

Ideal Operational Amplifiers: a Circuit for which Gain Still Matters*

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Presented here is a simple circuit involving ideal operational amplifiers that we have not located in any standard electronics textbook. Analyzing it using standard methods leads to an indeterminacy that can only be resolved by re-examining the fundamental assumption of infinite gain associated with such amplifiers. A re-examination of standard assumptions in an altered context is always educational, in this case for undergraduate students of electronic circuits. A proposed use for this circuit as a reliability tool is also discussed, and its associated analysis is an instructive exercise in itself.

INTRODUCTION

NEITHER a colleague of mine nor I have been able to locate the simple circuit illustrated in Fig. 1 in any electronics textbook. (See, for example, [1–4]).

Setting aside for the moment the question of its utility, a consideration of this circuit has educational value. What are V_1 and V_2 for ideal operational amplifiers, without assuming that the two amplifiers are matched? An ideal operational amplifier is one having an input impedance and an open-loop gain that can be considered infinite for all practical purposes. Two amplifiers may be considered ideal and not have the same (very large) gain, in which case they are unmatched. In pursuing a solution to this problem, a typical engineering student would immediately write down the following equations involving continuity of current and Kirchhoff's Voltage Law for ideal operational amplifiers:

$$I_0 = I_1 + I_2 \quad (1)$$

$$V_1 = -I_1 R_1 \quad (2)$$

$$V_2 = -I_2 R_2. \quad (3)$$

Now what? An additional condition appears to be missing that would allow one to solve this problem.

DISCUSSION

The apparent insolubility of this problem is a consequence of invoking ideality too early in the calculation, which is typically not a problem in textbook exercises involving operational amplifiers. What must be done is to assign finite gains,

A_1 and A_2 , to the operational amplifiers, solve for the two voltages, and then take the limit as the gains approach infinity. In addition, we must note that the input voltages to the amplifiers are equal, though unknown *a priori*. If we allow ourselves the assumption that the amplifiers contain infinite input impedance, then these considerations modify the second and third equations above, and add a fourth:

$$V_1 + I_1 R_1 + V_1/A_1 = 0 \quad (4)$$

$$V_2 + I_2 R_2 + V_2/A_2 = 0 \quad (5)$$

$$V_1/A_1 = V_2/A_2 \quad (6)$$

When these equations are solved for V_1 and V_2 , we obtain the following:

$$V_1 = -I_0 R_1 R_2 / [R_1(1 + A_2)/A_1 + R_2(1 + A_1)/A_1] \quad (7A)$$

and

$$V_2 = -I_0 R_1 R_2 / [R_2(1 + A_1)/A_2 + R_1(1 + A_2)/A_2] \quad (7B)$$

For large gain, $1 + A \rightarrow A$ in both cases. Thus, even in the limit of infinite gain, the final expression involves the ratio A_2/A_1 , which can be perfectly finite. A more enlightening form for the high-gain limit is as follows:

$$V_1 = -\{I_0(R_2/A_2)/[(R_2/A_2) + (R_1/A_1)]\} R_1 \quad (8)$$

This expression involves a current divider made up of resistors scaled down by their respective amplification. Such resistors are, of course, the input impedances of the two amplifiers containing a feedback resistor. Thus, this intuitive form for the result could have been guessed at the outset and allows for a ready generalization to more operational amplifiers. It also demonstrates that

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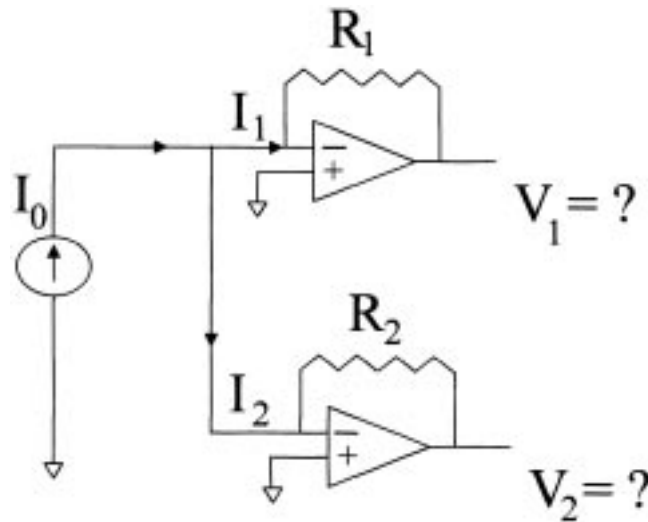


Fig. 1. Neither a colleague nor I have been able to locate this simple circuit in any electronics textbook.

the input impedance of the open-loop amplifiers need only be $\gg R/A$ for this treatment to be valid. If three amplifiers are used, with the relevant parameters of the third denoted by I_3 , R_3 , A_3 , and V_3 , then in the limit of high gain:

$$V_1 = -I_0(R_1R_2R_3/A_2A_3)/[(R_1R_2/A_1A_2) + (R_1R_3/A_1A_3) + (R_2R_3/A_2A_3)] \quad (9)$$

with analogous expressions for the other two voltages obtained by cyclic permutation.

POTENTIAL APPLICATION

As for the utility of this circuit, it allows one to measure directly the ratio of large open-loop gains using a closed-loop configuration. This method seems preferable to measuring the open-loop gains individually, using extremely small input signals, and then taking the ratio. Obtaining this ratio allows one to determine whether two, or more, amplifiers are matched (or mismatched) and remain matched (or mismatched) under various environmental conditions, such as temperature, humidity, electrical overstress, and ionizing radiation. The gain and bandwidth of individual operational amplifiers have been studied under temperature and ionizing radiation, as described in [5]. In the case of the circuit proposed here, one of the amplifiers need not be exposed to the environmental stress for its associated voltage to be affected by it.

