	Slope f(h, t)	$h(t + \Delta t)$ (cm)
0	0					
2						
etc						

Table 2b. Tabular execution of level-pool reservoir routing, using the Euler-Heun method to solve Equation (4)

Time (sec)	Inflow Q_{in} (cm^3/s)	Head in tank 1 (cm)	Outlow Q_{out} (cm^3/s)	Area A_R (cm^2)	1st slope f(h, t)	Revised Q_{out}	Revised A_R	2nd slope f(h, t)	average $\bar{f}$	$h(t + \Delta t)$
0	0									
2										
etc										etc

independent variable. The nominal or apparent slope S_A associated with the linear regression outcome can be used to obtain K according to:

$$-\frac{1}{K} = S_A \quad (5b)$$

The nominal or apparent intercept I_A associated with the linear regression outcome can be used to obtain s_o/K found in Equation (5a), according to:

$$\frac{s_o}{K} = \exp(I_A) \quad (5c)$$

The student used about 30 data points, collected over 15 minutes, to compare independent estimates of s_o/K to those found using the actual initial volume s_o and measurements of the tank(s). Similar efforts were applied to the transforms and lumped parameters implied by equations C-11 and C-21.

USING SYSTEMS SIMULATION SOFTWARE (STELLA®)

STELLA® is an environment for constructing and interacting with models. It has two main layers, the 'high-level mapping' layer and 'model construction' layer. The former is used to create a system map that identifies the important operatives in a given system. The latter, which was the only layer used for the application described herein, makes it possible to develop a detailed representation of the individual processes in the system being considered. Instead of the lengthy and complex codes used by conventional programming environments, the STELLA® environment uses only four icons to achieve system representation. These four icons represent state variables (Stocks), activities in the system (Flows), conversion of inputs into outputs (Converters), and information transmission between the other three elements of the model (Connectors). This not only simplifies model building, it provides an enhanced interactive environment between the model and the model builder.

STELLA® has been used to model the learning process itself. Eftekhar *et al.* [16] used it to determine the effect that structure, time delays and policies have on the amount that undergraduate students learn (the stock in STELLA® was 'the amount learned'). The same authors subsequently investigated the effects of student

