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An Undergraduate CACSD Project: the Control of Mean Arterial Blood Pressure during Surgery*

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An appreciation of the basic ideas behind the tuning of conventional proportional-integral-derivative (PID) controllers should be a fundamental requirement of any introductory control course whether it is delivered in an Electrical, Mechanical or Chemical Engineering Department. This contribution presents a computer-aided control system design (CACSD) assignment that the authors use to teach students about system identification from process step-response data and subsequent PI/PID controller design using simple tuning relationships. The biomedical system considered here, namely the control of mean arterial blood pressure (MAP) in patients during surgery, embodies a number of interesting practical considerations that need to be taken into account when carrying out the control design.

INTRODUCTION

AN APPRECIATION of the basic ideas behind the tuning of conventional proportional-integral-derivative (PID) controllers should be a fundamental requirement of any introductory control course whether it is delivered in an Electrical, Mechanical or Chemical Engineering Department. The justification for inclusion in the Chemical Engineering syllabus is trivial as at least 90% of all control loops in the process industries use PI/PID controllers. With regard to Electrical Engineering it is a fact that many engineering companies will seek to employ only electrical engineering graduates as control engineers. Traditionally Electrical Engineering Control Courses tend to emphasize more mathematical conceptual design methods, such as pole-zero placement and optimal control. Examining some of the most popular textbooks used on these courses any aspects of conventional PID controller tuning are usually relegated to the frequency response design chapters. This obviously leaves a gap in the students' education with regard to the use of simple PID tuning methods based on process step responses—the most popular practical approach to achieving a control design in the process industries. A Mechanical Engineering Control Course should consider both mechatronic and process systems: process systems because of the fluid dynamics and heat transfer elements in the syllabus.

This contribution presents a computer-aided control system design (CACSD) assignment that the authors use to teach students about system identification from process step-response data and subsequent PI/PID controller design using simple tuning relationships. Ideally this assignment is used to complement a control laboratory but unfortunately in these days of depleted laboratory resources in many universities the assignment can also be used to provide undergraduates with perhaps their only insight into the practical design of process control systems.

The main aspects of the assignment are:

- System identification of first-order-plus-time delay (FOPTD) process models from step response data.
- The use of the identified approximate models for PI/PID control design using simple controller tuning methods.
- The analysis, via simulation, of the different performance obtained with different approximate models.
- Investigating how the integration of practical control constraints affects the control performance.
- Examining the resultant controller performance when the underlying process characteristics change.

The basic CACSD assignment structure has been used with a number of processes, either chemical or biomedical in nature, biomedical systems being

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sometimes used to expose the students to a wider range of control applications. The biomedical system considered here, namely the control of mean arterial blood pressure (MAP) in patients during surgery, embodies a number of interesting practical considerations that need to be taken into account when carrying out control design.

The software tools used for the assignment are MATLAB and SIMULINK. Usually the basic SIMULINK file for the PID control of the biomedical system will be given to the students so they are able to concentrate all their energy on understanding the model identification and control design elements of the assignment.

THE CONTROL PROBLEM

There are many instances in healthcare in which a patient's mean arterial blood pressure (MAP) must be rapidly reduced. The most common situation is the need to reduce a patient's MAP following cardiac surgery to promote healing. The most prevalent means of lowering the MAP is by infusion of the vasodilator drug sodium nitroprusside (SNP). Because of the characteristics of SNP, that is it is fast acting and produces a toxic metabolite (thiocyanate), its infusion must be carefully monitored which can be extremely time consuming for clinical staff. Much research has been concentrated on investigating an automatic means of controlling MAP and has confirmed the improvement that a closed-loop control system can offer.

