

Reinventing the Teaching of Statics*

PAUL S. STEIF

Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA. E-mail: steif@andrew.cmu.edu

ANNA DOLLÁR

Mechanical and Manufacturing Engineering Department, Miami University, Oxford, OH 45056, USA. E-mail: dollara@muohio.edu

A substantial revamping of teaching engineering statics is proposed. Several common features of traditional instruction in statics are critiqued, including a general failure to focus students on the key concepts of the subject. The ultimate goal of instruction, empowering students to apply the methods and ideas of statics to real artifacts, is emphasized. This goal requires students to make clear connections between the symbols of the subject (primarily forces) and what they represent (the interactions between bodies). We also point out a major stumbling block to making such connections: the considerable difficulty many students have in envisioning forces between inanimate objects, a difficulty which is well documented in the physics education literature. This forms the backdrop to our proposed approach to statics instruction. We have reorganized statics to allow students to confront the major concepts one at a time, initially within the context of forces that students experience first hand – through the senses of touch and of sight. The proposed progression of topics and concepts, and how they are elucidated through artifacts that students manipulate in class, are presented. In addition, we describe highly interactive classroom methods that allow students to confront these concepts, test their understanding of them, and refine that understanding through spirited discussions with peers.

INTRODUCTION

MECHANICAL INTEGRITY, which remains an essential requirement for a vast array of technologies, new and traditional, is strongly rooted in the basic subject of statics. Statics lays the foundation for subsequent courses, namely dynamics and mechanics of materials. While there are new ideas which are emphasized in engineering dynamics, instructors in this subject inevitably find that some of the students' difficulties are rooted in ideas from statics, such as free body diagrams and working with forces. An essential idea in mechanics of materials is that of internal loads and its relation to external loads. This idea first emerges in statics, but failure to make the distinction between these loads is a common error committed by students in mechanics of materials. Statics, together with these courses, also forms the basis for much engineering design and practice. Along with many instructors, we are disappointed with the extent to which students are able to use statics in the analysis and design of mechanical systems and structures which they confront in their subsequent education [1] and later in their professional careers. Improving learning in the subject of engineering statics deserves significant attention.

In this paper, we propose a radical reworking of instruction in statics; both the content and its implementation. The proposed changes in the content are partially based on aspects of typical

statics textbooks (and courses) which we find troublesome. Proposed changes are also motivated by our contentions as to what *ought* to be emphasized in a statics course, given the concepts that are necessary for solving problems and the difficulties students have in learning these concepts. The proposed implementation is largely motivated by general lessons which can be gleaned from the literature on means of improving learning across domains. Thus, the thorough reworking of statics to be presented here rests on a solid foundation of existing research regarding the conceptual challenges that students face in statics and on strategies that have led to learning gains in other contexts.

CRITIQUE OF TRADITIONAL STATICS INSTRUCTION

One can have minor quibbles regarding the approaches of different textbooks. Here we critique those features which have serious negative impact on students' understanding of the prime concepts of statics. For example, much of the early portions of textbooks address the mathematical manipulation of force vectors. This sets a dangerous tone – forces are really interactions between bodies, usually in contact. These preliminary activities in statics tend to reinforce the reluctance of students to associate forces with interacting bodies, and instead to treat them as disembodied mathematical entities.

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By contrast, the interacting bodies are sometimes displayed in contact, and then the force of one on the other is drawn superimposed on the two bodies. This sets a terrible example for drawing free body diagrams. The force should only be drawn if one of the two bodies is displayed, while the other is removed. Recent studies indicate that the drawing on free body diagrams of internal forces, that is forces between two bodies both of which are displayed in the diagram, is an error frequently committed by students.

Statics differs from an introductory Newtonian physics course in a number of significant ways, including the emphasis on the rotational equilibrium of bodies. Thus, the concept of the moment about a point due to a force and the concept of a couple are extremely important and must be distinguished from one another. The couple is a convenient representation of a combination of forces which have a net tendency to rotate. Like the force, the couple represents an interaction between contacting bodies. The moment is only a calculation of the tendency for a force to cause rotation of a body about an arbitrarily chosen point; it is not an interaction. However, some textbooks draw the moment or couple vector (double-headed arrow, or straight arrow with an arrow-headed circular arc along its length) regardless of whether a moment or a couple is intended. This confuses students into thinking that the moment is applied at the point, whereas it is not a direct interaction, although the couple is.

