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Teaching Measurement Uncertainty to Undergraduate Electronic Instrumentation Students*

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Measurement uncertainty is an important concept for electronic instrumentation students. This paper presents an easy way to explain this difficult concept that has been developed in the telecommunication engineering school of the Polytechnic University of Catalonia (Barcelona, Spain). The lesson development is based on a number of references from metrological associations that can be obtained from the internet. Two examples illustrate the process of evaluation and calculation of uncertainty in simple measurement situations.

INTRODUCTION

ELECTRONIC INSTRUMENTATION (EI) is a mandatory subject in the telecommunication engineering course in Spain, which is a study program of five years divided into two cycles of five semesters each. Electronic instrumentation is studied in the fourth semester of the second cycle.

The aim of this course is to present students with the knowledge on measurement methods, design of test instrumentation systems and selection of test equipment, automatic test equipment and virtual instrumentation that they will need for their professional career.

By the end of this course, students should be able to: (a) develop a measurement procedure, (b) find the appropriate instruments for performing the measurements, (c) connect the instruments and evaluate the problems that result from a bad connection, (d) assemble the results and present them, and finally (e) evaluate the measurement uncertainty.

Most students believe, erroneously, that the measurement process ends with the fourth step (d) and never think about the measurement uncertainty. The first barrier to overcome is to convince the students that the measurement uncertainty is an important factor. With this in mind, it is a good idea to suggest they try a simple experiment from the UKAS web page [1]: give the students a sheet of paper with a line drawn on it and ask them to measure the length of this line with a ruler, then interchange the rulers between them and ask them to write down the results. They should then be asked: Do different people produce different results with the same ruler? Do different rulers give consistent results? The outcome of this experiment is amazing enough to motivate students regarding the concept of measurement uncertainty.

As an introduction to the concept, there are several articles about measurement uncertainty on the internet [2, 3]. The lesson plan is based on four textbooks [4–7], all of which are free documents that students can obtain from the internet.

The concept of uncertainty is explained on a metrological basis because this approach provides a systematic methodology to analyze measurement uncertainty, and this facilitates its study. The ISO guide on the expression of uncertainty in measurements (GUM) [8] and the American National Standard ANSI/NCSL Z540-2-1997 [9] provide the internationally agreed method for estimating measurement uncertainty, so the lesson description is based on this method. The documents recommended as textbooks are also based on GUM and the ANSI/NCSL Z540-2-1997.

The lesson is scheduled in four parts: the first part focuses on introducing some definitions. In the second part, the estimation of measurement uncertainty by statistical methods (type A uncertainty estimation) is explained. The third part is devoted to type B uncertainty estimation (estimation not related to statistical methods). Finally, the fourth part deals with uncertainty estimation in indirect measurements and the computation of expanded uncertainty.

SOME DEFINITIONS

The purpose of a measurement is to assign a numerical value to a property of an object or to a physical variable. The measurement result is a number that quantifies this property, but this number in itself gives little information. A parameter is also needed that quantifies the quality of

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the result of the measurement. This parameter is the measurement uncertainty. That is: a measurement result *m* has an uncertainty u(*m*).

It is important not to confuse the terms 'uncertainty' and 'error'. While error is the difference between the measured value and the 'true value' of the measurand (so cannot be obtained), uncertainty is a quantification of the doubt about the measurement result [10].

A more rigorous definition of measurement uncertainty would be as follows: 'the uncertainty of measurement is a parameter, associated with the result of the measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand' [11].

The *measurand* is the particular quantity subject to measurement, for example the value of a certain resistor R. The value of the measurand depends on several input magnitudes. In the case of a resistor, the relationship between its value and the input magnitudes is given by Ohm's law:

$$R = \frac{V}{I} \qquad (1)$$

where V is the voltage and I is the current across the resistor.

In a more general way, a measurand Y is a function of several input quantities $(X_1, X_2, \ldots, X_n)$:

$$\mathbf{Y} = \mathbf{f}(\mathbf{X_1}, \mathbf{X_2}, \ldots, \mathbf{X_n}) \qquad (2)$$

An estimate of the measurand Y, expressed as lowercase (y), can be obtained from the input quantity estimates $x_1, \ldots, x_n$

$$y = f(x_1, x_2, \ldots, x_n) \qquad (3)$$

where the function *f* represents the procedure of measurement.

