Design Project for Advanced Mechanics of Materials*

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A semester-long team design project was implemented in an advanced mechanics of materials course for both undergraduate and graduate students. The project involved the mechanics of a bicycle crank arm and built on earlier coursework. Student teams developed specifications and design criteria, performed analysis and design, developed validation experiments, compared experimental results with predictions, and reviewed finite element model predictions from students in a different course. Students completed an anonymous survey at the end of the project, which indicated that learning experiences obtained from the project were valued as much as listening to lectures and working homework problems. The survey results also indicate that undergraduate and graduate students have different attitudes about learning methods; with graduate students valuing a much wider range of learning methods.

INTRODUCTION

ALMOST ALL engineering colleges in the United States have a department that teaches engineering mechanics courses, whether it is Engineering Science and Mechanics, Mechanical Engineering, Civil Engineering, or some other department. It is common for this department to offer an advanced mechanics of materials course that picks up where the elementary mechanics of materials course ends. Topics for such a course include:

- stress and strain
- material laws
- energy methods
- complex stress states
- failure prediction
- plane elasticity
- thick-walled cylinders and disks
- unsymmetrical bending of beams
- torsion of noncircular sections
- plates and shells
- beams on elastic foundations
- curved beams
- plastic buckling, etc.

Obviously, not all of these topics can be thoroughly covered in one semester so an instructor has plenty of leeway to pick and choose the topics that best fit the curriculum of their department.

The Department of Engineering Science and Mechanics at Penn State offers a senior level technical elective entitled, ‘E Mch 400—Advanced Strength of Materials and Design.’ Additionally, the department offers a graduate course, ‘E Mch 500—Advanced Mechanics of Materials.’ In the fall semester of 2001 these two courses were taught together, that is students attended the same lectures three times a week. In this semester, 15 students took E Mch 400 (two were graduate students) and 15 students took E Mch 500 (two were undergraduate students participating in an integrated undergraduate-graduate program). The course demographics are shown in Table 1.

These courses follow the elementary mechanics of materials course and introduce the field equations of mechanics, covering:

- principal stresses and maximum shear stress
- failure criteria
- energy methods
- torsion of noncircular sections
- shear flow
- unsymmetrical bending
- thick-walled cylinders and disks
- rudiments of plates and shells.

In addition to learning these topics, engineering students need to learn how to design [1, 2]. We have introduced design into a course traditionally laden with analysis by implementing a semester-long team design project. Students earned grades based upon: participation in class activities, homework assignments, in-class tests, the final exam, and the design project. E Mch 400 and E Mch 500 were different courses in that more advanced homework and test problems were given to the E Mch 500 students. Furthermore, E Mch 400 and E Mch 500 students were assigned different components of the team design project. The department also offers ‘E Mch 461—Applied Finite Element Analysis’. Students in this class conducted a simulation phase of the project.

The design project dealt with the crank arm of a bicycle. Our reason for choosing this project will be discussed later, but the bicycle is a wonderful tool for studying mathematics, science,
and engineering because students are familiar with its function. Bicycles are readily available, easily taken apart, and moderately inexpensive (at least some of them). For these reasons bicycles have been used as teaching tools for years [e.g., 3–5].

This paper discusses the design project and is subdivided into the following sections: development, implementation, and results. We close with conclusions. While the paper focuses on mechanics of materials and design of a crank arm, we hope that readers from other disciplines will be able to generalize the approach and apply at least some of the ideas to their particular area of interest.

**PROJECT SELECTION AND OBJECTIVES**

We defined the course objectives and set overall objectives for the design component in order to select a design project that will serve in the achievement of both. Then with the project chosen, we set detailed learning objectives to guide its completion.

**Course objectives**

Advanced mechanics of materials students will be able to:

- Develop models of mechanical components by making reasonable assumptions and writing appropriate equations and then solving them.
- Apply appropriate failure criteria.
- Formulate a design methodology and use it to do design.

**Overall design objectives**

The design component of the course will enable students to:

- Apply the principles learned during the course directly to a practical problem.
- Apply basic knowledge in addition to what is learned in the course to solve an engineering problem.

- Learn to design and conduct experiments, as well as analyze and interpret data.
- Function on multidisciplinary teams, hence communicate with team members and learn professional and ethical responsibility.