Statics textbooks also tend to introduce the concept of static equivalency (resultants), just after introducing the moment and the couple, but prior to equilibrium. But without equilibrium and the importance of the summation of forces and moments, the need for static equivalency is largely unmotivated. This adds to the tendency discussed below for statics to become largely a matter of mathematical manipulation, rather than a matter of modeling physical systems.

The notion of loads acting on connections, joints or supports is generally presented in statics textbooks with minimal explanation. Essentially, cookbook recipes are offered: if you see this diagram – e.g. a pin joint – then draw these forces. The approximations which are standard for joints are based on the common assumption of negligible friction on contacting surfaces. If the effects of friction have not even been acknowledged, students cannot be expected to make sense of approximation based on the neglect of friction.

Finally, despite the extensive mathematical preliminaries, students are thrown relatively rapidly into solving equilibrium problems which require many of the key concepts of statics. More than anything else, traditional instruction fails to acknowledge the distinct concepts in statics and to separate them out for focus and consideration.

Statics is an engineering tool which is ultimately of value if it is used, along with other theories, to predict the behavior of real objects. We cannot

expect students to learn statics in a very abstract way, largely through mathematical manipulations, and then apply those ideas to real artifacts. Rather, physical experiences with the forces and moments that act between, or within, objects must be part and parcel of the very earliest exposure to statics. Why must this be the case?

In statics, as in many engineering subjects, one hopes that experiences applying basic concepts and principles in a limited set of physical situations will enable students to apply those same concepts and principles to new situations in the future. This is referred to as *transfer* in the cognitive psychology literature. Transfer is promoted by, among other things, having learned the original material deeply, ‘with understanding’ [2]. What constitutes deep understanding in statics? In engineering science subjects, generally, insight into physical systems is gained by deducing relationships between variables which represent features of the physical system. We would argue that a deep understanding of a subject such as statics includes a firm connection between the variables that are used and what they represent. This is recognized in the physics education literature. For example, Trowbridge and McDermott [3] studied students’ facility with interpreting the concept of velocity in the case of masses moving along a track. Reif and Allen [4] considered students’ interpretations of acceleration. Laurillard [5] articulated this important aspect of learning in the context of university education more generally; she asserted that students must learn to ‘relate the sign to the signified.’

Clearly, a critical concept in statics is that of interaction between contacting bodies, and so success in statics relies on the ability to interpret the concepts of force and couple. Researchers studying learning in physics, where the concept of force is first introduced, have found that students have particular difficulty appreciating the existence of forces between unmoving, relatively rigid, inanimate objects [6, 7, 8]. As a simple example, students often have difficulty with the notion that the table on which a book rests actually exerts a force on the book. Now, in statics forces appear in a much wider variety of circumstances, as compared with Newtonian physics. Unfortunately, the vast majority of forces of interest to statics – between objects and their supports or between connected objects – are precisely such contact forces. Thus, from the start, students in statics are handicapped – we are hoping they will learn to relate the symbols representing forces to the actual forces, yet they don’t believe most of these forces exist! It is no wonder that problem-solving in statics has become largely an exercise in mathematics.

IMPLICATIONS FOR SEQUENCE OF STATICS INSTRUCTION

In summary, traditional instruction fails to allow students to confront, and get comfortable

with, concepts in isolation first; it thrusts students immediately into solving problems requiring many complex concepts. In addition, deep understanding of statics lies in being able to relate the symbols (forces) to the interactions between bodies which they represent, and yet students are known to have difficulty in perceiving the forces between inanimate objects, the forces of most concern in engineering. These two insights have been the prime motivators for our reformulation of statics instruction. We have reorganized the principal ideas of statics so as to *build gradually one upon the other*, and so as to be comprehended entirely in the context of situations in which the *forces are readily perceived by students*. In particular, forces are made real by focusing on forces which can be experienced through the senses of touch or sight. This involves students manipulating objects and exerting by hand those forces and couples which are necessary to provide equilibrium. Forces are experienced through the sense of sight by arranging for forces to cause visible deformation of readily deformable bodies, such as foam or extensible cords, or motion.

Thus, we separate out and address the basic concepts of statics (forces, moments, couples, static equivalency, free body diagrams, equilibrium in 2-D and 3-D, friction), all without the need to invoke forces between contacting inanimate objects. Only after this initial phase of statics are students gradually introduced to contacts between inanimate objects and connections. This gradual transition from manually exerted forces to contacts between inanimate objects prepares students for a far sounder understanding of the loads acting at connections between bodies.