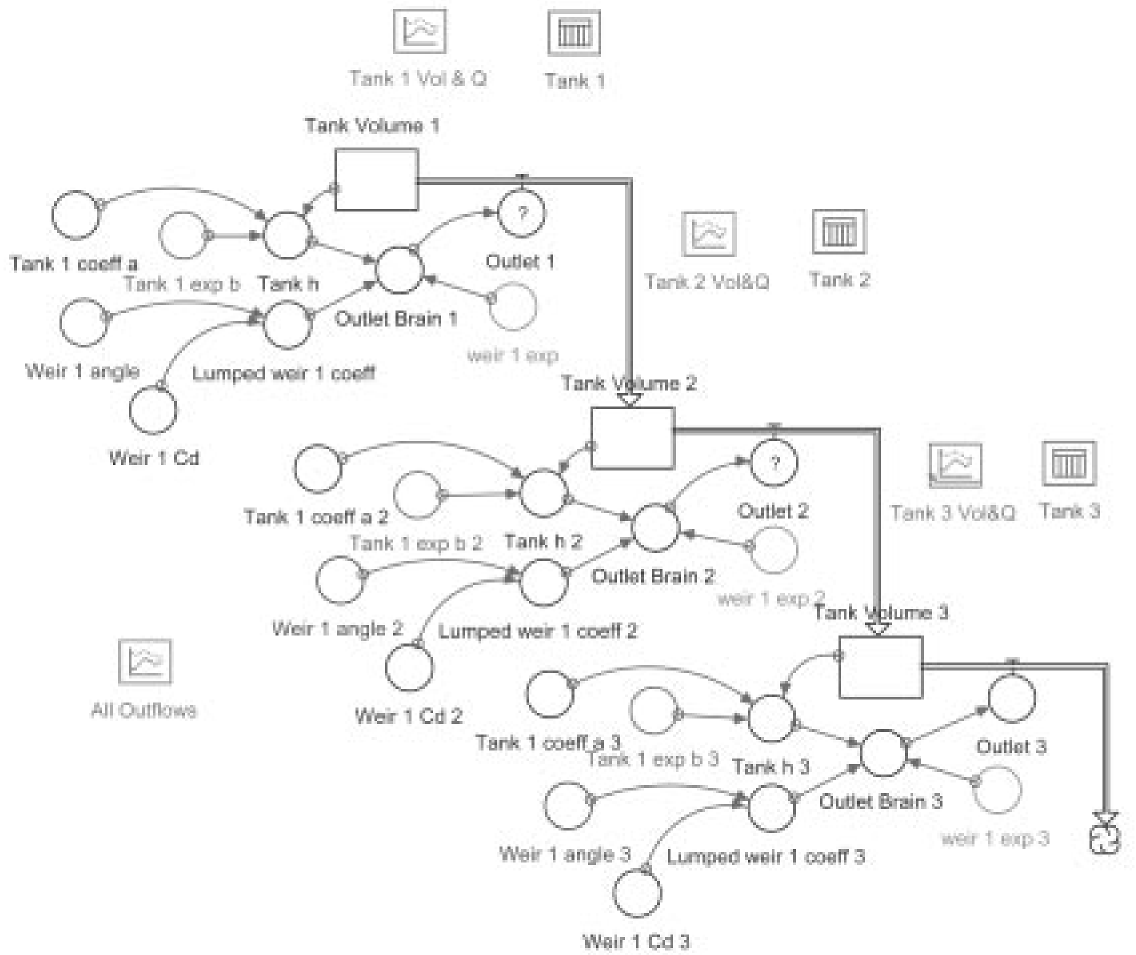

Fig. 5. Template for the reservoir cascade created using STELLA® systems simulation software. (The template becomes animated during simulation, with the tanks filling and emptying.)

values, external reinforcement, and student effort on learning. In a more environmental vein, Ndegwa *et al.* [17] used STELLA® to simulate the biological and nutrient kinetics of composted manure, successfully mimicking its cyclical changes in temperature and moisture content. The diversity in the applications cited underlines the fact that STELLA® falls in a class of relatively new generic software, one that has both significant pedagogic potential and practical use.

OUTCOMES

We were reasonably pleased with the students' reports, especially considering that this was the first time that this experiment had been attempted. The requirements as to what the report had to contain were stated in too general a fashion. Instructions to 'compare outcomes' were generally not well-executed by the students. It was assumed that the general idea of modelling a physically observable phenomenon using theories, as compared to approximations to theories, was already understood. This was apparently not uniformly the case. Many students seemed to treat all of the outcomes, including the physical modelling effort, as having completely equal validity and significance, in that many students did not seem to treat the experimental data as the ultimate basis for making comparisons. They were also weak in their appreciation of (i) the role of errors in their physical measurements on the outcomes, and of (ii) the idea that parameter estimates arising from modelling efforts might not be perfect, and might in fact be honed or calibrated in order to improve the agreement between computed and observed outcomes. Appendix A presents a statement of how the initial data collection and processing was to be executed. A progress report was required shortly afterwards, so as to spread out the work in a more explicit manner (students being notoriously poor at beginning the analysis of fresh data in a timely manner). Appendix B is the statement of what was expected of each group—a formal report in which all the methods used to model the routing phenomenon were to be compared. Appendix C is a classical mathematical derivation of the expression for the hydrograph from the n^{th} tank of a linear cascade (expanded from Viessman *et al.* [15]).

STUDENT REACTION

The experiment and associated report-writing requirement were not favourably rated in the formal evaluation of the course (conducted by the faculty). Informal discussions that took place some months later indicated that students felt that although the exercise was valuable, it was too much work for the marks allocated, and that the evaluations had been harsh. Recent comments from two of the more mature students in the class were as follows:

> 'Through completing the linear reservoir modelling exercise we uncovered many principles and concepts that could only be discovered through self-directed investigation and shared learning among the team. One reward was the exposure of the advantages and limitations of each modelling approach, and how these were controlled by varying model controls and input parameters.'

> 'All too often computer modelling becomes a poorly understood "black box" situation, but by applying the same numerical algorithm in spreadsheets as was used in STELLA®, a greater understanding of both the advantages and the disadvantages of the software was gained. This learning process was complimented by using non-deterministic methods to gain an understanding of their own usefulness and limitations. It was gratifying to find that the same approximate coefficients as were used in the theoretical solution could be recovered from the non-deterministic model.'

There were no complaints of cognitive fragmentation, possibly because most of the tools that were needed to do the modelling were provided along the way, in various forms. The students did not, however, successfully use all of the available tools in all cases. There did not appear to be any 'rhyme or reason' to this selectivity.

