The Role of Interactivity in Control Learning*

S. DORMIDO, S. DORMIDO-CANTO, R. DORMIDO, J. SÁNCHEZ and N. DURO
Dept. Informática y Automática, ETS Ing. Informática, UNED, Juan del Rosal 16, 28040 Madrid, Spain.
E-mail: sdormido@dia.uned.es

The scenario for control education is changing and we must adapt to the new situation. Information technology opens a whole new world of real opportunities. Computers show a great potential to enhance student achievement, but only if they are used appropriately as part of a coherent education approach. Computers do not change in the way books or labs do—they allow us to go deeper and faster. This paper presents the personal experience of the authors in the use of interactive tools in order to make students more active and involved in their own control engineering learning process. Some examples, with different degrees of complexity, have been selected in order to show how we can use the control visualization concept in a new family of interactive tools for control education.

INTRODUCTION

THE PRESENT SCIENTIFIC and technological environment offers unprecedented challenges and opportunities in order to apply the impressive advances produced in the last years in information technology to control education.

Because of the amazing progress made in computer technology, today it is possible to design 'control education tools' with the following characteristics:

- better man-machine interaction;
- natural and intuitive graphical user interfaces;
- high degree of interactivity.

Automatic control ideas, concepts and methods are so rich in visual content that they can be represented intuitively and geometrically. The visual content can not only be used for presenting tasks and handling concepts and methods, but also manipulated for solving problems. Our feeling is primarily visual and thus it is not surprising that visual support is so present in our teaching process. Control educators very often make use of symbolic processes, visual diagrams and other forms of imaginative processes in their explanations.

The basic ideas in automatic control often arise from very specific and visual situations. All teachers know how useful it is to go back to this specific origin when they want to explain skillfully to the students the corresponding abstract objects.

This way of acting with explicit attention to potential specific representations in order to explain the abstract relations that are of interest to the control expert is what we term *control visualization* The fact that visualization is an espe-cially important aspect in the control expert's activity is something completely natural if we bear in mind the applied mathematics nature of control theory.

Visualization thus appears to be something profoundly natural in the origins of automatic control, in the discovery of new relations between mathematical objects, and in the transmission and communication of our control knowledge.

One of the important tasks for teachers in control engineering is to transmit to our students not only the formal and logic structure of the discipline but also, and certainly with much more emphasis, the strategic and intuitive aspects of the subject [1]. These strategic and intuitive aspects are probably much more difficult to make explicit and assimilate for students precisely because they lie very often in the less conscious substrata of the expert's activity.

Interactive tools, which are accessible to students at any time on the Internet, are considered a great stimulus for developing the student's engineering intuition. These interactive tools attempt to 'demystify' abstract mathematical concepts through visualization for specifically chosen examples. At the present time, a new generation of software packages have created an interesting alternative for the interactive learning of automatic control [2–4].

This paper presents our personal experience in the use of interactive tools in order to make students more active and involved in their own control engineering learning process. The strategy that we will follow will be to show different examples of how the control visualization concept can be used in a new family of interactive tools for control education. The examples have a different degree of complexity but all share a common philosophy: the use of interactivity to improve

the learning process in control engineering. The ultimate goal is to facilitate comprehension of the concepts that we are trying to transmit to our students.

INTERACTIVITY IN CONTROL EDUCATION

Bearing in mind these general considerations, the computer can be regarded as a tool that allows us to visualize and manipulate, in an interactive way, control objects.

In order to design technical systems or simply to understand the physical laws that describe their behavior, scientists and engineers often use computers to calculate and graphically represent different magnitudes. In control engineering, these quantities include among others: time and frequency responses, poles and zeros on the complex plane, Bode, Nyquist and Nichols diagrams, phase plane, etc. Frequently these magnitudes are closely related and constitute different visions of a single reality. The understanding of these relationships is one of the keys to achieve a good learning of the basic concepts and it enables students to carry out control systems design accurately.

Many tools for control education have been developed over the years. Innovative and interesting ideas and concepts were implemented by Prof. Åström and colleagues at Lund. In this context we should highlight the concepts of *dynamic pictures* and *virtual interactive systems* introduced by Wittenmark [5]. The main objective of these tools is to make students more active and involved in control courses.

In essence, a dynamic picture is a collection of graphical windows that are manipulated by just using the mouse. Students do not have to learn or write any sentences. If students change any active element in the graphical windows an immediate recalculation and presentation automatically begins. In this way they perceive how their modifications affect the result obtained. Dynamic pictures cannot only be effective in presenting engineering concepts in the classroom but also beneficial in extending student experience in analysis and design assignments. This invitation to creativity can be most useful where specialized control engineering student projects are concerned.

This strategy causes us to 'think small and simple'. This is justified by a frank assessment of our limited knowledge for designing educational software as well as by practical considerations about how to manage incremental innovation. As dynamic pictures are fairly easy to create and deploy, they provide a means for rapidly prototyping and testing control principle ideas. In particular, they can be used as sharp tools for investigating precisely what it takes to make a control concept known to students. In this way, the 'virtue of simplicity' becomes an issue in

learning research on the design and use of this kind of tool.

These tools are based on objects that allow direct graphic manipulation. During these manipulations, the objects are immediately updated, so that the relationship among the objects is continuously maintained. *Ictools* [6] and *CCSdemo* [5], developed at the Department of Automatic Control at Lund Institute of Technology, and *SysQuake* at the Institut d'Automátique of the Federal Polytechnic School of Lausanne, [7, 8] are good examples of this new educational philosophy for teaching automatic control.