GENERAL APPROACHES TO IMPROVING LEARNING WHICH ARE RELEVANT TO STATICS

To improve students' understanding of statics, instructors of statics must gather all the tools at their disposal. While we have discussed the content of statics above, several generally accepted approaches to improving learning outcomes are potentially quite relevant to the classroom implementation of this new content. Students who are actively engaged in learning learn more [9–11]. While it can be more time-consuming, ideas that are reached through discovery may be more firmly grasped than those that are acquired through typical lectures or textbooks. Students learn through a constant iterative process of assimilating new information and testing out their evolving understanding with feedback from instructors; thus the integration of assessment into the learning process can be of great benefit [12–14].

The process of learning is also aided when new information is placed in the context of knowledge which students have previously acquired; that is, students build on what they already know [15].

Students can learn a great deal from one another; collaboration, if harnessed appropriately, is a powerful tool in learning [16]. Finally, for many subjects in the sciences or technologies, physical referents or manipulatives can serve to enhance learning [17]. Moreover, the use of manipulatives, as well as the more interactive techniques, can serve to accommodate students with a greater range of learning styles, as compared to only traditional lectures.

CLASSROOM IMPLEMENTATION

To implement an object-centered approach in the classroom we have formed learning modules [18–20] that link readily perceivable physical situations with the modeling or representational approaches of mechanics. Learning modules include classroom desktop experiments or demonstrations, PowerPoint presentations and, often, concept questions. When the experiment involves an object, there is either a single copy of the object for the instructor to demonstrate in front of the class or there are enough objects for every two or three students to share a copy. The instructor controls the PowerPoint presentations, which explore ideas introduced by the experiments and facilitate the transition from real objects to their models and representation with symbols. A presentation may depict a physical object and how it can be held in equilibrium, deformed or otherwise manipulated under the action of forces. The presentations also serve to illustrate the free body diagrams corresponding to the configurations depicted.

Many of the presentations contain concept questions, akin to Mazur's ConcepTests [21]. These are multiple-choice questions that assess student understanding of concepts, and which typically require little or no analysis. Through the raising of colored note cards or through electronic classroom communication systems, each student votes for a particular answer among the multiple choices. Depending on the votes, we may invite students to argue the question with one another and/or to manipulate the object. Students vote again and any remaining discrepancies are discussed. Additional details may be found in the companion paper [22].

In the remainder of the paper, we focus on the logical sequence in which the concepts of statics are developed and how these concepts are elucidated through a combination of objects and conceptual questions. This development is explained in a concrete way by showing slides of PowerPoint presentations through which we address these concepts in the classroom. In the accompanying paper, we put additional emphasis on classroom implementation and we display several modules in detail. In a number of figures, there is a small colored box with lettering (e.g. Pi, for pink). This shows the correct choice from

among the multiple-choice answers offered in a previous slide. While we focus here on statics *concepts*, it must also be pointed out that the requisite mathematical skills, such as vector algebra, also need to be developed as appropriate if the desired level of problem-solving ability is to be attained.

SEQUENCE OF CONCEPTS

2-D equilibrium of bodies under action of forces

Equilibrium of bodies requires consideration of both the translational and rotational effects of forces; that is, the forces must balance and the moments must balance. Simple situations are considered (Fig. 1), based on an L-shaped object, or parts thereof; students exert forces that maintain this body in equilibrium. Also, the concept of center of gravity for a compound body is introduced naturally.

Couples and static equivalence

Statics involves combinations of forces; one combination of forces can have the equivalent effect to many other combinations. In particular, we focus on combinations of forces which have a tendency only to rotate a body, and the representation of this combination as a couple (Fig. 2). These concepts are pursued, first in 2-D situations, and later in 3-D. The same elongated object can be supported at various locations in multiple ways, all of which are declared to be statically equivalent. Loads can also be observed to be statically equivalent by virtue of the equivalent deformations they cause, which adds additional physical justification for the concept of static equivalence. Also, the mathematical statement of static equivalence, in terms of the summation of forces and moments, is more firmly motivated, we believe, by placing this topic *after* 2-D equilibrium under forces, contrary to the usual sequence in statics textbooks.