The practical control design considerations concern both the desired performance specification and constraints on both the rate of change and the maximum value of the infused drug. Additionally both the time-delay and gain associated with the MAP model can be expected to change throughout the course of the operation. A minimum desired control performance for set-point changes in MAP of 20 mmHg or above was proposed by Slate [1] and includes a 20% settling time of less than 10 minutes, a maximum overshoot of less than 10 mmHg, and a steady-state error within ± 5 mmHg. Martin, et al. [2], subsequently reduced the overshoot specification to 10% and this will be used here in addition to the other specifications. Along with these performance characteristics the controller also has some clinical constraints. The first of these is a maximum allowable infusion rate. The reason for this is that SNP is metabolised by the body into cyanide, and hence too much SNP can end up being toxic to the patient. The other constraint is that incremental increases in the infusion rate should be limited in magnitude—this is to prevent rapid decreases in MAP which can cause diminished blood flow or circulatory collapse.

THE SYSTEM MODEL

The patient blood pressure model used here was developed by Martin, et al. [2], and is a modification of the model initially proposed by Slate [1] for use in automatic control studies. The transfer function for this model is:

$$G_{MAP}(s) = \frac{Y(s)}{U(s)} = \frac{K(1+\tau_3 s)e^{-\theta s}}{[(1+\tau_3 s)(1+\tau_2 s)-\alpha](1+\tau_1 s)},$$

$$P(s) = P_O - Y(s).$$

where $P(s)$ is the arterial pressure, P_O is the initial pressure, $Y(s)$ is the drop in the pressure due to SNP and $U(s)$ is the rate of infusion of SNP into the patient. The remaining parameters are as follows: K is the system gain, θ is the system delay, τ_1 is the time constant of the SNP action, τ_2 is the time constant of the flow through pulmonary circulation, τ_3 is the time constant for flow through a systematic circulation and α is the fraction of SNP recirculated.

Many biomedical systems, like process systems, are characterised by time delays and the response of a patient's MAP to drug infusion using SNP is no exception. There are three contributors to the system delay;

1. the drug infusion rate through the catheter;
2. internal patient circulation and perfusion delay;
3. the SNP recirculation characteristics of the patient.

The following representative parameter values are to be used [2]:

$$K = 2.5(\text{mmHg/Cml/h});$$

$$\tau_1 = 50(\text{sec}); \quad \tau_2 = 10(\text{sec});$$

$$\tau_3 = 30(\text{sec}); \quad \alpha = 0.5; \quad \theta = 60(\text{sec}).$$

Giving the following system transfer function:

$$G_{MAP}(s) = \frac{5.0(1+30s)e^{-60s}}{(1+130s+4600s^2+30000s^3)}.$$

SYSTEM IDENTIFICATION FROM STEP RESPONSE DATA

After the students are introduced to the control problem, and the system model, $G_{MAP}(s)$, it is indicated that they will be using first-order plus time delay (FOPTD)-based PI/PID controller tuning. Therefore the first part of the assignment requires that a FOPTD approximate system model be obtained from the defined system. The FOPTD model has the following transfer function structure:

$$G(s) = \frac{K\ e^{-\theta s}}{(1+\tau s)}.$$

To emphasise to the students the importance of system identification and ultimately good models

in control design they are told to try two different step response-based approaches to obtaining the FOPTD model. The first method is the standard modelling technique based on drawing a tangent at the point of inflection on the process reaction curve and using this to find values of the time delay and time constant [3]. The model gain is easily found from the steady-state response. This approach is very simple but quite approximate in the sense that it uses only one point on the transient part of the step response. Usually, within each class there are many different interpretations of where the point of inflection occurs on the process step response. This tends to lead to a wide range of identified FOPTD models from the same data. For a unit step change in drug infusion rate the following model was obtained using this approach:

$$G_1(s) = \frac{5\ e^{-66.9s}}{1 + 130s}.$$

The second identification approach is also step response-based but avoids the use of the point of inflection on the process step response. The method of Sundaresan and Krishnaswamy [4] provides design equations that relate the time delay, θ, and time constant, τ, (of the FOPTD model) to two time points, t_1 and t_2, of the transient part of the step response. These correspond to 35.3% and 85.3% of the final process step response value:

$$\theta = 1.3t_1 - 0.29t_2; \quad \tau = 0.67(t_2 - t_1).$$

Using this approach the following FOPTD model was obtained:

$$G_2(s) = \frac{5e^{-78.6s}}{1 + 84.4s}.$$

When the two FOPTD models have been obtained the students are asked to produce a plot that shows the responses of the underlying biomedical system model and the two identified FOPTD models to the same input. They are then asked to comment on the relative accuracy of the identification methods.