CONCLUSIONS

Hydrologic routing was successfully used as a vehicle to introduce civil engineering students to the idea that there are many ways to simulate a phenomenon. The thought processes and decisions associated with such modelling efforts appear to be analogous to the thought processes associated with design, which is highly PBL in its essence. The data collection of time-varying water levels via a digital camera was reasonably successful (achieving the right lighting was found to be difficult), but might in future be simplified by creating a digital video clip that could be mounted on the WebCT site for the course. Many of the students were very intrigued by the possibilities implied by the STELLA® systems simulation software. The new experiment was a qualified success and will be used on a regular basis. The MS-Word® file for this article, as well as the AutoCAD files for the linear tank design, can be obtained for free from the first author.

REFERENCES

1. D. Hansen and G. H. George, A multi-facetted approach to teaching modelling, *via* hydrologic routing, *12th Canadian Conf. on Eng. Education*, (2001) pp. 63–73.
2. G. E. Dieter, *Engineering Design, a Materials and Processing Approach*, McGraw-Hill, NY (1991).
3. M. Savin-Baden, *Problem-Based Learning in Higher Education: Untold Stories*, Society for Research into Higher Education and Open University Press, UK (2000).
4. W. E. Whiteman and K. P. Nygren, Achieving the right balance: properly integrating math software packages into engineering education, *ASEE J. of Eng. Educ.*, **89**(3), 2000, pp. 331–336.
5. R. L. Daugherty and J. B. Franzini, Fluid Mechanics with Engineering Applications, McGraw-Hill, NY, (1977).
6. *High Performance Systems*, *STELLA® Systems Simulation Software*, High Performance Systems, Inc., Lebanon, NH 03755 (2000). See also http://www.hps-inc.com/edu/phy_and_eng/stella.htm.
7. W. J. Orvis, *1-2-3 for Scientists and Engineers*, Sybex, San Francisco (1987).
8. D. Hansen, Using electronic spreadsheets to solve heat conduction and seepage problems, *Proc. of 1st Prov. Conference on Computer-Aided-Learning and Computer-Managed-Instruction*, Memorial University of Nfld, (1992) pp. 23–25.
9. D. Hansen and R. Droste, Counteracting numerical dispersion when using an electronic spreadsheet to perform 1-D advection-dispersion studies, *First Canadian Environmental Eng. Conference*, Vol. II-2, (1990) pp. 834–850.
10. T. N. Olsthoorn, Computer notes—the power of the electronic worksheet: modelling without special programs, *Groundwater*, May-June, (1985) pp. 381–390.
11. D. R., Townsend, V. K. Garga and D. Hansen, Finite difference modelling of the variation in piezometric head within a rockfill embankment, *Canadian Journal of Civil Engineering*, **18**(2), (1991) pp. 254–263.
12. T. F. Wolff, *Spreadsheet Applications in Geotechnical Engineering*, PWS Publishing Ltd, Boston (1995).
13. S. C. Chapra and P. Canale, *Numerical Methods for Engineers*, 2nd ed., McGraw-Hill, NY (1988).
14. P. B. Bedient and W. C. Huber, *Hydrology & Floodplain Analysis*, 2nd ed., Addison-Wesley, Reading MA (1992).
15. W. Viessman and G. L. Lewis, *Introduction of Hydrology*, 4th ed., Harper-Collins, NY (1996).
16. N. Eftekhar, D.R. Strong and O. Hawaleshka, Towards dynamic modelling of a teaching-learning system: 1. The unified theory, *Int. J. Eng. Educ.*, **12**(6), 1997.
17. P. M. Ndegwa, S. A. Thompson and W. C. Merka, A dynamic simulation model of *in situ* composting of caged layered manure, *Composting Science and Utilization*, **8**(3), (2000) pp. 190–202.

APPENDIX A

Data collection from a physical model of a linear reservoir cascade

CIVL4720 Civil Engineering Computations Laboratory Assignment
Today you will photographically record the behaviour of a physical model that represents a reservoir cascade. You are not restricted to recording the temporal variation in water levels by this method only. In this case the cascade is a series of three aluminium tanks arranged horizontally in a series. These reservoirs have a special shape and each has a weir at the outlet.