For those that begin learning in this field some concepts are initially difficult to grasp, due to the fact that their properties are expressed in two different domains: time and frequency. Transient behavior, such as settling time, overshoot, and the risk of saturation are analyzed typically in the time domain, while concepts like stability, noise rejection and robustness are expressed more easily in the frequency domain. The basic mechanisms that connect them and other phenomena like, for example, the effects of sampling and nonlinear elements, to mention just a few, can be illustrated in a very effective way using these tools [9].

THE PARADIGM OF INTERACTIVE DESIGN

Design involving a human being also benefits from a fast and intuitive approach because it lets the designer understand what is happening. Boring trial and error search from the best set of parameters is avoided. From a pedagogical point of view, effects deduced from simple systems and controllers can often be generalized and lead to an intuitive understanding of the underlying mathematical basis. From the point of view of the control design, the evolution in the use of interactive graphics as help can be divided into three phases.

1. *Manual calculation*. This period is previous to the availability of digital computers as a tool in the process design. The procedure consisted in the calculation of a few numerical values. Control engineers developed sets of rules in order to draw graphics by hand.
2. *Computers as an auxiliary tool*. With the advent of digital computers it was possible to make the creation of graphics much easier. Control engineers had the possibility of tuning the design parameters using a trial and error procedure following an iterative process. Specifications of the problem are not normally used to calculate the value of the system parameters because there is not an explicit formula that connects them directly. This is the reason for dividing each iteration into two phases, as it is shown in Fig. 1. The first one, often called *synthesis*, consists in calculating the unknown parameters

of the system taking a group of design variables (that are related to the specifications) as a basis. During the second phase, called *analysis*, the performance of the system is evaluated and compared to the specifications. If they do not agree, the design variables are modified and a new iteration is performed.

3. *Computers as an interactive tool*. It is possible however to merge both phases into one and the resulting modification in the parameters produces an immediate effect (see Fig. 2). In this way, the design procedure becomes really dynamic and the students perceive the gradient of change in the performance criteria given for the elements that they are manipulating. This interactive capacity allows us to identify much more easily the compromises that can be achieved.

Interactive design with instantaneous performance display goes one step further. In many cases, it is not only possible to calculate the position of a graphic element (be it a curve, a pole or a template) from the model, controller or specifications, but also calculate a new controller from the position of the element. For instance, a closed loop pole can be computed by calculating the roots of the characteristic polynomial, itself based on the model and controller; and the controller parameters can be synthesised from the set of closed loop poles if some conditions on the degrees are fulfilled [10].

This two-way interaction between the graphic representation and the controller allows the manipulation of the graphical objects with a mouse in a very natural form. Since a good desi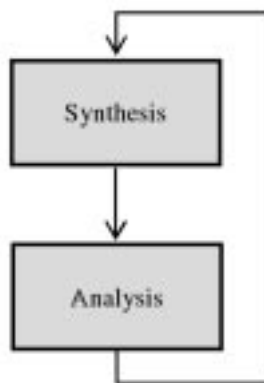gn usually involves multiple obj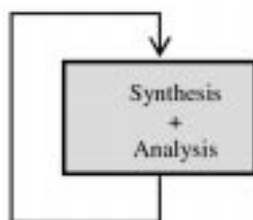ectives using different representations (time domain or frequency domain), it is possible to display several graphic windows that can be updated simultaneously during the manipulation of the active elements.

The philosophy of interactive design with instantaneous performance display offers two main advantages when compared with the traditional procedure (non-interactive approach). First, it introduces from the beginning the control engineer to a tight feedback loop of iterative design. The designers can identify the bottlenecks of their designs in a very easy way and can attempt to fix them. Second and this is probably even more important, not only is the effect of the manipulation of a design parameter displayed, but its direction and amplitude also become apparent. The control engineer learns quickly which parameter to use and how to push the design in the direction of fulfilling better tradeoffs in the specifications. Fundamental limitations of the system and the type of controller are revealed [11] and [12], giving a way to find an acceptable compromise between all the performance criteria. Using this interactive approach the students can learn to recognize when a process is easy or difficult to control.

Here are the main characteristic of the interactive design:

- The modification of the parameters produces immediately an update of the graphical scopes.
- The design process is completely dynamic.
- Students feel the gradient of the performance change in relation with the elements that they are handling.
- This interactivity allows to identify more easily the compromises can be reached.

Fig. 1. Non-interactive approach.

Fig. 2. Interactive approach.

ALIASING

Sampling is a fundamental property of computer-controlled systems because of the discrete-time nature of the digital computer. Sampling a continuous-time signal simply means to replace the signal with its values in a discrete set of points. The sampling instants are often equally spaced in time, that is $t_k = kh$. This case is called *periodic sampling* and h is called the sampling period. The corresponding frequency $\omega_s = 2\pi/h$ (rad/s) is called the sampling frequency. It is also convenient to introduce a notation for half the sampling frequency $\omega_N = \pi/h$ (rad/s), which is called Nyquist frequency.

The sampling process and the reconstruction of the signal can be described schematically as in Fig. 3. A sinusoidal signal of frequency ω and unit amplitude, $u(t) = \sin(\omega t)$, is applied to the A/D converter. This continuous-time signal is thus converted to a sequence of number, which is again converted to a continuous-time signal $y(t)$ by the D/A converter (signal reconstruction).
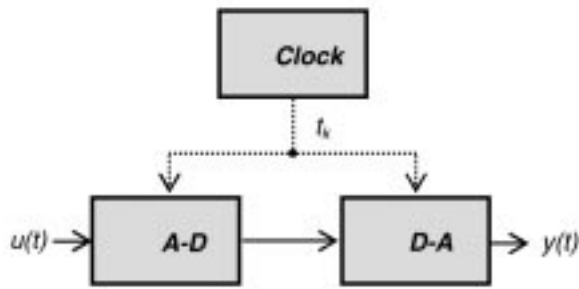
Fig. 3. Sampling process.