CONTROLLER TUNING METHODS

There are an enormous number of available controller tuning methods based on low-order approximate, such as FOPTD, system models. To help the students to more easily understand the concepts of control tuning and performance evaluation only time-domain approaches to PI/PID controller tuning are used in this assignment. In addition only the controlled response to system set-point changes are considered because this is the case for which the performance specification is defined. The two PID controller tuning methods used here are:

- Cohen and Coon—tuning relationships [5] which claim to give closed-loop responses with a decay ratio of ¼.
- Integral error criteria based tuning methods [6] that consider the performance of the whole controlled response instead of just the transient part.

It is recognised that nowadays with stricter control specifications the Cohen and Coon tuning relationships provide responses, for set-point changes, that are too oscillatory and the students are told this. For the controller tuning relationships based on integral error criteria the students are introduced to two performance indices:

Integral of the absolute value of error (IAE)

$$IAE = \int_0^\infty |e(t)|dt$$

Integral of the squared error (ISE)

$$ISE = \int_0^\infty [e(t)]^2 dt$$

where t is the time and the error signal $e(t)$ is the difference between the set point and the controlled output.

These indices are those most commonly considered for controller design in the process and electrical industries. At this point in the assignment the students are asked to choose which integral error criteria tuning method they will be using. In this description of the assignment only the IAE-based approach is considered. Relevant controller tuning formulae are given below:

Cohen and Coon: PI-controller

$$K_c = \frac{1}{k}\frac{\tau}{0}\left(0.9 + \frac{\theta}{12\tau}\right), \quad \tau_1 = \frac{\theta(30 + 3(\theta/\tau))}{9 + 20(\theta/\tau))}.$$

IAE: PI-controller

$$K_c = \frac{a_1}{K}\left(\frac{\theta}{\tau}\right)^{b_1}, \quad a_1 = 0.758 \quad b_1 = -0.861$$

$$\tau_1 = \frac{\tau}{a_2 + b_2\left(\frac{\theta}{\tau}\right)}, \quad a_2 = 1.02 \quad b_2 = -0.323$$

IAE: PID-controller

$$K_c = \frac{a_1}{K}\left(\frac{\theta}{\tau}\right)^{b_1}, \quad a_1 = 1.086 \quad b_1 = -0.869$$

$$\tau_1 = \frac{\tau}{a_2 + b_2\left(\frac{\theta}{\tau}\right)}, \quad a_2 = 0.74 \quad b_2 = -0.13$$

$$\tau_D = a_3 \quad \tau\left(\frac{\theta}{\tau}\right)^{b_3}, \quad a_3 = 0.348 \quad b_3 = 0.914$$

where K_c is the proportional gain, τ_I the integral reset time and τ_D the derivative time.

The students, in their control lectures, will have already been introduced to simple transient time-domain performance measures, such as percentage overshoot, rise time, and settling time. Another objective of the assignment is to extend their knowledge and understanding of this area. As well as using a controller tuning method based on an integral error criterion the students are also asked to use an integral error method to calculate the controller performance, as well as using simple time-domain criteria.

The students are told to use the integral error measure of performance that matches their choice of integral error criteria-based tuning method. The integral error measure of performance is also used for the Cohen and Coon control studies.

CONTROLLER DESIGN AND SIMULATION STUDIES

At this point each student should have two identified FOPTD models, $G_1(s)$ and $G_2(s)$, an idea about which is the most accurate one as well as having chosen an integral error criteria-based tuning method to use in the control studies. The students are now asked to do the following calculations:

- Design a PI controller using Cohen and Coon for both of their identified FOPTD models.
- Design a PI controller using the chosen integral error criteria (IAE or ISE) for both of the identified FOPTD models.

For the identified models the following values are found:

- *C-C PI Controller* for $G_1(s)$ $K_c = 0.366$, $\tau_I = 109.4$; for $G_2(s)$ $K_c = 0.210$, $\tau_I = 93.3$.
- *IAE PI Controller* for $G_1(s)$ $K_c = 0.269$, $\tau_I = 152.3$; for $G_2(s)$ $K_c = 0.161$, $\tau_I = 117.4$.