Every input quantity estimate x_i has an associated *standard uncertainty,* $u(x_i)$, that is the estimated standard deviation of x_i. The measurement standard uncertainty is defined as a combination of the standard uncertainties of the input quantity estimates and is called *combined standard uncertainty*:

$$u_c(y) = g(u(x_1), u(x)2), \ldots, u(x_n), x_1, x_2, \ldots, x_n) \qquad (4)$$

where *g* is a function related to *f*.

Returning to the resistor example, the uncertainty in the resistor value will be a function of voltage and current uncertainties.

$$u_c(R) = g(u(V), u(I), V, I) \qquad (5)$$

Regardless of which are the uncertainty sources, the international consensus method for estimating measurement uncertainty provides two ways of evaluating it: *Type A Evaluation*, where the uncertainty is estimated by using statistical methods, and *Type B Evaluation*, where the uncertainty is estimated from other information (past experience, calibration certificates, manufacturer specifications, etc.) [4–7].

Before attempting to estimate the measurement uncertainty, it is necessary to know how the effects are contributing to it. There are three kinds of effect that influence the measurement result:

a) *Random effects:* When the measurement is repeated the result is randomly different (with a zero mean).
b) *Systematic effects:* The effect is the same for each measurement. These are usually factors related with the measurement instrument like bias or aging, or related with the measurement procedure like load effects. They must be corrected prior to estimating the measurement uncertainty.
c) *Mistakes or aberrations*: These should not be considered in measurement uncertainty estimations, so they should be rejected.

Figure 1 shows these three kinds of effects. Random effects are distributed around the central point without following any definite pattern. Systematic effects are all biased from the central

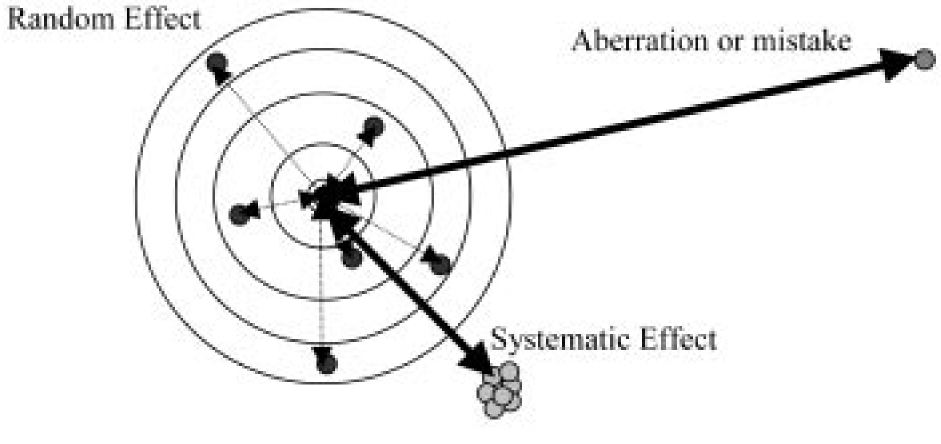


Fig. 1. The concepts of random effect, systematic effect and aberration or mistake.

point. Aberrations or mistakes are far away from the central point and can be easily detected as a mistake.

TYPE A EVALUATION OF UNCERTAINTY

Type A evaluation of uncertainty is performed by a statistical analysis of experimental data. The procedure for the evaluation is as follows:

1. From all possible results obtained when measuring a magnitude q (*the population)*, a set of n independent results q_k is extracted (*the sample)*. The expectation of the population is the true value of q.
2. To estimate the value of q, the arithmetic mean is calculated as follows:

$$\overline{q} = \frac{1}{n}\sum_{k=1}^{n} q_k \qquad (6)$$

3. The experimental standard deviation is used to estimate the dispersion of results:

$$s(q) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(q_j - q)^2} \qquad (7)$$

In fact s(q) gives information about the dispersion of the sample. If a new set of n measurements are extracted from the population, the value obtained for the arithmetic mean will be different. So a parameter for estimating the dispersion of values of the arithmetic mean is needed.