Although no information processing has been made by an algorithm inserted between the A-D and D-A converters, the output $y(t)$ presents in certain circumstances a very different form from that of the input $u(t)$.

If we consider sinusoids only, the basic fact is that to reconstruct a sinusoid from samples alone it is necessary and sufficient to have more than two samples per cycle (sampling theorem). If we have much more than two samples per cycle the task of fitting the sinusoid is easier; but if we have less than two samples per cycle, then a lower frequency sinusoid can be fitted to the same data and we cannot tell which is the true signal from the samples alone.

Most reconstructive processes will fit the lowest frequency that is consistent with the data. The process whereby a fast signal is sampled and then reconstructed with a lower frequency signal having the same samples is called '*aliasing*' [13, 14].

We have the surprising observation that the sampling procedure does not only introduce loss of information but may also generate new frequencies. The new frequencies are due to interference between the sampled continuous-time signal and the sampling frequency ω_s. This interference can also introduce *beating* in the sampled-data signal

To explain this phenomenon by using interactive concept we need to describe in a simple way the modification of the signal frequency ω and the sampling frequency ω_s. The metaphor has been a '*frequency map*' where the user can drag and modify both frequencies ω and ω_s and see the effects on the output $y(t)$ in an immediate way.

This is illustrated in Fig. 4, which shows the output for four different frequencies. For frequencies smaller than the Nyquist frequency ($\omega < \omega_N$), the sampling theorem is fulfilled (Fig. 4a). At frequencies close to the Nyquist frequency, ($\omega = \omega_N - \epsilon$), there is a substantial interaction with the first alias, $\omega_N + \epsilon$. Typical beats of ϵ (rad/s) are thus obtained (Fig. 4b). For frequencies higher than the sampling frequency ($\omega > \omega_s$) creates a signal component with the alias frequency ($\omega_s - \omega$). The phenomenon of aliasing has a clear meaning in time. Two continuous sinusoids of different frequencies appear at the same frequency when sampled (Fig. 4c). We cannot, therefore distinguish between those based on their samples alone. In the case that $\omega = \omega_s$ the output of the D-A converter $y(t)$ is identically equal to zero (Fig. 4d).
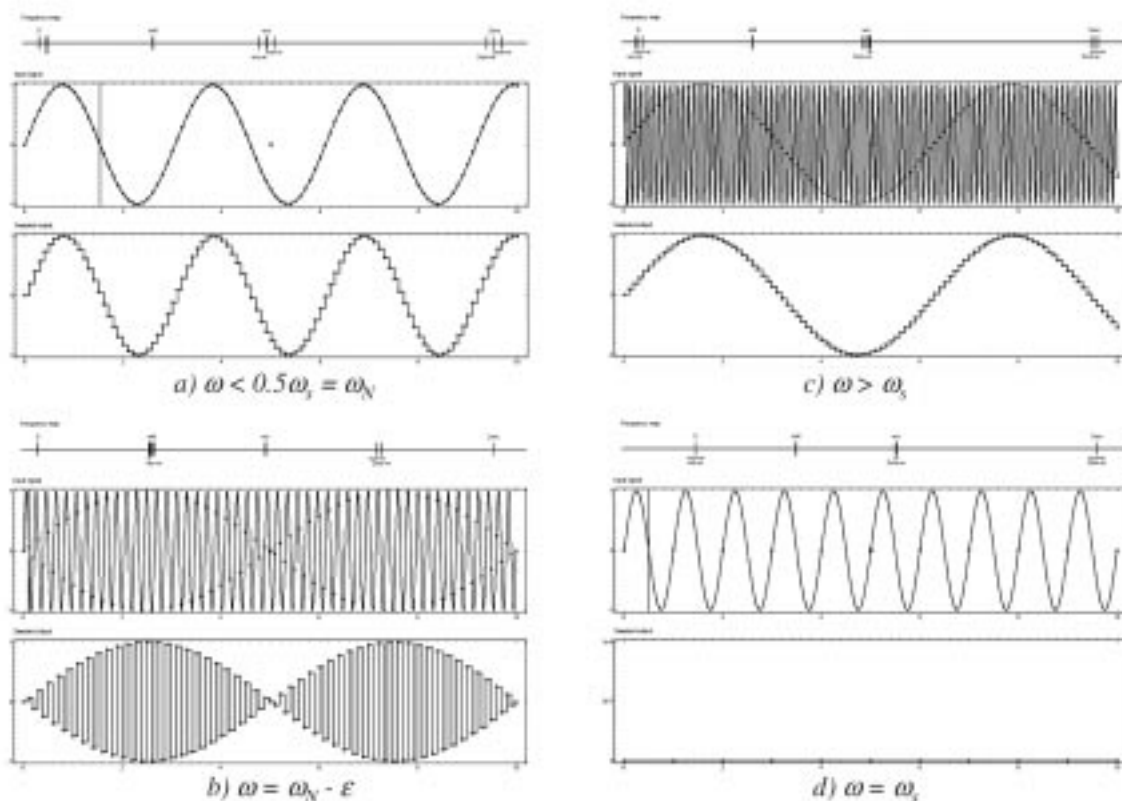

a) $\omega < 0.5\omega_s = \omega_N$

b) $\omega = \omega_N - \varepsilon$

c) $\omega > \omega_s$

d) $\omega = \omega_s$

Fig. 4. Sinusoidal excitation and sampled output of the system in Fig. 3.