Equilibrium of bodies with forces and couples

We next consider the simultaneous action of forces and couples on bodies and their effects on

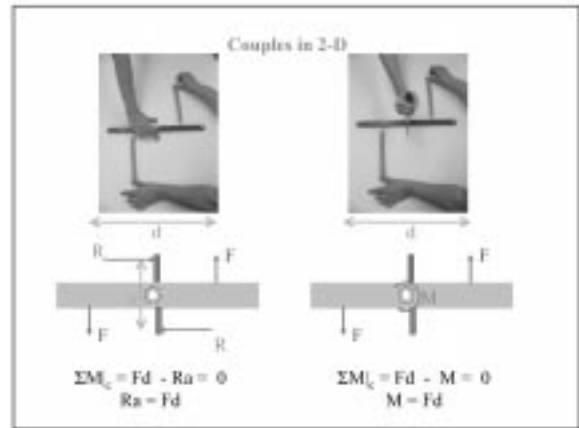


Fig. 2. Couples and static equivalence.

equilibrium (Fig. 3). In particular, we emphasize that the moment summation must include couples, but that the couples do not contribute to force summations. These modules seek also to develop intuition as the directions of the forces and their locations and the senses of couples which can balance a body. While we first treat 2-D cases (not shown), with 3-D cases (Fig. 3) tipping is seen to occur about many axes; hence, equilibrium needs to include moments balancing about multiple axes. The concept of center of gravity in 3-D emerges naturally in the context of where the body should be balanced, along with the means of predicting it in the case of a composite body.

Separation of bodies

Fewer concepts are more pivotal to statics, and the basis for more errors, than the ideas that (i) every force acts between two bodies (usually in contact) and that (ii) equations of equilibrium always pertain to an explicitly identified body. In the previous examples, utilizing the L-shaped object, the agent of each force, *and that there is an agent at all*, is not highlighted. Here, we seek to highlight this idea, but, again, with recourse only to forces that can be *felt*. Thus, we have students gain experience in dismembering systems,

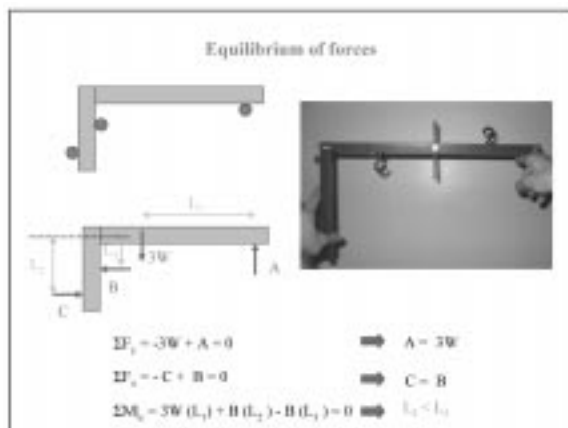


Fig. 1. Equilibrium in 2D.

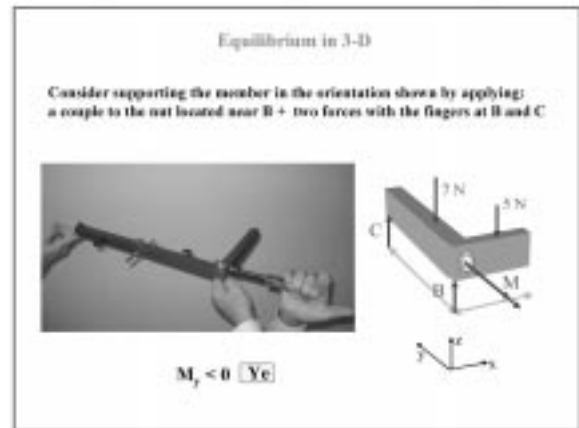


Fig. 3. Equilibrium with forces and couples in 3-D.

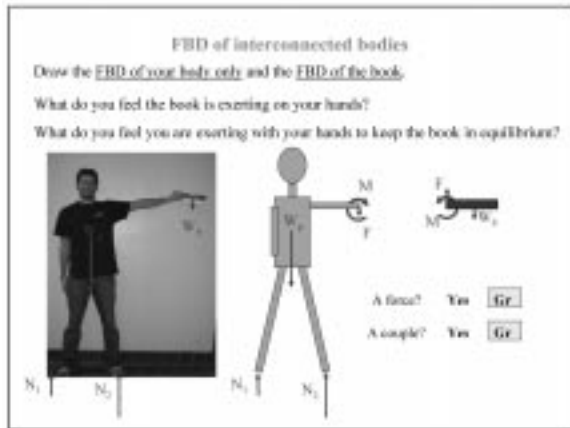


Fig. 4. Separating bodies in free body diagrams.

associating with each force the two acting bodies, and considering equilibrium for various subsets of the system. The example in Fig. 4 depicts a person supporting an object; when imposing equilibrium, the person and the book can be considered as a single body, or can be separated.