4. The experimental standard deviation of the arithmetic mean is defined as:

$$s(\overline{q}) = \frac{s(q)}{\sqrt{n}} \qquad (8)$$

this parameter is an estimator of the standard uncertainty so:

$$u(\overline{q}) = s(\overline{q}) = \frac{s(q)}{\sqrt{n}} \qquad (9)$$

Equation (8) is only valid if the n measurements are statistically independent. If the measurements are correlated, the mean and the experimental standard deviation of the mean may be inappropriate estimators of the desired statistics. In such cases the data should be analyzed by statistical methods adequate for treating a series of correlated randomly varying measurements [8, 12].

Figure 2 shows graphically the type A standard uncertainty evaluation. Two sets of n independent measurements (n = 10) are shown. The mean, the standard deviation and the standard uncertainty for the two sets of measurement are also shown.

Example 1

In order to measure an unknown voltage, an acquisition card for a PC is used. Eight measurements of the unknown voltage are performed obtaining the following readings: 10.110V, 10.107V, 10.119V, 10.105V, 10.111V, 10.108V, 10.108V, 10.109V. The PC card offset is 0.003v and its gain is 1.003.

a) Obtain an expression for the correction of systematic effects and correct the readings.
b) Obtain an estimation of the unknown voltage and the standard deviation of the electrical noise superimposed on it.
c) Calculate the standard uncertainty of the unknown voltage.

Solution

a) The offset and gain of the PC acquisition card are systematic effects. Equation (10) gives the

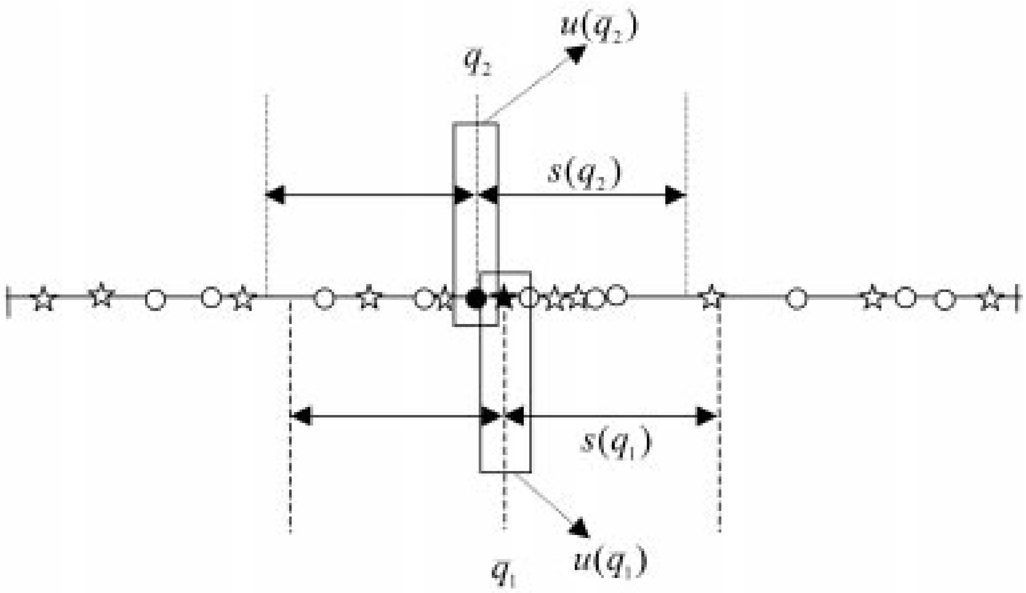


Fig. 2. Graphical illustration for type A uncertainty evaluation. Two sets of measurements (☆, ○) are shown. The mean value for each set of measurements (★, ●) and its uncertainty are also presented.

measured voltage as a function of input voltage and systematic effects:

$$V_{measured} = G \cdot V_{input} + V_{offset} \tag{10}$$

where G is the gain of the acquisition card and V_{offset} is the offset voltage. The corrected readings are expressed as:

$$V_{input} = \frac{V_{measured} - V_{offset}}{G} \tag{11}$$

Table 1 contains the corrected readings.