1. Ensure that the above apparatus is level, so that the tank outlets are vertical.
2. Use a black dry-erase marker to write the date and the Trial No. on the small white-board above the piezobank.
3. Insert the triangular dam into the weir opening of the (empty) top reservoir.
4. Use the hose to fill the top reservoir nearly to the top. Use the tap (hand-valve) in the SE corner of the lab to control the fill-up.
5. Put the clock back on the stand, plug it in, and zero it. Turn on the light and hold it near (but not in front of) the clock. The light is important in illuminating the clock face, so as to be clearly seen in the photos (ie. you will use a series of jpg's to read off the times so the clock face needs to be well-lit).
6. Get the photographer in position. You need a photo at $t=0^-$. Measure the starting water level in the top tank.
7. Have someone simultaneously pull the plug and flick the start-switch on the clock.
8. Take a sequence of pictures to document what happens with respect to water level variation in the three reservoirs, through time. Note any unusual behaviour.

Before you leave:

- Make sure you understand how to read the clock-face. (The major divisions are not minutes and the smallest divisions are not seconds).
- Make sure you understand how to read the scale on the piezobank.
- Determine what piezometer reading is associated with zero flow in each case.
- Measure all relevant physical dimensions that, in your opinion, affect the behaviour of the cascade (using a ruler, calipers, etc.)

Fig. 6. The cascade of linear reservoirs (weir plates not installed).

Initial processing and presentation of raw data:

1. Find the governing equation for a weir in a fluid mechanics or hydraulic structures textbook (stated as flow, Q, as a function of depth *h* over the invert). These books are in the TA347 and TC5 sections of our library. Write down the reference for the book(s) in formal academic citation style. Study the part of the text associated with the governing equation and write down all of the assumptions that are inherent to it.
2. The TA will e-mail the jpg images to you. Use a paper printout of these jpg's to obtain a series of water levels (heads) and times (by reading the clock in each view). Separate your data-sets by tank. Your jpg's can be viewed by MS Photo-editor or Paint. Increasing the brightness and contrast (along with the percent magnification) may assist you in reading the times from the clock.
3. Convert the heads to outflows using the weir equation from step (i).
4. Plot the variation in water level, over time, for all three tanks (on one page). Make the ordinate depth in cm and the abscissa time in minutes. Also plot the three outflow hydrographs (differentiating them by tank) on a single page. Make the ordinate flow in cm^3/s and the abscissa time in minutes.
5. Prepare a schematic of the experimental set-up, showing dimensions and distances. Pass in (to the office, with the TA's name on it) the two graphs, your schematic, and just one of your jpg views by Friday of the same week, for a preliminary evaluation. Use this handout as your cover page.

APPENDIX B

Modelling a linear cascade via level-pool routing

Laboratory #2

1.0 *Introduction.* Explain what the words 'level-pool routing' mean. Give real-world example(s) of a cascade. Contextualise your laboratory work by indicating the general applicability of the phenomena being observed. Who would be interested in this phenomenon, in general? Indicate how practitioners would use this type of data if it had been collected from a prototype-scale case.

2.0 *Objectives*. Present your objectives as a list. Use this list to explain the scientific issues being addressed. Essentially, your purpose is to simulate the behaviour of a sequence of small reservoirs. See the material below regarding the additional details that are of interest. State all goals, great and small, in the language of 'Objectives'. Use a narrower point of view than that found in section 1.0.

3.0 *Method of Investigation.* Describe the physical model (apparatus) both qualitatively and quantitatively. Present a labelled schematic of your apparatus and show a representative photo. Describe how the apparatus works. Give the procedure that you followed for your experiments.

4.0 *Analysis.* Present the relevant theories. State and explain the meaning of the equations that you used to interpret your data (see guidelines on presenting equations). State any implicit limitations or assumptions associated with the equations/theory.

- Deterministic Approach 1. Briefly present the mathematical basis for the analytical solutions, using nomenclature appropriate to this particular problem.
- Deterministic Approach 2. Present the mathematical basis for the Euler-Heun Method, using nomenclature appropriate to this particular problem.
- The STELLA® software package. Present your template and explain the meaning and function of its components. Describe how you applied this software to model this phenomenon and its sub-components.
- Non-deterministic Approach. State the transformations (if any) that you used to obtain OLS-based 'best-fit' equations to describe your experimental outcomes. In some cases did you to minimise the sum of the squared errors without the use of transformations, and if so, why? Mathematically demonstrate the physical significance of the regression constants, where applicable.

5.0 *Presentation and Discussion of Results***.** Interleave within this section those representative figures/tables that support your most important results (the rest of the figures may be placed in an appendix).