Contacting bodies – distributed forces

The concepts developed in the remainder of the paper are important in their own right. They also pave the way for a deeper understanding of the forces and couples that act between connected bodies: the ultimate application of engineering statics. After having experience with the concept of static equivalence, students are prepared to understand the idea of a uniformly distributed force and its equivalence (statically) to a single force acting through the center. This helps explain the sense in which the weight force acts through the center of a rectangular body. With the concept of a uniformly distributed force, we offer students their first experiences with problems that require separation of contacting *inanimate* bodies (Fig. 5a). Such exercises must serve to dispel the false idea that the normal force is equal to the weight, rather than providing whatever force that is necessary to maintain equilibrium.

Students next confront the idea that bodies in contact with one another over an extended surface (rather than at a point) may interact with a *non-uniform* distribution of force. Through various means (static equivalence arguments and observing deformation), students learn that the net force may not act through the center, but somewhere else along the area of contact. Students then address problems in which the position of the net force of contact is to be determined from equilibrium (Fig. 5b).

Frictional contact forces and their net effects

Up to this point, we have focused on situations in which the frictional forces are negligible. However, the early treatment of concepts involving forces must prepare the ground for understanding the forces and couples that act between inanimate

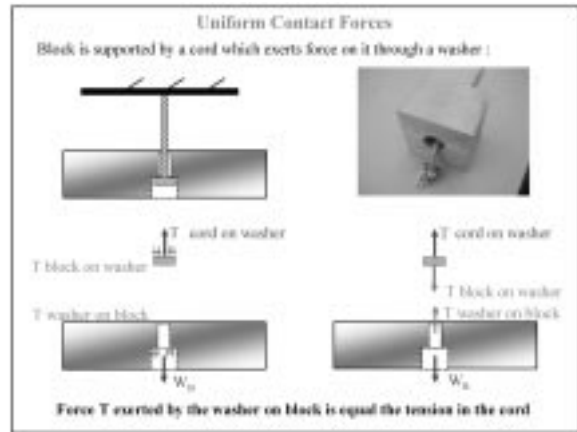


Fig. 5a. Uniformly distributed forces.

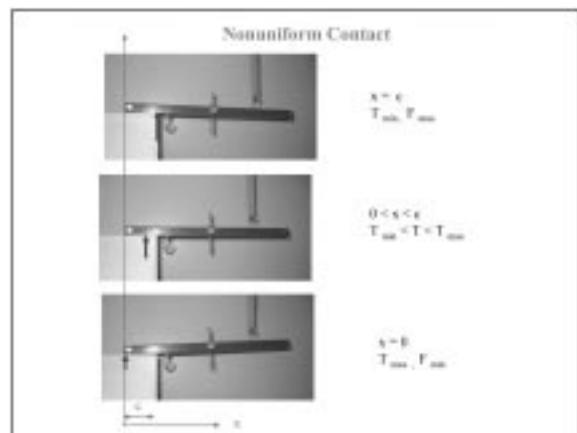


Fig. 5b. Changing position of net force associated with distributed loading.

connected bodies. We argue that only if students have had experience observing what friction forces *can* produce are they able to understand the implications of the *neglect* of friction, which underlies common models for loads at connections. Thus, we want students to appreciate that the friction force is exerted by two contacting bodies on one another; it is the component of the force acting tangentially to their common surface. Second, like the normal force, the friction force adjusts its value seeking to maintain equilibrium (to resist motion). This lays the groundwork for the distinction between the actual frictional force and the *upper limit* on the frictional force (μN), which are regularly confused by students. Concepts involving friction are conveyed again using simple objects that can be balanced by forces or couples applied by hand. Besides contributing to the net force on a body, we emphasize that combinations of friction forces acting at discrete points or in a distributed fashion can provide couples (Fig. 6). As a prelude to the later treatment of pin joints, we also focus on the hand gripping a cylinder loaded in various ways; we identify which reactions exerted by the gripping hand require friction and which do not.