b) The estimation of the unknown voltage and the electrical noise superimposed can be obtained respectively from the arithmetic mean equation (6) and the standard deviation equation (7) of the corrected values:

$$\overline{V}_{input} = \frac{\sum V_{input}}{8} = 10.075\,V$$
$$V_{noise} = \sqrt{\frac{\sum (V_{input} - \overline{V}_{input})^2}{7}} = 4.6\,mV \tag{12}$$

c) The standard uncertainty of the measurement can be obtained from equation (9):

$$u(\overline{V}_{input}) = 1.7\,mV \tag{13}$$

TYPE B EVALUATION OF UNCERTAINTY

Type B evaluation of uncertainty is performed by means other than statistical analysis of a series of observations. If only an estimate x_i from an input quantity X_i is available, the evaluation of standard uncertainty is based on all available information on the possible variability of the input quantity (previous measurement data, knowledge of behavior of materials and instruments, manufacturer's specifications, data provided in calibrations etc.). The more common methods for type B standard uncertainty evaluation are:

a) From previous knowledge of the measured input quantity X_i a probability distribution of X_i can be supposed. In this case, the expectation of X_i, $E(X_i)$, is an estimate of the input quantity and the standard deviation of X_i, $s(X_i)$, is an estimate of the measurement uncertainty.

b) If only the limits of the distribution are known, two distributions are commonly used: rectangular or uniform distribution or triangular distribution. Rectangular distribution is the simplest option for the analysis and it represents the worst case. Triangular distribution is used when it is known that there is a central tendency for the values of the variable of interest. Although these two distributions are the most commonly used, other distributions are sometimes employed. Some of these distributions are described below.

Rectangular distribution

All the possible values have the same probability. Typical situations for using this distribution are, among others, digital resolution, component tolerances, quantization error or RF phase angle [13]. In a more general way, a rectangular distribution should be used when only the two limits of possible values are known, or when the probability distribution function is unknown. The uniform distribution leads the most conservative uncertainty estimation; that is, it gives the largest standard deviation.

Normal distribution

This distribution is very important because it represents the statistical behavior of most phenomena that occurs in nature. It is a less conservative way to estimate uncertainty because it gives the lowest standard deviation. Usually, the calculation of the standard deviation is based on the assumption that the end-points, $\pm a$, encompass 99.7 percent of the distribution, if the number of observations is high enough [14].

Triangular distribution

This distribution is used when there is a 100% minimum containment limit and when it is known that there is a central tendency for the values of the variable of interest. Triangular distribution leads to a less conservative estimate of uncertainty than uniform distribution but a more conservative estimate than normal distribution; that is to say, it gives a smaller standard deviation than uniform distribution, but larger than normal distribution.

U distribution

This distribution is used when the most likely value of the measurand is near to the containment limits. For instance, it applies to sinusoidal interference.

Poisson distribution

This is used to model the number of random occurrences in a given unit of time or space. In this case the interest variable must be a count of events.

Table 2 contains the most common distributions and their relevant parameters.

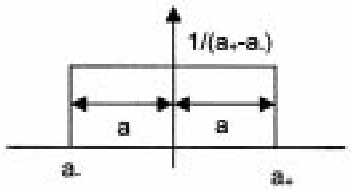
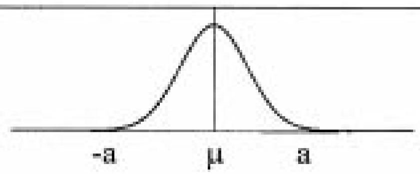
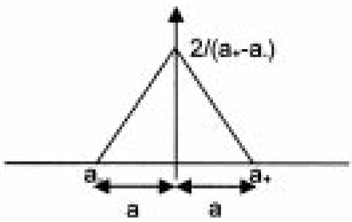
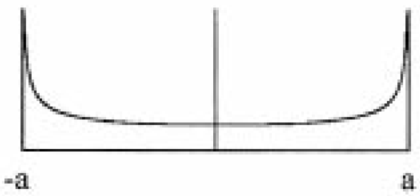
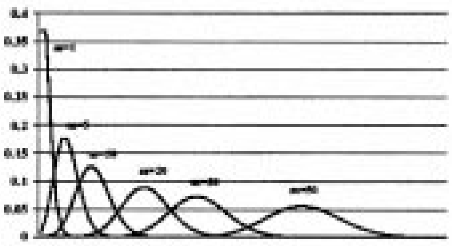
Other ways to evaluate type B standard uncertainty

Sometimes the estimate of x_i is taken from a manufacturer specification, or a calibration certificate, etc., and its uncertainty is stated to be a multiple of a standard deviation. In this case, the