Control studies using Cohen and Coon

Initially the students are asked to compare the PI controller performance using the Cohen and Coon parameters calculated from $G_1(s)$ and $G_2(s)$. The set-point changes in MAP are 20 mmHg to match those given in the desired performance specification (see above). The responses, both decreases in MAP, $Y(s)$, and drug infusion rate, $U(s)$, have to be plotted with the students also being asked to record the time response performance criteria, rise-time, percentage overshoot and settling time in addition to the calculated integral error performance for $Y(s)$.

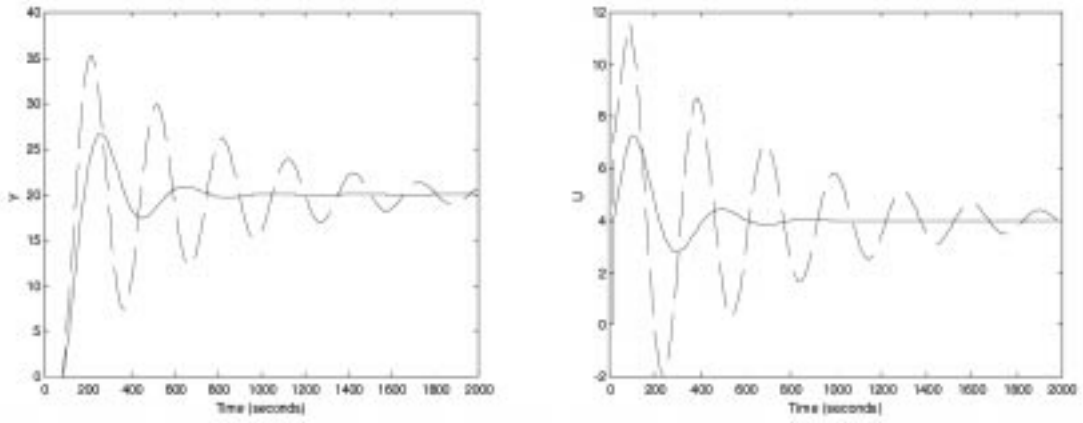
Figures 1(a) and 1(b) show the responses for $Y(s)$ and $U(s)$ respectively. The responses of the $G_1(s)$-based controller are shown by the dashed line plot. They are asked to comment specifically on the difference in control performance for the two model-based designs as well as also making general comments on the effect of model approximation on the controlled performance.

Control studies using integral error criteria

The students are then asked to repeat the previous exercise but instead use their integral error criteria-based PI controllers. They are again asked to record the time response performance criteria as well as calculate the integral error performance for $Y(s)$ as well as comment on the effect of model approximation on the controlled performance. Figure 2(a) and (b) show the responses for $Y(s)$ and $U(s)$ respectively. The responses of the $G_1(s)$-based controller are shown by the dashed line plot.