BIFURCATIONS

One of the main goals of this example is to show how the students can be helped to develop a sound and practical understanding of the simplest bifurcations of fixed points for flows on the line [15].

Saddle-node bifurcation

The saddle-node bifurcation is the basic mechanism by which fixed points are created and destroyed. When a parameter is varied, two fixed points move toward each other, collide, and mutually annihilate.

The prototypical example of a saddle-node bifurcation is given by the first-order system:

$$\dot{x} = r + x^2$$

where r is a parameter, which may be positive, negative or zero. When r is negative, there are two fixed points, one stable and one unstable. When $r = 0$, the fixed points coalesce into a half-stable fixed point at $x = 0$. This type of fixed point is extremely delicate because it vanishes as soon as $r > 0$, and then there are no fixed points at all. In this case, we say that a bifurcation occurred at $r = 0$, since the vector fields for $r < 0$ and $r > 0$ are qualitatively different. The most common way to depict the bifurcation is to use the bifurcation diagram. Fig. 5 shows the bifurcation diagram for the saddle-node bifurcation. The parameter r is regarded as the independent variable, and the fixed points are shown as dependent variables.

Transcritical bifurcation

There are certain scientific situations where a fixed point must exist for all values of a parameter and can never be destroyed. For example, in the logistic equation and other simple models for the growth of a single species, there is a fixed point at zero population, regardless of the value of the growth rate. However, such a fixed point may change its stability when the parameter is varied. The transcritical bifurcation is the standard mechanism for such changes in stability. The normal form for a transcritical bifurcation is:
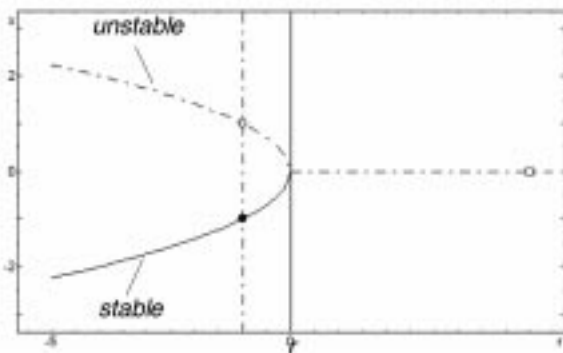
$$\dot{x} = rx - x^2$$

where we allow x and r to be either positive or negative.

For $r < 0$, there is an unstable fixed point at $x = r$ and a stable fixed point at $x = 0$. As r increases, the unstable fixed point approaches the origin, and coalesces with it when $r = 0$. Finally, when $r > 0$, the origin becomes unstable, and $x = r$ is stable. We can say that an exchange of stabilities has taken place between the two fixed points.

There is an important difference between the saddle-node and transcritical bifurcations: in the transcritical case, neither of the two fixed points disappears after the bifurcation, instead they just switch their stability. Figure 6 shows the bifurcation diagram for the transcritical bifurcation.

Supercritical pitchfork bifurcation

This bifurcation is common in physical systems that have *symmetries*. For example, many problems have a spatial symmetry between left and right. In such cases, fixed points tend to appear and disappear in symmetrical pairs.

There are two different types of pitchfork bifurcation. The simpler type is called supercritical. The normal form of the supercritical pitchfork bifurcation is:

$$\dot{x} = rx - x^3$$

Note that this equation is invariant under the change of variables $x \rightarrow -x$. When $r < 0$, the origin is the only fixed point, and it is stable. When $r = 0$, the origin is still stable, but much
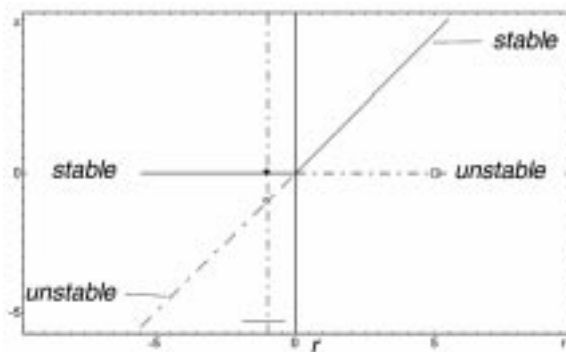

Fig. 6. Transcritical bifurcation diagram.

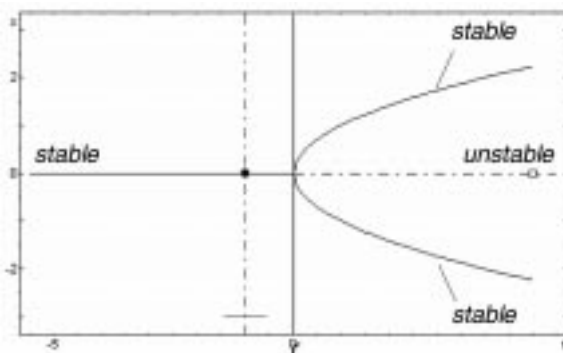Fig. 5. Saddle-node bifurcation diagram.

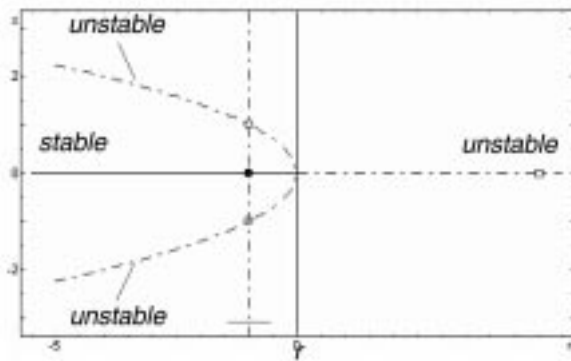Fig. 7. Supercritical bifurcation diagram.