Table 1. Corrected offset and gain readings for example 1

Reading (V)	10.110	10.107	10.119	10.105	10.111	10.108	10.108	10.109
V_{input} (V)	10.076	10.073	10.086	10.071	10.077	10.074	10.074	10.075

Table 2. Most common distribution probability functions used in type B uncertainty evaluation

Rectangular or Uniform		$f(x)=\frac{1}{a_+-a_-}$ $u(x)=\frac{a_+-a_-}{\sqrt{12}}$ if $a_+=a_-=a$ $f(x)=\frac{1}{2a}$ $u(x)=\frac{a}{\sqrt{3}}$
Normal	 ± a, encompass 99.7 % of the distribution.	$f(x,\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $u(x)=\frac{a}{3}$
Triangular		$f(x)=\begin{cases}\frac{2}{(a_+-a_-)a_-}(x+a_-) & -a_-\le x\le 0\\ \frac{2}{(a_+-a_-)a_+}(a_+-x) & 0\le x\le a_+\end{cases}$ if $a_+=a_-=a$ $f(x)=\begin{cases}\frac{x+a}{a^2} & -a\le x\le 0\\ \frac{a-x}{a^2} & 0\le x\le a\end{cases}$ $u(x)=\frac{a}{\sqrt{6}}$
U distribution		$f(x)=\frac{1}{\pi\sqrt{a^2-x^2}}$ $-a\le x\le a$ $u(x)=\frac{a}{\sqrt{2}}$
Poisson		$P(x,m)=\frac{m^x e^{-m}}{x!}$ $u(x)=\sqrt{m}$

standard uncertainty is the value of the uncertainty divided by the multiplier.

The uncertainty in the estimate of x_i can also be given by an interval with a 90, 95 or 99% level of confidence. In this case it may be assumed that there is normal distribution (unless there are other indications). The standard uncertainty is calculated by dividing the stated value by the corresponding factor for normal distribution and the confidence level. The factors for the confidence levels of 90, 95 and 99% are 1.64, 1.96 and 2.58 respectively.

STANDARD UNCERTAINTY IN INDIRECT MEASUREMENTS

The result obtained from an indirect measurement is a combination of several magnitudes. In the section 'Some Definitions', equation (3) gives the measurement result as a function *f* of several input quantities, and equation (4) gives the measurement uncertainty as a function *g* of input quantity uncertainties. In this section the relationship between the functions *f* and *g* will be analyzed.

Independent magnitudes

If the different magnitudes used to obtain the measurement result are independent, the combined standard uncertainty can be calculated using a first-order Taylor series approximation as follows:

$$u_c(y) \cong \sqrt{\sum_{i=1}^{N} u_i^2(y)} \tag{14}$$

where $u_i(y)$ is the contribution to the standard uncertainty of y resulting from the standard uncertainty of the input quantity x_i

$$u_i(y) = c_i \cdot u(x_i) \tag{15}$$

c_i is the sensitivity coefficient associated with the input quantity

$$c_i = \frac{\partial f}{\partial x_i} \tag{16}$$

Substituting equations (16) and (15) in equation (14) the combined uncertainty can be expressed as:

$$u_c(y) \cong \sqrt{\sum_{i=1}^{N} u_i^2(y)} = \sqrt{\sum_{i=1}^{N} c_i^2 u^2(x_i)} = \sqrt{\sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i)} \tag{17}$$

Dependent magnitudes

We say that two magnitudes are dependent when a variation in one of them causes a variation in the other. In this case the combined standard uncertainty can be calculated as follows:

$$u_c(y) \cong \sqrt{\sum_{i=1}^{N}\sum_{k=1}^{N} c_i c_k u(x_i, x_k)} = \sqrt{\sum_{i=1}^{N} c_i^2 u^2(x_i) + 2 \cdot \sum_{i=1}^{N-1}\sum_{k=i+1}^{N} c_i c_k u(x_i, x_k)} \tag{18}$$

where c_i is the sensitivity coefficient and $u(x_i, x_k)$ is the covariance between the input magnitudes x_i and x_k. This covariance can be obtained from the standard uncertainty of input quantities and their correlation coefficient:

$$u(x_i, x_k) = u(x_i) \cdot u(x_k) \cdot r(x_i, x_k) \tag{19}$$

Note that $r(x_i, x_i) = 1$.