Clearly we want to expose students to the three primary areas of mechanics—experimentation, theory, and computation—in an integrated fashion through an active exploration of a common product. With all of these things in mind, we brainstormed to identify a list of potential topics [6] and then condensed it to four: automotive connecting rod, golf club, bicycle frame, and drive shaft. However, a conversation with Mark Laplante [7], the lead test engineer at a bicycle assembly plant, revealed learning opportunities offered by studying a bicycle crank arm. To start with, there are material selection issues. Since it is a rotating part, least weight is important, but so is durability, wear resistance, corrosion resistance, and fatigue resistance. Fabrication methods and costs are also major factors, as are aesthetics. Predicting the mechanical response of a crank arm is difficult because of the complex stress state associated with combined axial force, bending about two axes, transverse shear, and torsion all being present in a member that is often nonprismatic and unsymmetrical. The stress state at any point in the crank arm is dependent on the current crank orientation angle. Furthermore, the loading is cyclic and highly variable depending on the terrain. Design of a crank arm requires one to define the specifications on expected operating load, target lifetime, overload conditions, and factor of safety—a significant challenge! (Interestingly enough, Mr. Laplante uses ability to describe the stress state in a crank arm as a way to screen job applicants.)

For this course, a project involving the design of a crank arm for a bicycle was selected because of the familiarity that students have with it, its simple function, its interesting and common design dilemmas, and because the analysis can range from very simple to very complex. We recognize that many engineering failures are associated with connections. However, design of the connections of a crank arm to the pedal and the crank shaft was not part of the project due to the focus on topics chosen for the course.

**Detailed project learning objectives**

Students working on the project will be able to:

1. Formulate a model for a bicycle crank arm having a straight, prismatic cross-section.
2. Analyze the model to determine stresses.
3. Design and conduct experiments to validate analytical predictions.
4. Design a suitable cross-section by applying appropriate failure criteria.
5. Develop interaction skills for working efficiently in teams.
This particular project helps achieve all of the above objectives and eight of the eleven outcomes listed by the Accreditation Board for Engineering and Technology (ABET).

**PROJECT IMPLEMENTATION**

A bicycle crank arm is a critical component of the drive mechanism as it transmits the force generated by the rider and his or her weight to the crankshaft on which the chain wheel is mounted. Failure of a crank arm can cause serious injury to the rider. In 1997, Shimano American Corporation of Irvine, California received more than 630 reports of crank arm failures in North America resulting in 22 injuries, including cuts and fractures leading to a recall of more than 1 million crank arms installed on bicycles in North America [9, 10]. Replacement of these cranks was a significant cost burden to the company and highlights the importance of good design.

For logistical reasons, the project is subdivided into four significant tasks (see Project Tasks below). The division is such that tasks requiring general knowledge and application of fundamentals of mechanics of materials can be initiated at the beginning of the course, while subsequent tasks build on these as students progressively apply principles learned in class, such as failure criteria, fatigue, torsion analysis and finite element analysis. This also facilitates assigning different tasks to students with different experience.

Teams were chosen during the first two weeks of class by the instructor with the aim of making them as multidisciplinary as possible. Each team had two E Mch 400 students and two E Mch 500 students. An attempt was made to pair up international students from the same country to enhance communications and ease cultural differences. This strategy worked well in general, but one team with two Chinese and two American students experienced difficulties. Early in the semester class time was allotted for team discussion of design related issues. Later this was curtailed.

Each team was required to keep a written record of all work done on the design project. This record contains notes from meetings, lists from brainstorming sessions, work from individual team members, analysis calculations, reference material, and a record of who did what; and was turned in periodically for grade. The record was kept in a notebook intended to help keep the team’s work organized and shared equally amongst the members as well as provide a safe means of transport for design documents.

**PROJECT TASKS**

Each set of task requirements is stated as presented to the students and followed by the basis for it.

**Part A: Design of crank arm of circular cross section**

A.1. Students were required to: Develop specifications for design of the crank arm. Include for example: estimated loads, size restrictions, material and manufacturing, possible failure modes.

Basis—The first step in a design problem is to develop design criteria based on the target market, function, materials, mechanics, and cost of the process. It helps define clear objectives for the design and enables the selection of a simplified model for analysis based on the requirements of design.

A.2. Students were required to: Analyze a left crank arm having a circular cross section; including the following:

- a clear and complete free body diagram;
- internal force calculations and load diagrams;
- stress calculations for any point in the cross-section due to axial force, bending about both axes, shear from transverse load, torsion, and effective stress;
- plot von Mises effective stress as a function of crank orientation angle for 8 points on the circumference of the cross-section;
- determine the critical condition, i.e., if effective stress is the critical quantity, for what crank orientation angle and where in the cross-section does the maximum effective stress occur.