- Deterministic approach 1. Present graphical comparisons* (observed and simulated hydrographs), and discuss.
- Deterministic approach 2. Present graphical comparisons (observed and simulated hydrographs) and discuss.
- Simulation software. Show how your Stella-based model properly demonstrated how the physical characteristics of the cascade affect its simulated behaviour. Explain how these 'control outcomes' make sense.
- Non-deterministic approach. Compare constants having physical significance. What can you infer about the system?
- Compare the above outcomes. Show the effect of changing Δt (where applicable). Use the analytical solutions to quantify the magnitude of your numerical errors. Use an objective measure of overall goodness-of-performance to quantitatively evaluate your degree of success in simulating the observed phenomenon. Is there a best model? Where possible, use the modelling techniques to infer the values of any physical constants or lumped parameters. Compare the inferred and observed constants. Discuss the effect of adjusting those model controls that appear to improve the predictions, such as Δt and the type of numerical method. In light of how the data was collected and your imperfect knowledge of the physical system tested, is there a point at which you would not be able to recognise an outcome as being better, had it been an experimental outcome? Did any refinements to the underlying physics prove not to be necessary or helpful?

6.0 *Summary and Conclusions*

7.0 *References*

8.0 *Appendices*

- Supporting Mathematical Derivations (if any).
- Supporting Figures.
- Further details on Laboratory Set-up (if still required, in light of Section 3.0).
- Supporting Tables and Computer Printouts (computer printouts are best summarised as neatly-done and easily-read tables, do not include them 'raw').

* Important: Follow the guidelines provided on the presentation of graphs and tables.

APPENDIX C

Deriving a general expression for the outflow from a cascade of linear reservoirs

Adapted by D.H. and G.H.G. from pp. 223–4 of Viessman & Lewis, Introduction to Hydrology, 4th ed., Harper-Collins.

In general the conservation of the volume in a reservoir dictates that:

$$Q_{in} - Q_{out} = \frac{ds}{dt} \tag{C-1}$$

where Q_{in} is the inflow, Q_{out} is the outflow, and ds/dt is the rate of change of storage volume in the reservoir. For a linear reservoir:

$$s = KQ_{out} \tag{C-2}$$

If Equation (C-2) governs the behaviour of the reservoir (not the case for most reservoirs), from Equation (C-1) we may write:

$$Q_{in} - Q_{out} = K\frac{dQ_{out}}{dt} \tag{C-3}$$

First reservoir. Let the first reservoir have an instantaneous volume appear in it, so $Q_{in} = 0$ after $t = 0$.

$$-Q_{out} = \left.\frac{ds}{dt}\right|_{t>0} \tag{C-4}$$

Substituting (C-2) into (C-4) gives:

$$-Q_{out} = K\left.\frac{dQ_{out}}{dt}\right|_{t>0} \tag{C-5}$$

$$\text{or:}\quad \frac{Q_{out}}{dQ_{out}} = -K\left.\frac{1}{dt}\right|_{t>0} \qquad \text{or:}\quad \frac{dQ_{out}}{Q_{out}} = -\frac{1}{K}\,dt \tag{C-6}$$

Integrating (C-6) from $t = 0^+$ to time t:

$$\int_{0^+}^{t} \frac{1}{Q_{out}}\,dQ_{out} = -\frac{1}{K}\int_{0^+}^{t} dt$$

$$\ln Q_{out} - \ln Q_{out}|_{t=0^+} = -\frac{1}{K} \qquad \text{or:}\quad \ln\left(\frac{Q_{out}}{Q_{out}|_{t=0^+}}\right) = -\frac{t}{K} \qquad \text{or:}\quad \frac{Q_{out}}{Q_{out}|_{t=0^+}} = \exp\left(-\frac{t}{K}\right) \tag{C-7}$$

From (C-2) $Q_{out} = s/K$ so it is also true that:

$$Q_{out}|_{t=0^+} = \frac{s_o}{K} \tag{C-8}$$

Substituting (C-8) into (C-7) gives:

$$\frac{Q_{out}}{s_o/K} = \exp\left(-\frac{t}{K}\right)$$

$$\text{or:}\quad Q_{out} = \frac{s_o}{K}\exp\left(-\frac{t}{K}\right) \tag{C-9}$$

Note: If we want to know the volume drained after a given elapsed time for tank 1, from equation (C-9):

$$\forall = \int Q_{out}\,dt = \int_0^t \frac{s_o}{K}\exp\left(-\frac{1}{K}t\right)dt$$