Uncertainty budget

The uncertainty analysis for a measurement should include a list of all sources of uncertainty and the associated standard uncertainties of measurement and the methods for evaluating them. A good method of showing all this information is the uncertainty budget of measurement. Table 3 shows a typical uncertainty budget.

Expanded uncertainty

In some applications it is often necessary to give a measure of uncertainty that defines an interval around the measurement result that may be expected to encompass a large fraction of the distribution of values expected for the measurand [8].

This measure of uncertainty is called expanded uncertainty and is denoted by U. The expanded uncertainty is obtained by multiplying the combined standard uncertainty by a *coverage factor k*:

$$U = k \cdot u_c(y) \tag{20}$$

The result of a measurement is now expressed as:

$$Y = y \pm U \tag{21}$$

Table 3. Typical uncertainty budget: an ordered arrangement of the quantities, estimates, standard uncertainties, assumed probability density functions, sensitivity coefficients and contributions to total uncertainty calculation

Input quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$	pdf	Sensitivity coefficient c_i	Contribution to the standard uncertainty $u_i(y)$
X_1	x_1	$u(x_1)$	...	c_1	$c_1 \cdot u(x_1)$
X_2	x_2	$u(x_2)$	...	c_2	$c_2 \cdot u(x_2)$
...	...	...	...	...	...
X_N	x_N	$u(x_N)$	...	c_N	$c_N \cdot u(x_N)$
Y	**y**				$\mathbf{u_c(Y)}$

where y is the best estimate of the value attributable to the measurand Y, and $y-U$ and $y+U$ define an interval about the measurement result that includes a large fraction p of the probability distribution characterized by the result and its combined standard uncertainty, and p is sometimes referred to as the level of confidence of the interval.

The coverage factor k is chosen for providing an interval $Y=y\pm U$ corresponding to a particular level of confidence p. This is not easy to do in practice because it requires an extensive knowledge of the probability distribution characterized by the result and its combined standard uncertainty. But, by the Central Limit Theorem the probability distribution of a variable obtained from a combination of several variables with different probability distributions can be approximated by a normal distribution. The larger the number of variables, the more exact the approximation.

In order to obtain a stricter approximation of expanded uncertainty, we should consider that the calculation of an interval having a specific level of confidence requires the distribution of a variable $[y-Y]/u_c(y)$. If the measurand Y is a normally distributed quantity and y is the best estimation of Y (that is, the arithmetic mean of n independent observations of Y) with experimental deviation of the mean $s(y)=u_c(y)$, the variable $[y-Y]/u_c(y)$ follows a student t-distribution with $\nu=n-1$ degrees of freedom.

The coverage factor is now obtained from the t-distribution tables for a determinate confidence level and $\nu=n-1$ degrees of freedom.

When the degrees of freedom tends to ∞ be the student t-distribution tends to be a normal distribution.

Example 2

In order to characterize a sensor that can be modeled as a voltage source, a digital multimeter is used for measuring its output voltage. The manufacturer of the sensor specifies an output resistance of 10 k$Q\pm$1kQ with a coverage factor $k=2$. The manufacturer of the digital multimeter specifies an input impedance of 10 M$Q\pm$0.1 MQ with a coverage factor $k=3$, and a measurement accuracy of 0.0035% of the reading +0.0005% of the range. The multimeter reading is 99.9v when a range of 100v is selected.

a) Draw the equivalent circuit for the measurement.
b) Obtain the equation for estimating the value of the voltage source and calculate it.
c) Calculate the standard uncertainty of each input quantity.
d) Obtain the uncertainty budget and calculate the voltage source combined standard uncertainty.

Solution

a) Figure 3 shows the equivalent circuit for the measurement.

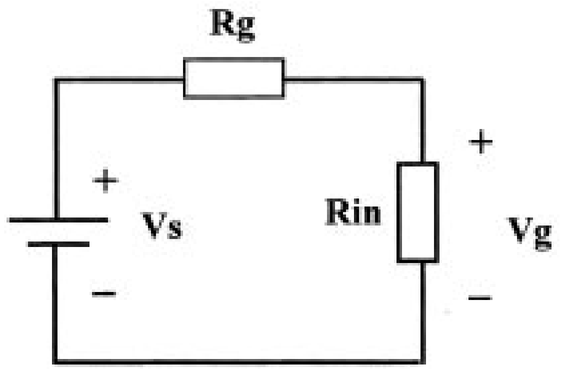


Fig. 3. Equivalent measurement circuit for example 2.

b) The measured voltage source is:

$$V_m=\frac{R_{in}}{R_{in}+R_s}V_s \qquad (22)$$

where V_s and R_s are the source voltage and source resistance respectively and R_{in} is the multimeter input resistance.