Fig. 8. Subcritical bifurcation diagram.

more weakly so, since the linearization vanishes. Finally when $r > 0$, the origin becomes unstable. Two new stable fixed points appear on either side of the origin, symmetrically located at $x = \pm\sqrt{r}$. The reason for the term 'pitchfork' becomes clear when we plot the bifurcation diagram (see Fig. 7)

Subcritical pitchfork bifurcation

In the supercritical case, the cubic term is *stabilizing*: it acts as a restoring force that pulls $x(t)$ back toward $x = 0$. If instead the cubic term were *destabilizing*, as in:

$$\dot{x} = rx + x^3$$

then we'd have a *subcritical* pitchfork bifurcation. Figure 8 shows the bifurcation diagram. Compared to Fig. 7, the pitchfork is inverted. The nonzero fixed points $x = \pm\sqrt{-r}$ are unstable, and exist only below the bifurcation ($r < 0$), which explains the term 'subcritical'. More importantly, the origin is stable for $r < 0$ and unstable for $r > 0$, as in the supercritical case, but now the instability for $r > 0$ is not opposed by the cubic term. This effect leads to *blow-up*: one can show that $x(t) \rightarrow \pm\infty$.

LINEAR SYSTEMS ANALYSIS AND DESIGN

Classical control design methods normally use simplifying assumptions in the design procedure; for example, root locus design is mainly based on the location of the dominant second-order poles. In many occasions the controllers designed are good for giving students some insight into the problem and for designing an initial compensator. However the effects of the neglected dynamics can produce a significant deviation from the desired specifications. For this reason, students must simulate the system and iterate on their compensator until they get the fulfilment of the specifications. Unfortunately, most control text-books ignore this iteration scheme, concentrating instead on the initial design methods.

From the student's point of view, the iteration can be very time-consuming and confusing. Students would benefit a lot from an interactive

tool that automates much of the iteration and gives good insights about the progress of the design.

LSAD (short for Linear System Analysis and Design) is an interactive tool where a great part of the iteration involved in the design of classical controllers can be carried out in a simple way using an interactive and friendly graphical environment. Figure 9 shows the GUI windows of LSAD. The GUI windows can be modified in a very easy way in order to adapt the interface to the problem that must be solved. With this tool the students can solve in a graphical environment most of the exercises proposed in an introductory course on automatic control.

Compensator poles, zeros and integrators can be added or deleted through the Bode diagram, root locus, and Nyquist or Nichols plot. A pole or zero can be moved by first clicking on the 'Move' button in the 'Controller parameters' window, then clicking on the desired pole or zero (in whenever representation) and dragging it to another location. All the diagrams (Bode, Root locus, Nichols and Nyquist) update simultaneously during the move as well as the open loop and closed loop step response. Also some performance variables of the system are visualized: gain margin, phase margin, critical frequency, cross-over frequency, peak value, time to peak, rise time, settling time, steady-state error and velocity error.

In order to determine the compensator, different time and frequency specifications can be introduced using interactive elements. For example in Fig. 9 the activation of the following specifications is shown: low frequency gain, phase margin, high frequency attenuation, M_p peak and absolute and relative damping. Thus students get an immediate feeling on how moving a compensator pole or zero influences the design procedure. This enables them to gain a deeper knowledge into the tradeoffs in the design process rather than using a manual time-consuming procedure.

Next an illustrative example is presented that shows the interactive design technique and use of the LSAD tool in enhancing the classical control design methods as they are now commonly taught.

Gain adjustment

The basic concept underlying both the gain-adjustment and lag compensation design is that the desired degree of stability (that is, phase margin) may often be achieved by a reduction gain. In order to initiate the example, let us consider the use of a simple gain adjustment, although it is seldom an adequate solution to a design problem. The system to be considered is shown in Fig. 10a where the transfer function of the uncompensated plant is:

$$G_p(s) = \frac{s}{s(s + 10)}$$

The problem is to select the value of K_c such that the velocity-error constant K_v must be equal to
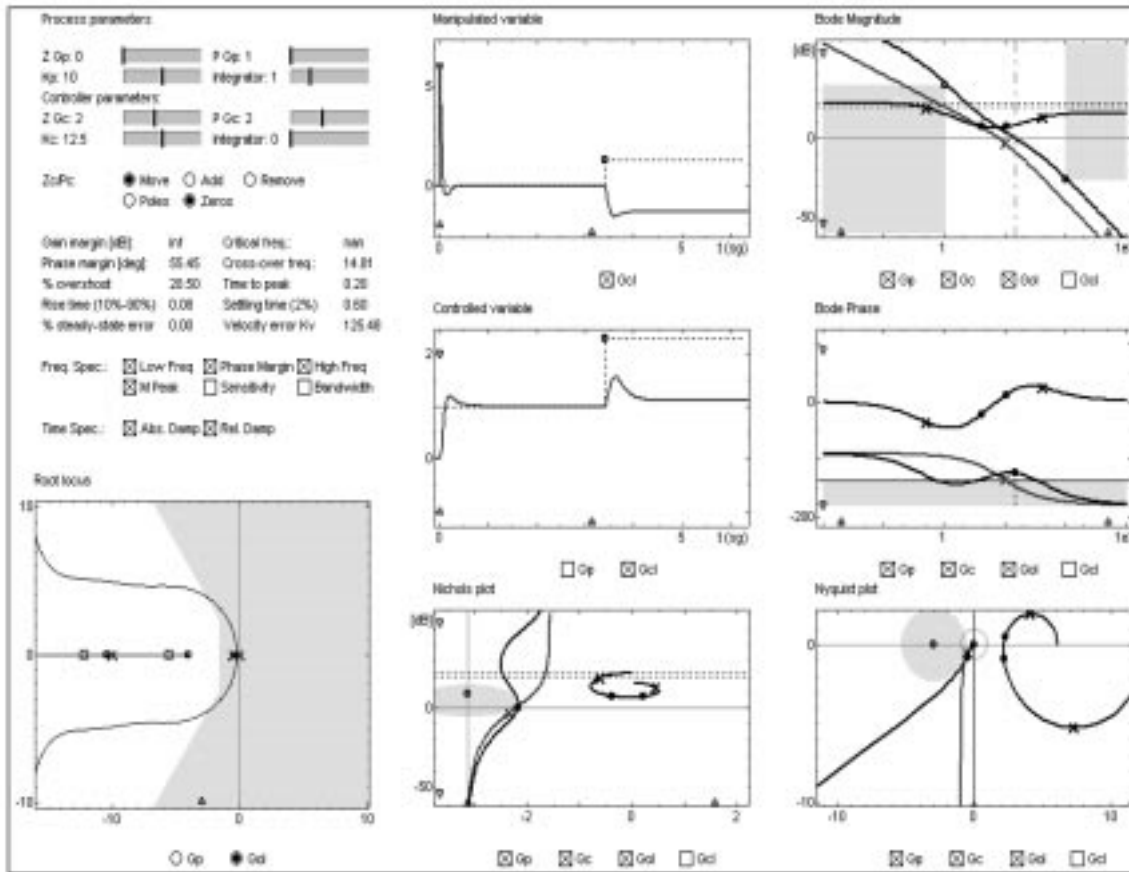
Fig. 9. Main window of linear system analysis and design.

100. Since this is a type 1 system, K_v is found by taking the limit of $sG_p(s)$ as s goes to zero; the velocity-error constant of the uncompensated plant is equal to 10. In order to increase K_v to 100, K_c must be equal to 10. Using the GUI shown in Fig. 9, the student can select the 'Move' option (by clicking on the 'Move' button). Once the button is clicked, the compensator gain can be dragged, in the Bode Magnitude, to obtain the desired specification (see Fig. 10a).

Lag compensation

To establish the details of the lag-compensation design, let us consider the previous example in which the following specification is added: the phase margin must be equal to 45°. With the simple gain adjustment the phase margin is 17,9°; however, it is observed from the phase shift diagram in Fig. 10a that for frequencies less than 10 rad/s the phase shift is less than 135°. Thus in addition to a gain adjustment, a lag compensation can be used to reduce the crossover frequency to about 8 or 9 rad/s.

The student can select the 'Add' option (by clicking on the 'Add' button) and then use the Bode diagram (magnitude or phase) to introduce the lag-compensator directly on the graph. Now the compensator pole and zero can be moved to obtain the phase margin specification by first clicking on the 'Move' button in the window,

then clicking on the desired pole or zero and finally dragging it to another location (see Fig. 10b).

Lead compensation

To illustrate the lead compensation technique, let us consider the previous example including the following specification: sinusoidal inputs of up to 1 rad/s should be reproduced with ≤ 2 percent error. Our first thought is the application of a lag compensator. However, the specification of a middle frequency gain requirement needs that $|G_c(j\omega)G_p(j\omega) \geq 34|$ for $\omega \leq 1$ rad/s. This requirement is indicated by the yellow area in the Bode magnitude in Fig. 10c. The use of lag compensation cannot be used, not because the phase lag is too large at all frequencies but rather because the phase lag is too large in the frequency range that is necessary to meet the middle-frequency specification.

Although lag compensation may not be used as a solution to the current design problem, the situation is almost ideal for the application of lead compensation. Lead compensation may be successfully applied here because no high-frequency attenuation specification is given and because the phase lag of the uncompensated plant increases slowly after crossover.