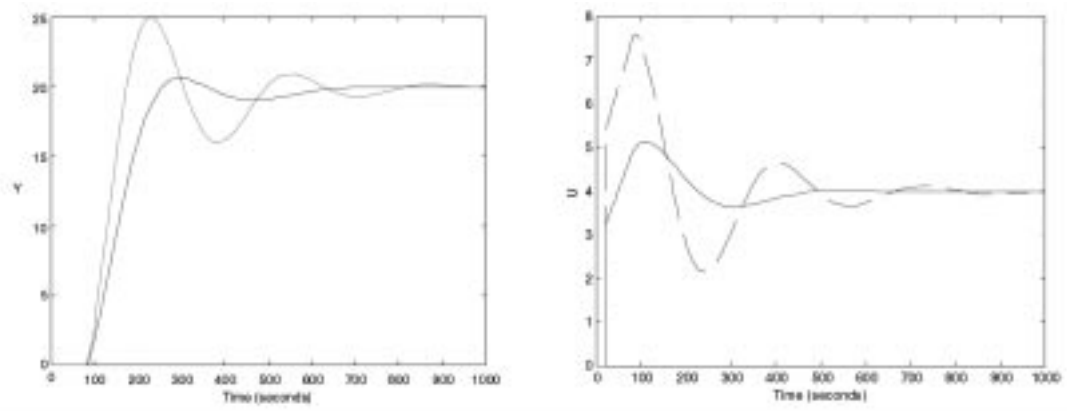
PI controller design comparison

The students are then asked to compare the performance of the Cohen and Coon tuning method with that of their integral error criteria approach. They are asked to plot, on the same graph, the 'best' PI controlled set-point response from the Cohen and Coon study and also from the integral error method study. They are asked to comment on the relative performance of the two



(a) (b)

Fig. 1. Comparison of Cohen-Cohen tuned PI control using the two different identified FOPTD models: (a) MAP response; (b) drug infusion rate.



(a) (b)

Fig. 2. Comparison of IAE tuned PI control using the two different identified FOPTD models: (a) MAP response; (b) drug infusion rate.

design methods, using the performance indicators and the given desired performance specification, and choose the 'best' approach. Both of the $G_2(s)$-based controller designs satisfy the settling-time performance specification but only the IAE-based controller satisfies all specifications.

PI/PID controller comparison

The next stage is to compare the relative performance of PI and PID control. The students are asked to select what they consider to be their best PI controller and then use the same FOPTD model and design method to find the corresponding PID controller values. Assuming that the model $G_2(s)$ and the IAE-based tuning approach are chosen the corresponding PID controller parameters are found to be $K_c = 0.231$, $\tau_I = 136.4$ and $\tau_D = 27.49$.

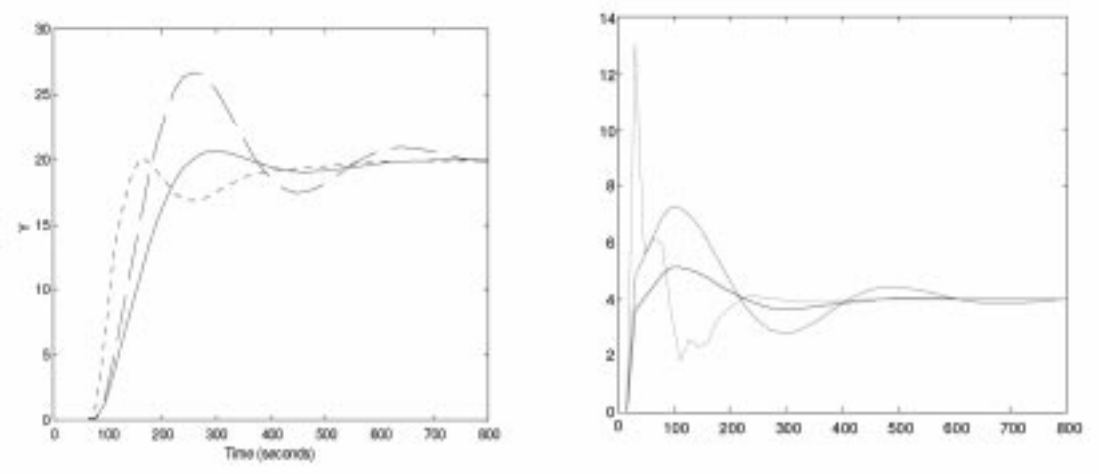
The students are asked to comment on the relative performance of the two controllers.

Figure 3 compares the PI and PID controller responses and drug infusion rates using the IAE tuning method and model $G_2(s)$. The responses for the 'best' Cohen and Coon tuned PI controller are also included for comparison purposes. The PID controller improves the rise-time and settling time of the MAP response over the PI controlled response but at the expense of a sharp spike-like increase in the drug infusion rate. The tighter control provided by the PID controller is also reflected in the recorded IAE performance—IAE-PID is 2356 while for IAE-PI it is 2993. The IAE performance of the Cohen and Coon PI controller is 3595.