Obtaining Vs from equation (22):

$$V_s=\left(1+\frac{R_s}{R_{in}}\right)\cdot V_m \qquad (23)$$

The value of Vs is obtained by substituting the nominal values in equation (23)

$$V_s=\left(1+\frac{10\,kQ}{10\,MQ}\right)\cdot 99.9\,V=100\,V \qquad (24)$$

c) In equation (23) there are three input magnitudes for obtaining V_s: V_m, R_s and R_{in}.

c-1 R_s standard uncertainty: the manufacturer gives the value of R_s with its expanded uncertainty. In this case:

$$u(R_s)=\frac{U(R_s)}{k}=\frac{1000}{2}=0.5\,kQ \qquad (25)$$

c-2 R_{in} standard uncertainty: this is the same as for c-1; the manufacturer gives the expanded uncertainty as follows:

$$u(R_{in})=\frac{U(R_{in})}{k}=\frac{100\,000}{3}=0.033\,MQ \qquad (26)$$

c-3 V_m standard uncertainty, from the multimeter specification:

$$\text{multimeter accuracy}=0.0035\%\text{ of reading}+0.0005\%\text{ range}$$

$$accuracy=\frac{0.0035}{100}\cdot 99.9\,V+\frac{0.0005}{100}\cdot 100\,V=3.9\,mV \qquad (27)$$

Because the probability distribution function is unknown, a uniform distribution is considered

Table 4. Uncertainty budget for example 2

Input quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$	pdf	Sensitivity coefficient c_i	Contribution to the standard uncertainty $u_i(y)$
V_m	99.9 V	2.25 mV	uniform	1.001	2.25 mV
R_s	10 kΩ	0.5 kΩ	normal	9.99 μA	4.99 mV
R_{in}	10 MΩ	0.0033 MΩ	normal	−9.99 nA	−0.03 mV
$\mathbf{V_s}$	**100 V**				**5.47 mV**

for the worst case. In this case the standard uncertainty is:

$$u(V_m) = \frac{\text{accuracy}}{\sqrt{3}} = \frac{3.9\,\text{mV}}{\sqrt{3}} = 2.25\,\text{mV} \qquad (28)$$

d) The voltage source combined standard uncertainty is obtained applying equation (17), because all input quantities are independent. First, the sensitivity coefficients must be calculated:

V_m sensitivity coefficient:

$$\frac{\partial V_s}{\partial V_m} = 1 + \frac{R_s}{R_{in}} = 1.001 \qquad (29)$$

R_s sensitivity coefficient:

$$\frac{\partial V_s}{\partial R_s} = \frac{V_m}{R_{in}} = 9.99 \cdot 10^{-6}\,\text{A} = 9.99\,\mu\text{A} \qquad (30)$$

R_{in} sensitivity coefficient:

$$\frac{\partial V_s}{\partial R_{in}} = -\frac{R_s}{R_{in}^2} V_m = -9.99\,\text{nA} \qquad (31)$$

Note that R_s and R_{in} coefficients have electrical current units and the V_s sensitivity coefficient is dimensionless.

Table 4 contains the uncertainty budget and the combined standard uncertainty for the voltage source. With this example, the standard uncertainty calculations using a type B evaluation of uncertainty are shown. The contribution of each uncertainty source can be seen in Table 4.

CONCLUSIONS

Measurement uncertainty is an important concept for undergraduate electronic instrumentation students to know. In this paper a simple way of explaining this subject has been presented. Reference materials for this subject can be obtained from the web pages of metrological associations. Some important measurement concepts have been introduced in this paper, then the two types of uncertainty evaluation are explained, and finally the calculation of combined standard uncertainty is presented. Two examples have been provided to illustrate type A standard uncertainty evaluation and combined standard uncertainty calculation using a number of input quantities with type B standard evaluation. The use of this approach to explain measurement uncertainty has been a positive experience for our electronic instrumentation students.

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