All the students need to do in order to interact with the tool is to interchange the relative position
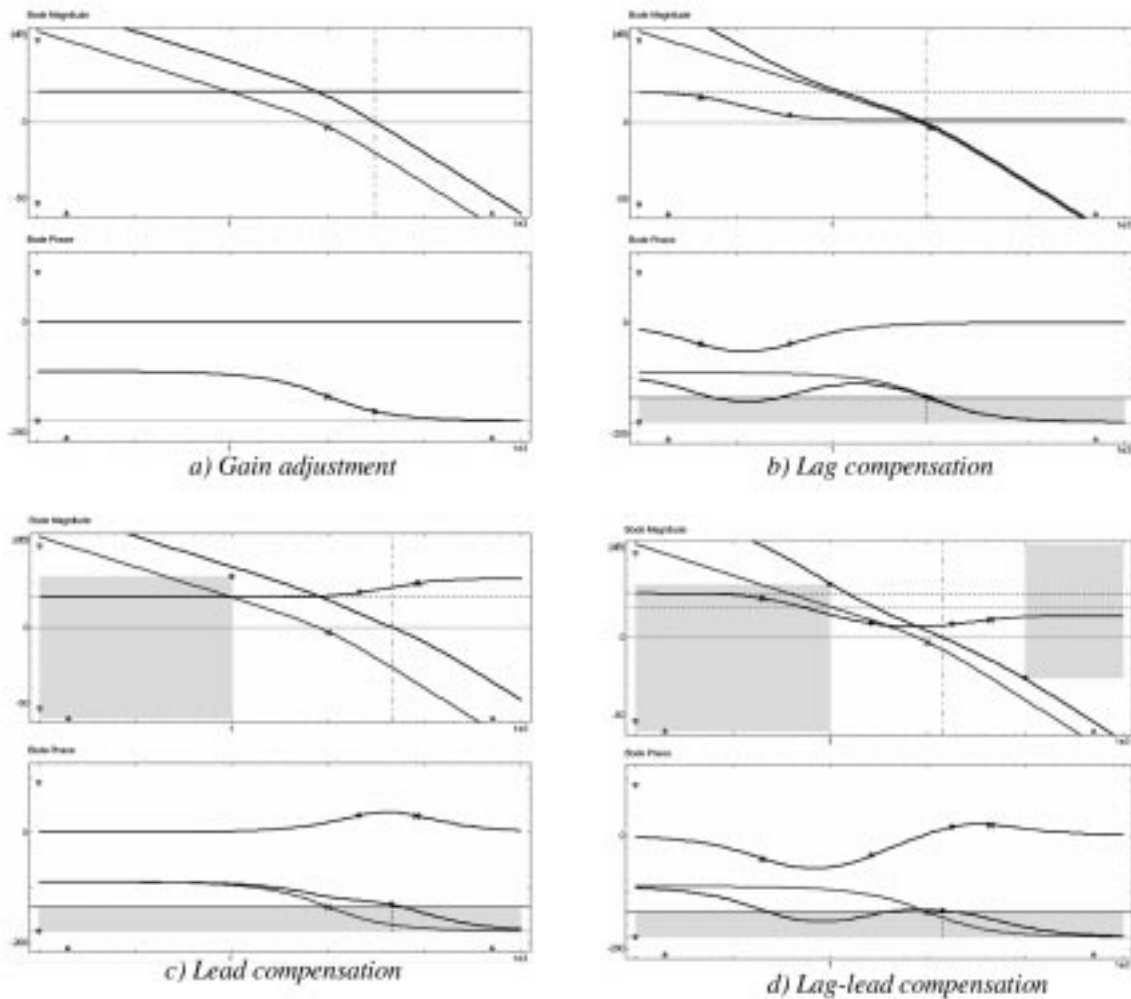
a) Gain adjustment

b) Lag compensation

c) Lead compensation

d) Lag-lead compensation

Fig. 10. G_p, G_c and G_pG_c Bode diagrams for different classical compensators.

of the zero and the pole and move the location to the high-frequency region (see Fig. 10c).

Lag-lead compensation

In the two previous examples lag- and lead-compensation techniques have been showed as separate approaches. Here, by combining the two approaches to generate a composite lag-lead compensation method, it is showed how it is possible to achieve better results than when either of the methods is used separately.

To illustrate the design procedure for lag-lead compensation, let us consider once again the plant discussed in terms of lag or lead compensation. To the previous set of specifications, let us add the high-frequency attenuation: sinusoidal inputs of greater than 100 rad/s should be attenuated at the output to 5% of their value at the input.

This specification requires that $|G_c(j\omega)G_p(j\omega) \leq 0.05|$ for $\omega \geq 100$ rad/s. Figure 10d shows this specification. In this case, the high-frequency attenuation specification rules out the use of lead compensation since no high-frequency gain may be used. We have seen above that lag cannot be used.

Either. However, this problem can be solved quite easily with the use of lag-lead compensation.

Following the same interactive design procedure the student can now select the 'Add' option (by clicking on the 'Add' button) and then use the Bode diagram (magnitude or phase) to introduce the lag compensator directly on the graph of the lead compensation. Now changing to the *move option* it is possible to interact with the lag-lead compensator poles and zeros in order to fulfil all the specifications.

Thus LSAD tool provides an excellent means of enhancing the standard 'textbook' design approach for classical compensators. The images and the immediate feedback enable students to gain insight into the design very quickly.

POLE PLACEMENT DESIGN: A STATE-SPACE APPROACH

Pole placement refers to the fact that the design is formulated in terms of obtaining a closed-loop system with specified poles. In this section a simple

regulation problem is presented. Further, only the single input/single output case will be discussed. Because computer control is considered, the control signals will be constant over the sampling period. If the sampling period h is given, the process can be described by the following discrete-time system:

$$x(kh + h) = \Phi x(kh) + \Gamma u(kh)$$

The disturbances are assumed perturbations in the initial state of the system. The objective is to find a linear feedback law of the form: $u(kh) = -Lx(kh)$, so that the closed-loop system has a specified characteristic equation. With n parameters in the state-feedback vector, it is possible to place n poles arbitrarily, if the system is reachable [13]. This will guarantee that the disturbances decay in a specified way.

Figure 11 shows the basic screen of the developed interactive tool for the pole placement design. The student can modify interactively the following parameters:

- The process plant.
- The state variable representation.
- The sampling period.
- The specified closed loop poles.
- The initial state.

A deadbeat control of a second-order system is depicted in Fig. 11. To put the pole placement design method into practice, it is necessary to understand how the properties of the closed loop system are modified by the design parameters that

are the closed-loop poles and the sampling period. Using this tool the student can obtain a clear comprehension on how these parameters influence the response of the system.

JUMP RESONANCE

When nonlinear systems are excited by a periodic forcing function, it is sometimes observed that the amplitude of the periodic output changes very suddenly, or 'jumps'. A physical description of the observed phenomena is best given with the aid of Fig. 12. If a constant-amplitude low-frequency sinusoidal forcing function is applied, the steady-state output response is periodic with the same fundamental frequency. When the input frequency is slowly increased the steady-state output amplitude normally increases slowly in a manner quite similar to that found in a linear system. However, at some critical frequency ω_1, it is found that only a small increase in frequency is required to cause a jump in amplitude from A to B (see Fig. 12).