Basis—A simplified model of the crank arm having a straight circular cross-section is considered for analysis. Since elementary mechanics of materials is a prerequisite to the E Mch 400 and E Mch 500 courses, the students have the tools necessary to analyze this simplified model at the start of the semester. Each subtask listed above helps define the steps involved in the analysis of a straight circular crank arm. Analysis of the circular cross-section enables students to understand kinetics of the crank arm.

A.3. Students were required to: Design a prismatic crank having a circular cross section. As a minimum, consider fatigue under normal operating conditions and ensure that the crank remains elastic during occasional overloads. Present the final design in a report that includes all final specifications and design criteria, calculations and analysis, as well as dimensions. E Mch 400 students should lead the design for overload and E Mch 500 students should lead the design to prevent fatigue failure.

Basis—Students must efficiently use fundamental analysis tools to determine critical conditions and select an optimal diameter given the loading conditions specified and the material chosen.

**Part B: Analysis validation—E Mch 400 only**

B.1. Suggest a laboratory experiment or set of experiments that will validate your crank arm analysis. (Note that this is not intended to be field testing of a prototype.) The solid model of the 25 mm diameter crank arm is shown in Fig. 1.
B.2. (Assigned after B.1. was submitted.) Validate the crank analysis with experimental results. Compare measured strains with predicted strains at points A and B shown in Fig. 1. Consider crank orientation angles of 0, 45, 90, 135, and 180 degrees as measured from the vertical.

*Basis*—Pro/Engineer [11] solid modeling software is used to create an engineering design of the cross-section achieved from the analysis (although this is done by a teaching assistant, not the students). This is converted into a machining drawing and a prototype is built using CNC machining. Tests are done to measure strains at certain points on the circumference of the crank arm for different loading conditions to compare with the theoretical strains achieved from the analysis. The cost of the prototype was $500 whereas the commercial part is probably not worth more than $10. This highlights the need for accurate modeling and analysis to reduce the number of prototypes that need to be made and tested.

Part C: Design of elliptical and rectangular cross sections

*C1.* E Mch 500 only. Develop equations and an algorithm for calculation of shear stress due to torsion at any point in elliptical and rectangular cross sections. Determine the rate of twist and the shear stress components at 8 points around the perimeter of elliptical and rectangular cross sections having an aspect ratio of 2:1 (with a 0.5 inch width) for a constant internal torque of 1000 lb-in.

*Basis*—in order to model more accurately the cross-section of an actual bicycle crank arm, a straight crank arm with rectangular or elliptical cross-section is considered. This task follows discussion on torsion analysis for non-circular cross-sections in the class. Since the analysis of these sections under torsion is a complex topic, this task is assigned to the E Mch 500 students.

*C2.* Combine the torsion analysis with bending and axial force to design an elliptical cross-section crank arm and a rectangular cross-section crank arm. E Mch 500 students should provide the analysis tools and E Mch 400 students should use the tools to do the design.

*Basis*—Combining the results of the torsion analysis with the rest of the stress analysis involves the entire team and requires E Mch 500 students to instruct E Mch 400 students on how to use their analysis tools for design.

Part D: Finite element comparison

Finite element consultants (E Mch 461 students) have analyzed 3D models of the prototype and a commercial crank. Examine the E Mch 461 report and answer the following:

*D1.* Describe why the strains on the prototype at the strain gage locations either are or are not qualitatively correct for each of the three crank orientations.

*D2.* Explain why the von Mises stress distributions are as shown in the E Mch 461 report at the 25%, 50%, 75%, and longitudinal sections of the prototype.

*D3.* Based on the von Mises stress distributions shown in the report at the 25%, 50%, 75%, and longitudinal sections, which crank orientation results in the critical condition for each crank?

*D4.* Based on the von Mises stress distributions shown in the report at the 25%, 50%, 75%, and longitudinal sections, what geometric feature is the most detrimental to the service life of the commercial crank (i.e., causes the worst stress concentration)?

*Basis*—The stresses from the prototype model were compared to the stresses in a commercial crank arm using finite element analysis. Solid models of the prototype and commercial cranks made using Pro/Engineer solid modeling software were discretized and analyzed by the E Mch 461 (Applied Finite Element Analysis) students using...
The results from the finite element analysis were provided to the E Mch 400 and E Mch 500 students to give them familiarity with finite element modeling. The three students enrolled in both E Mch 461 and E Mch 400/500 shared their analysis with the other students by giving presentations during class.