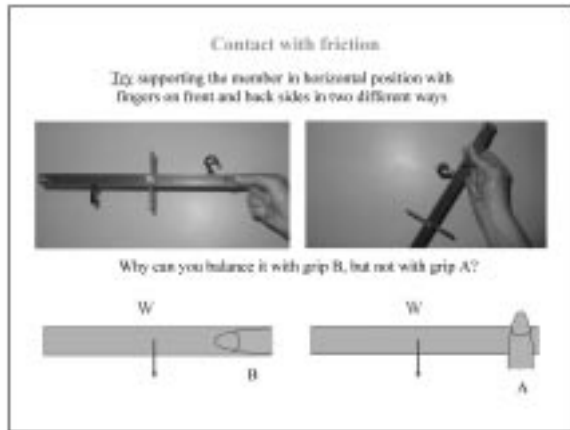


Fig. 6. Distributed friction forces acting over different areas create distinct balancing couples.

Equilibrium with normal and frictional forces

With the above set of concepts, in particular the ideas of static equivalence, distributed forces, friction and equilibrium, the student is now armed to confront a variety of problems in statics. For example, we can consider traditional problems, such as the block tipping or slipping on an incline plane, to which greater insight can now be brought. Namely, associated with contact is a potentially non-uniform distribution of force, which may have a net force acting at any possible point along the contact region. Moreover, the friction force seeks to maintain equilibrium, rather than necessarily to equal μN . We can also consider more unusual problems, such as the equilibrium of a human body in the position shown (Fig. 7). Such a problem ultimately draws together many concepts, including equilibrium in 3-D, the change in position of the net normal force, and that frictional forces can provide both a force and a couple needed for equilibrium.

SUMMARY

Engineering statics is critical to many majors, forming the core prerequisite for a number of other key courses. Yet, most students emerge from

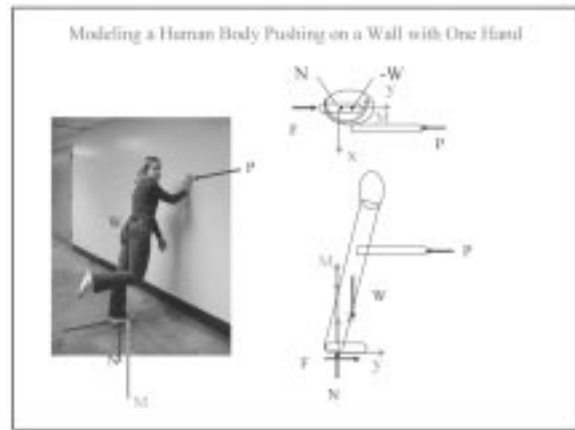


Fig. 7. Net forces and couples maintain the human body in equilibrium.

statics unable to effectively use its ideas and methods to solve engineering problems. Based on a critique of traditional statics instruction, on a recognition of what must be at the core of learning in statics, and on the difficulty students have in perceiving forces between inanimate objects, we have undertaken a radical reorganization of the sequence of topics and concepts addressed in engineering statics. In particular, we have reorganized topics so as to build systematically on one another, and to be addressed entirely in the context of situations where all relevant forces can be perceived through the senses of touch and sight. With this approach we seek to better prepare students to address the situations of traditional interest in statics, where forces are exerted by inanimate parts of machines and structures. The sequence of concepts is addressed in class through the use of learning modules which involve collaboratively manipulating objects and responding to conceptual questions. Examples of several learning modules and additional details of classroom implementation are given in the accompanying paper.

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Paul S. Steif received undergraduate and graduate degrees from Brown University and Harvard University in engineering mechanics. He is currently Professor of Mechanical Engineering at Carnegie Mellon University. He has been active as a teacher and researcher in the field of engineering mechanics, for which he has received a number of awards. Steif is currently involved in research studying student learning in basic engineering subjects, measuring student conceptual progress, and constructing educational materials that facilitate learning. Many of these developments have reached an international audience, including educational software which is published with widely selling textbooks.

Anna Dollár received both her Master's degree and doctorate in applied mechanics from Krakow University of Technology in Krakow, Poland. She was an Assistant Professor at the Illinois Institute of Technology in Chicago, and is currently Associate Professor at Miami University in Ohio. At IIT she received the departmental Excellence in Teaching award, and the University Excellence in Teaching Award. At Miami she received the School of Engineering and Applied Science Outstanding Teacher award. Her research focuses on the mechanics of solids and engineering education.