LABORATORY EXERCISE

An examination of data sheets on commercial operational amplifiers from various companies indicates that a considerable variation can exist in the open-loop gain of operational amplifiers,

presumably because their high gain is particularly sensitive to unavoidable manufacturing variations. For example, Advanced Linear Devices, Inc. in Sunnyvale, California. For typical closed-loop applications, these variations are unimportant because the gain is so high. In fact, this is a primary virtue of these devices when operated in such a fashion. When used as a comparator, however, variations in gain will directly affect the performance of the device. The circuit presented here allows a student to examine these variations in a closed-loop configuration, at least ratiometrically.

The circuit in Fig. 1 would not be difficult to set up experimentally. The current I_0 could be established at a convenient level with a resistor in series with a power supply. The feedback resistors could be conveniently chosen so that the output signals are relatively easy to measure and saturation of the amplifiers is avoided. Then from Equation 8, we obtain for the ratio of the open-loop gains:

$$A_2/A_1 = -I_0R_2/V_1 - R_2/R_1 \quad (10)$$

with a similar expression for A_1/A_2 in terms of V_2 . Both can be used to test for consistency. One can look at a variety of amplifiers, using one of them as a reference. In addition, as an example of an easily producible environmental stress, the effect of temperature on the open-loop gain of any one could be studied in a student laboratory. The results could then be compared to those obtained with microvolt-size voltages directly applied to such amplifiers with no feedback resistor. The increased difficulty of the latter measurement method would be noted.

CONCLUSION

We have presented a circuit that can be analyzed only after a re-examination of the assumption of

infinite gain of an operational amplifier. In addition, a laboratory exercise involving this circuit has been proposed to examine variations in the open-loop gain of nominally identical operational amplifiers.

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REFERENCES

1. R. C. Jaeger, *Microelectronic Circuit Design*, McGraw-Hill, New York (1997).
2. A. R. Hambley, *Electronics, 2nd ed.*, Prentice-Hall, Paramus, N. J. (1993).
3. S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, McGraw-Hill, New York (1998).
4. A. S. Sedra and K. C. Smith, *Microelectronic Circuits*, Oxford University Press, Oxford (1998).
5. D. Guckenberger and D. M. Hiemstra, Simultaneous cryogenic temperature (77K) and total dose ionizing radiation effects on COTS amplifiers, *2001 IEEE Radiation Effects Data Workshop*, July 2001, pp. 14–18.
6. D. A. Neamen, *Electronic Circuit Design and Analysis*, Irwin, Chicago (1996).

APPENDIX

The presence of input offset voltages

The presence of input offset voltages, which can also change under environmental stress, complicates the analysis yielding the fractional change in both the gain and the input offset voltage of the environmentally stressed amplifier. However, performing the analysis is an instructive exercise.

The input offset voltage of a differential amplifier is defined as the voltage that must be applied to its non-inverting input to produce a zero output when the negative input is grounded, and its existence is the result of imbalances in various pairs of nominally matched components [6]. Let us define V_{off1} and V_{off2} as the offset voltages of the two amplifiers in Fig. 1. Then, by linear superposition:

$$V_1 = -(V_i + V_{\text{off1}})A_1 \quad (\text{A1})$$

and similarly for V_2 . Since the same input voltage appears across both amplifiers, Equation 6 becomes:

$$V_1/A_1 + V_{\text{off1}} = V_2/A_2 + V_{\text{off2}} \quad (\text{A2})$$

which transforms Equation 7 into the more complicated expression:

$$V_1 = -[I_0 R_1 R_2 + V_{\text{off1}} R_2 + V_{\text{off2}} R_1 + (1 + A_2)(V_{\text{off1}} - V_{\text{off2}})]/D \quad (\text{A3})$$

where

$$D \equiv [R_2(1 + A_1)/A_1] + [R_1(1 + A_2)/A_2] \quad (\text{A4})$$

This equation may be complete, but it is complicated and not obviously useful in connection with the proposed application.

Let us reconsider Equation A2, while assuming that amplifier 2 undergoes the environmental stress and that the current I_0 can be ramped rapidly on the time scale over which any appreciable change in A_2 can occur. The relationship between V_1 and V_2 is a straight line with slope and intercept M and B , respectively. These are:

$$M = A_2/A_1 \quad (\text{A5})$$

$$B = A_2(V_{\text{off1}} - V_{\text{off2}}) \quad (\text{A6})$$

Then the ratio of the open-loop gains can be obtained at any time during the environmental exposure by determining the slope of the curve. Alternately, the fractional change in A_2 can be obtained from the fractional change in the slope. In addition:

$$(V'_{\text{off2}} - V_{\text{off1}})/(V_{\text{off2}} - V_{\text{off1}}) = (B'M/BM') \quad (\text{A7})$$

where the primes denote the value of the associated quantity at any time after the start of the exposure and the absence of a prime denotes the initial value of the quantity. This equation tells us how the difference in input offset voltage changes with accrued environmental damage.

Jonathan D. Weiss is a physicist and senior member of the technical staff at Sandia National Laboratories. He received his Ph.D. from the University of Illinois in high-pressure solid-state physics, a field in which he worked for several years. He later studied at the Optical Sciences Center of the University of Arizona, after which he was with the Air Force Research Laboratory in Albuquerque, New Mexico. While there, he did theoretical work on thermal blooming, an effect associated with the propagation of high power laser energy through the atmosphere. Since coming to Sandia National Laboratories, Dr. Weiss has worked mainly on fiber-optic sensors, first for underground nuclear test diagnostics and later for commercial applications. He holds twelve patents for such sensors, with several more pending. One has been licensed to industry. At Sandia, he has also worked in the area of flat-panel displays and has done some electronic design.