Schedule

Part A was completed over a period of six weeks, then after a short respite, parts B and C were worked on in parallel over a five week period. The final part, D, could not be assigned until after the E Mch 461 class completed it, which left about two weeks at the end of the semester for the teams to finish. (We note that traditional homework assignments and in-class tests were not omitted from the course, but that fewer homework problems were assigned to keep the students overall level of effort the same.)

PROJECT TECHNICAL RESULTS

The project teams first selected appropriate specifications, including the design load, for their design. The design load was determined by one of the following methods:

- rider weight times a dynamic load factor;
- rider mass times a maximum expected g-force;
- field testing (see Fig. 2).

A pedal dynamometer based upon the design of Hull and coworkers [13] was constructed prior to the class to measure the force applied to the pedal by the rider. The dynamometer is a protective aluminum box that encloses a potentiometer to measure the pedal angle with respect to the crank arm, and a pedal spindle that is instrumented with eight strain gages (Measurements Group EA-06-125BZ-350 [14]) connected in two full Wheatstone bridge circuits that measure force components normal and tangent to the pedal. Potentiometer and strain gage signals are acquired by a field computer system (Somat 2100, [15]). A sample record of the measured resultant force is shown in Fig. 2 for a 750 N rider traversing smooth terrain.

Some teams selected two design loads; one for fatigue analysis under normal operating conditions (e.g., 1020 N) and one for an occasional overload (e.g., 2040 N). Design load values ranged from 450–2670 N. While in reality the applied force

Fig. 2. Pedaling force measured by dynamometer for normal operation.
varies during the pedal cycle, most teams assumed it to be constant for simplicity and found the worst-case crank orientation angle, $\theta$, by von Mises stress analysis. The stress analysis includes bending about two axes, axial force, torsion, and transverse shear. The von Mises stress for this application:

$$\sigma_v = \sqrt{\sigma_{xx}^2 + 3(\tau_{xy}^2 + \tau_{zx}^2)}$$

is normalized with respect to $P/A$ and plotted as a function of $\theta$ during the down stroke in Fig. 3. Here the $x$-axis is oriented along the crank arm.

We chose to fabricate the 175 mm long, 25 mm diameter prototype shown in Fig. 4 from 7075-T6 aluminum. Two strain gauge rosettes were bonded to the prototype 125 mm from the pedal centerline to enable validation of the analysis. As shown in Table 2, the measured strains agreed quite well with the predicted strains for a 445 N force applied to the pedal 95 mm from the centerline of the crank.

Changing the crank cross-section from circular to elliptical or rectangular significantly complicates the torsion analysis. E Mch 500 students developed analysis tools, then had to explain how to use them to their E Mch 400 colleagues, which developed communication skills for all students. This appeared to be one of the more difficult tasks in the whole project. The series solution for torsion of a rectangular cross-section is complicated, but the problems were more likely related to having insufficient time budgeted for teaching and learning how to use the software tools.

The FEA models of the prototype crank and commercial crank are shown in Fig. 5. The teams were asked to qualitatively evaluate the stress distributions predicted by these models. This required students to understand what they were looking at, develop intuition about what the stress distributions should be (based on what they learned in the course), find the critical condition, and discuss the results with teammates. These are extremely important skills for structural design engineers.

### LEARNING OUTCOMES

Team grades were based on quality of the work, thoroughness, and creativity. Also, peer evaluations of teammates were done twice during the semester. Salamon and Engel [16] discuss grading

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Fig. 3. Normalized von Mises stress predicted at five points around the perimeter of the crank arm during the down stroke.

Table 2. Comparison of predicted (Pred) and measured (Exp) strain components (in microstrain) on prototype crank

<table>
<thead>
<tr>
<th>Crank Angle (degrees)</th>
<th>$\varepsilon_x$</th>
<th>$\varepsilon_y$</th>
<th>$\gamma_{xy}$</th>
<th>$\varepsilon_x$</th>
<th>$\varepsilon_y$</th>
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<td>Exp</td>
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<td>$130$</td>
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<td>$-110$</td>
<td>$0$</td>
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</table>
mechanics of materials design projects in more detail.

An anonymous survey was given to the students at the end of the semester. Twenty (83%) of the students participated in the survey. One question on the survey asked:

‘Using the scale below, rank each of the following teaching methods according to how useful they are in helping you learn the material for the course.

1 = Not at all useful, 2 = Somewhat useful, 3 = Useful, 4 = Very useful.’