Denoting κ as s_o/K and $\beta = 1/K$:

$$\forall = \int_0^t \kappa\exp(-\beta t)\,dt = -\frac{\kappa}{\beta}\exp(-\beta t)\Big|_0^t = -\frac{\kappa}{\beta}\exp(-\beta t) + \frac{\kappa}{\beta}$$

$$\forall = \frac{\kappa}{\beta} - \frac{\kappa}{\beta}\exp(-\beta t)$$

Using the definitions:

$$\forall = s_o\left[1 - \exp\left(-\frac{1}{K}t\right)\right]$$

Second reservoir. Let us now route this exponentially-decaying flow coming out of reservoir #1 (described by equation (C-9)) through reservoir #2 (i.e. 'pour' this outflow into tank #2). Substituting (C-9) into (C-3) with the Q_{out} of (C-9) as the new Q_{in} gives:

$$\frac{s_o}{K}\exp\left(-\frac{t}{K}\right) - Q_{out} = K\frac{dQ_{out}}{dt} \tag{C-10}$$

Solving using an integrating factor gives:

$$Q_{out} = \frac{s_o}{K^2}t\exp\left(-\frac{t}{K}\right) \tag{C-11}$$

In the limit (3rd reservoir and beyond). Repeating the entire procedure *n* times gives the following equation for the general outflow Q:

$$Q = \frac{s_o}{(n-1)!K^n}t^{n-1}\exp\left(-\frac{t}{K}\right) \tag{C-12}$$

Note that the well-known two-parameter Γ function (with $s_o = 1$) is:

$$Q = \frac{1}{\Gamma(n)K^n}t^{n-1}\exp\left(-\frac{t}{K}\right) \tag{C-13}$$

The same function with an initial volume s_o is then:

$$Q = \frac{s_o}{\Gamma(n)K^n}t^{n-1}\exp\left(-\frac{t}{K}\right) \tag{C-14}$$

The PMF of the Γ distribution (associated with equation (C-13)) is more commonly written:

$$f(x;\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right) \tag{C-15}$$

Comparing (C-14) and (C-15), it is obvious that time t is x, $n = \alpha$ and $K = \beta$.

Applying (C-14) to the peak flow of a given hydrograph:

$$Q_{max} = \frac{s_o}{\Gamma(n)K^n}t_{Q\,max}^{n-1}\exp\left(-\frac{t_{Q\,max}}{K}\right) \tag{C-16}$$

Dividing (C-14) by (C-16):

$$\frac{Q}{Q_{max}} = \left(\frac{t}{t_{Q\,max}}\right)^{n-1}\exp\left(\frac{t_{Q\,max} - t}{K}\right) \tag{C-17}$$

The expected value of equation (C-15) is

$$t_g = \alpha\beta = nK \tag{C-18}$$

The mode of the gamma distribution is:

$$x_{mod\,e} = (\alpha - 1)\beta$$

or, in hydrograph routing nomenclature:

$$t_{Q\,max} = (n-1)K \tag{C-19}$$

The difference between the mean and the mode is then:

$$t_g - t_{Q\,max} = K$$

The exponent $n - 1$ in equation (C-17) is:

$$n - 1 = \frac{t_{Q\,max}}{K} = \frac{t_{Q\,max}}{t_g - t_{Q\,max}} \tag{C-20}$$

Equations (C-17) and (C-18) lead to:

$$\frac{Q}{Q_{max}} = \left(\frac{t}{t_{Q\,max}}\right)^m \exp\left(\frac{t_{Q\,max} - t}{t_g - t_{Q\,max}}\right) \quad \text{where} \quad m = \frac{t_{Q\,max}}{t_g - t_{Q\,max}} \tag{C-21}$$

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William Cavers, B.Eng. Civil Engineering (2002), has very diverse interests in civil engineering and is now a graduate student in the Department of Engineering Mathematics at Dalhousie University. His senior undergraduate project involved the stress analysis of capped pile foundations and he is presently working on the application of random field modelling to geotechnical problems, such as differential settlement. He hopes to become involved in the offshore oil and gas industry in the near future.

Glyn George is an associate professor of mathematics in the Faculty of Engineering and Applied Science at the Memorial University of Newfoundland. He earned a B.Sc. in mathematics from the University of Southampton (England) and a Ph.D. in astronomy from the University of Wales. He holds a certificate in education (further education, CNAA, U.K.) and has been a Fellow of the Royal Astronomical Society (U.K.) since 1984. His research interests have been in probability, as applied to problems in astronomy, hydrology and engineering education. He has teaching experience in the United Kingdom, Bahrain, and Canada. He has served for over seven years as a school board member and elected officer in Newfoundland and most recently as the provincial secretary of the Newfoundland and Labrador Federation of School Councils.