Clinical constraints

By this point in the assignment the students should, hopefully, have an appreciation of the relative attributes of Cohen and Coon versus IAE tuning rules and PI versus PID controller performance. The next step is to introduce the practical control considerations, namely the clinical constraints, and determine what effect, if any, they have on the performance of the designed controllers.

The first consideration is to limit the maximum rate of drug infusion, which is needed to negate the possibility that too much SNP can be toxic to the patient. This value is a function of the patient



(a) (b)

Fig. 3. (a) MAP response. PI-IAE (full line), PI-C-C (dashed) and PID-IAE (dotted). (b) corresponding drug infusion rates. All designs use FOPTD model $G_2(s)$.

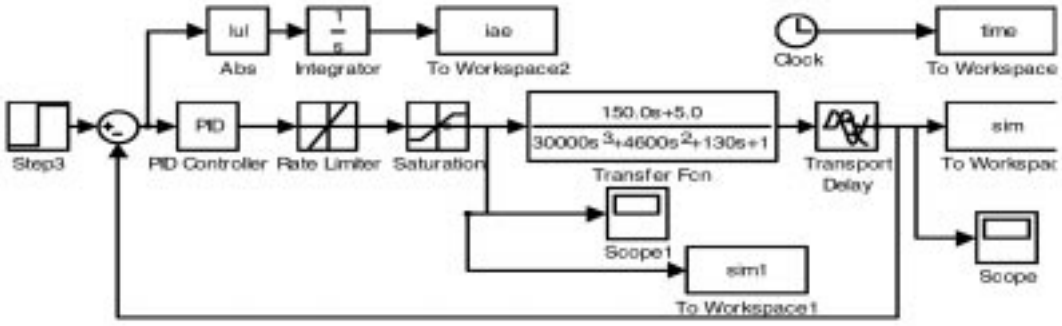


Fig. 4. SIMULINK block diagram for the constrained MAP control system.

weight and drug concentration and can be calculated from the following expression:

$$U_M = W_p\, i_M\, C_s^{-1}$$

where W_p is the patients weight (kg), C_s is the drug concentration (5000 μg/ml) and i_m is the maximum recommended dose (600 μg/kg hr). A patient of weight 70 kg (each student is given a different value of patient weight in the range 50 kg to 130 kg to consider) will give a maximum drug infusion rate of 8.4 ml/hr. It can be seen from Figs 1(b), 2(b) and 3(b) that the only control designs not affected by this constraint are the IAE-based PI designs.

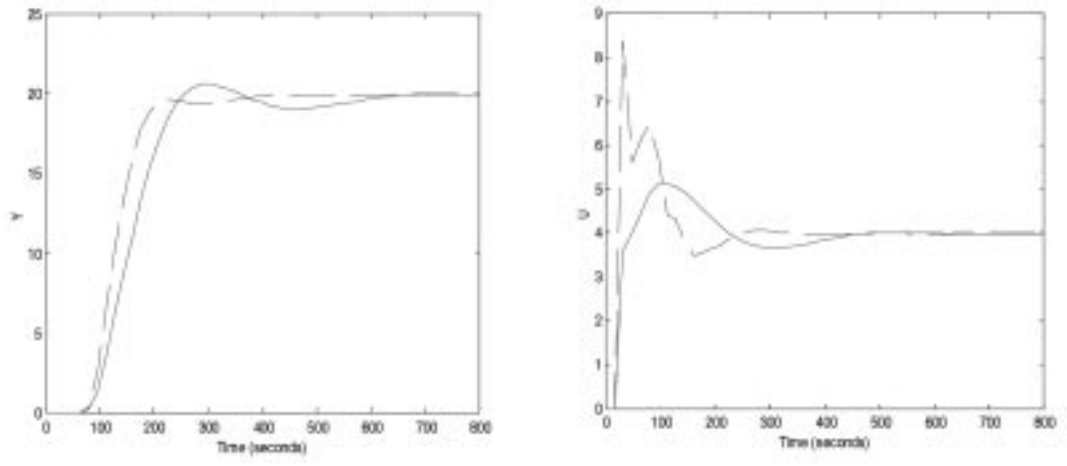
The second constraint is a limit on the rate of change of drug infusion rate. This is required so that there are no large drug-induced changes in the patient's MAP. In this case the maximum allowable incremental change in the drug infusion rate, δU_M , is limited to 20% of the maximum allowable drug infusion rate calculated above, that is 1.68 ml/hr.