Table 3 shows the student responses to questions related to learning techniques in terms of the mean as well as percentage of respondents. The design project, listening to the lectures, and working alone on the homework problems all had the same—and the highest—mean response. Clearly, the students felt that the design project was useful. The average amount of time spent on the project per week ranged from 0–6 hours, with 36% of the students indicating that they spent an average of 2 hours/week. This is comparable to the amount of time that they spent on homework and on reviewing notes.

Survey results were statistically analyzed to determine differences between E Mch 400 students and E Mch 500 students. Table 4 shows that the E Mch 500 students felt that the design project, lectures, and the textbook were all useful/very useful for learning, while the E Mch 400 students felt they were somewhat useful/useful. These differences are statistically significant, which means that the chance of obtaining these outcomes by chance
is very low. For example, a significance, $p$, of 0.010 means there is a 1 out of 100 probability that the outcome was obtained by chance. This suggests that graduate students value cooperative learning experiences, such as the design project, even more than undergraduates, possibly because they are more mature intellectually. Similar clear differences were obtained when comparing students expecting a grade of B or higher with students expecting a grade of C or lower. One interpretation of this result is that better students value opportunities to learn more than other students do.

Along these lines, we note that while we are aware of many efforts to incorporate cooperative learning techniques into undergraduate engineering education, we are not aware of similar efforts for graduate engineering education.

Perhaps a more informative way to look at the student responses to the survey questions shown in Table 5 is to focus on the extreme responses. The percentages of E Mch 400 and E Mch 500 students marking a ‘1 = Not at all useful’ or a ‘4 = Very useful’ response are shown in Table 5. This indicates openness to a variety of learning methods on
the part of E Mch 500 students that is not present in the E Mch 400 students.

It was apparent to the instructor that the project increased the students' interest level in the course, which as Wankat and Oreovicz [17] discuss is very beneficial for retention. Some students provided additional comments on the survey that related to the design project:

- 'The project work for the class was very good. The project taught us some things, which wouldn't have occurred if we did not have a project. Unfortunately the project was limited to torsion and bending problems. Therefore, I suggest to include several small projects in other areas like shear centre, FEA, unsymmetrical bending etc.'—E Mch 500 student

- 'The group project was a little too long. This should be a project that lasts only about ½ of the semester. The project should be more open-ended in the fact that each group should pick something they want to design/analyze. There is no need to add torsion of elliptical/rectangular cross-sections because the project was structured such that the methodology and ideas about how to approach the design was formulated early on. Experimental validation part was interesting and necessary part of the project.'—E Mch 500 student

- 'I thought when I enrolled in this class I would be learning material that is applicable to the real world. But apparently I was wrong most of the material is geared toward grad work or upper level research. The only practical thing was the project.'—E Mch 400 student

The last comment indicates technical immaturity of an E Mch 400 student: a professional would not view E Mch 400 or E Mch 500 as 'upper level research' courses.

While the mixed undergraduate/graduate student format of the class contributed to the tone of the third comment, it is indicative of the mindset of too many undergraduates. Some, not all, have a limited perception of and little experience with the ‘real world’ and if they can not link something like course material with that perception, they all too often do not see value in it. This could explain reluctance of some undergraduate students to value learning concepts as opposed to merely applying formulas. Graduate students have a broader experience base to draw on, many having already spent considerable time in the profession and have an easier time connecting concepts to applications.

At the end of each course at Penn State the students complete Student Rating of Teaching Effectiveness (SRTE) surveys. It is typical for SRTE scores to drop when new learning tools are implemented, especially if they require more effort from the students. This was in fact the case for this course.

### CONCLUDING REMARKS

A semester long team design project was implemented in an advanced mechanics of materials course. By focusing the initial part of the project on related extant knowledge, the students were able to start the project at the beginning of the semester. The project also provided an application for knowledge gained during the course and linked it with a computational analysis course. The project was clearly related to course objectives and helped fulfill ABET outcomes. Student surveys show that the project was a useful learning tool. Furthermore, graduate students responded even more positively to the project (and a variety
of learning methods in general) than did undergraduate students, and although the data is very limited, this may suggest that the value of these types of cooperative learning experiences and links between theory and applications are more valuable as the academic maturity of the students increase.

Acknowledgments—This project was partially funded by the National Science Foundation (Career Award 9875414) and the Leonhard Center for the Enhancement of Engineering Education at Penn State University. We would like to acknowledge Sam Heriegel for helping develop the project as part of his capstone design project, Mark LePlante for sharing his time and expertise, and Stephani Bjorklund for developing the student survey and analyzing the results.

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