Figure 5 compares the responses for the constrained PI and PID controllers. The spike in the drug infusion for the PID controller has been removed but this seems to have little effect on the controlled response when compared with the unconstrained response. The same can be said for the constrained and unconstrained PI controlled responses.

Process model mismatch

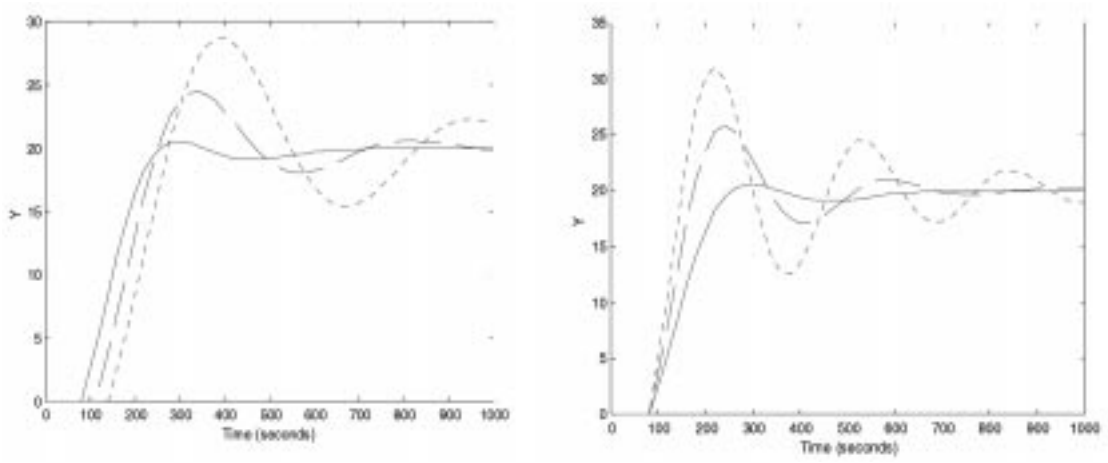
The final part of the assignment involves investigating how robust the designed controller is to errors in both the gain and then the time delay. A patient's characteristics can change fairly rapidly during an operation due to the effect of vasoactive drugs leading to possibly a doubling of the system gain and a substantial increase in the system time delay. In this case the student is asked to use their 'best' PI controller and investigate the performance in the face of these types of changes in $G_{MAP}(s)$.

Figure 6 shows the result of these studies using the constrained IAE-PI controller. Figure 6(a) examines changes in time delay in the system of +50% and +100% and compares these with the original response. It can be seen that both the overshoot and settling time increase with the increase in time delay error. The effect of applying a gain error of +50% and +100% to the controlled system is shown in Fig. 6(b). Compared with the system time-delay increases the controlled overshoot increases more although the frequency of oscillation of the response is faster leading to slightly faster settling times. The constrained PI controller still works fairly well despite these changes.



(a) (b)

Fig. 5. (a) MAP responses for constrained PI (full) and PID (dashed) control. (b) Corresponding drug infusion rates.



(a) (b)

Fig. 6. Process model mismatch studies on the constrained PI controller design (a) +100% time delay error (dotted line), +50% time delay error (dashed line). (b) +100% gain error (dotted line), +50% gain error (dashed line).

DISCUSSION

This CACSD assignment is intended to provide students, during an introductory control course, with the opportunity to try out modelling, control analysis and design knowledge on a meaningful and interesting control problem. It is also an attempt to bring various disparate elements (from the students point of view) together to show how these seemingly totally different elements combine to tackle the control of process systems.

The basic structure of the assignment is demonstrated using the example of the control of a biomedical system. Depending upon the actual application used and emphasis that the lecturer wants to give to different parts of the control course then elements can be introduced (for example investigating the accuracy of using Pade approximations in analyzing time-delay systems, the use of proportional control as well as controlling against disturbances) or removed.

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