Further increases in frequency result in a gradual decrease in output amplitude as shown. Next, if the frequency is slowly reduced, the output amplitude repeats itself at the various frequencies until the critical frequency ω_1 is regained with amplitude at B. Now it is discovered that ω_1 is not a critical value for the decreasing frequency; i.e., the amplitude does not decrease in jump fashion at ω_1 but continues along an apparently normal resonance curve from B toward C. A second critical frequency ω_2 is discovered as the
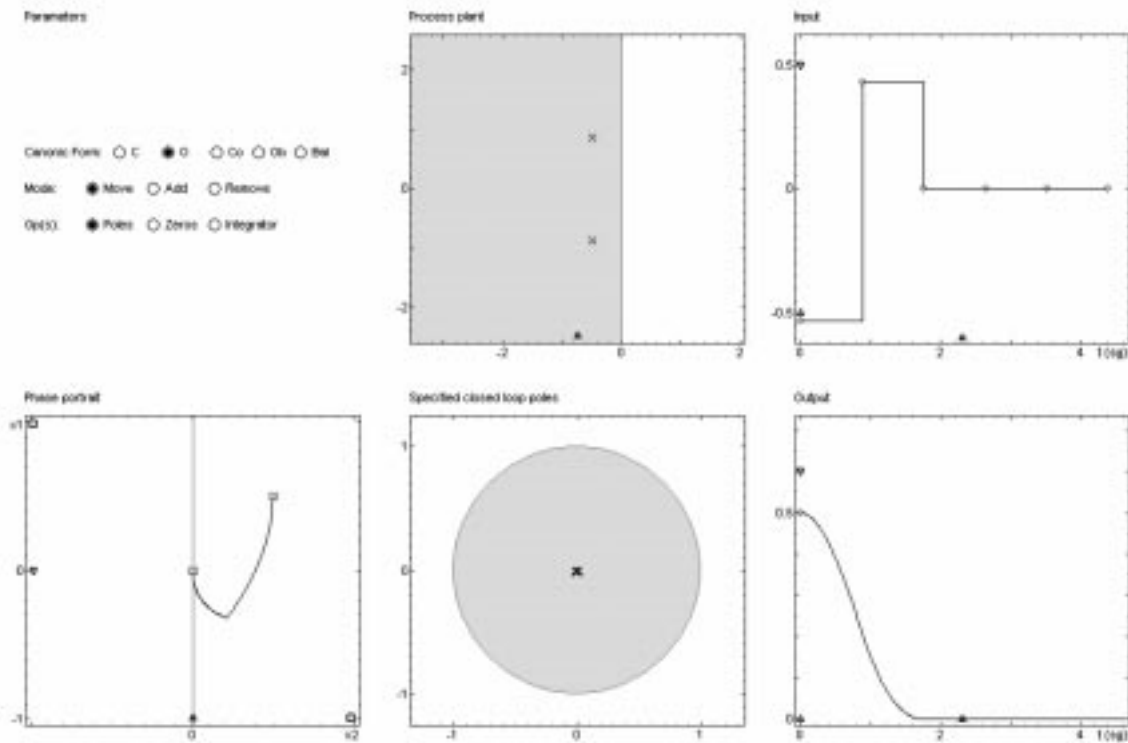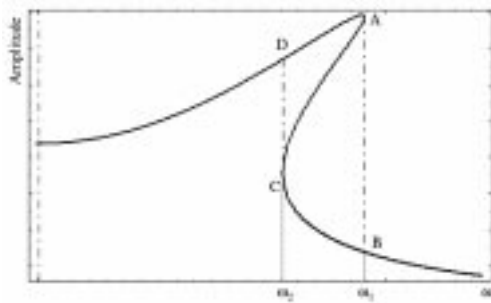

Fig. 11. Pole-placement design.

Fig. 12. The jump-resonance phenomenon.

forcing function is decreased from B to C. At frequency ω_2 there is another discontinuous jump from C to D. D is found to lie on the original increasing-frequency section of the curve, and for further decreases in frequency this curve is retraced. The observed jumps in the amplitude of the frequency response curve have given rise to the name 'jump resonance' [16].

The explanation for the jump resonance phenomenon is the existence of a frequency range ω_2 to ω_1 for which the solution of the nonlinear differential equation is triple-valued. The upper and lower curve segments, from D to A and from B to C, represent stable solution values, and the segments from A to C represent unstable solutions which cannot exist physically. Thus, as

the frequency is raised from D to A, the operation of the system is stable, and there is no tendency for the output amplitude to change, even though two other solutions are possible in the frequency range from ω_2 to ω_1.

The best-known cases of jump resonances are associated with Duffing's equation, which is in the form:

$$\ddot{y} + \dot{y} + f(y) = B\sin(\omega t)$$

Note that in Duffing's equation the nonlinearity is in the $f(y)$ term, which is commonly associated with the spring constant in a mechanical system. For a closed loop control system, this term is associated with the system gain constant (see Fig. 13)

The nonlinear differential equation of this system may be written as:

$$\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y(1 + \lambda y^2) = B\sin(\omega t)$$

where:

$$\alpha = \frac{1}{2\tau}, \quad \omega_0^2 = \frac{Kb}{\tau}, \quad \lambda = \frac{a}{b}, \quad B = \frac{KA}{\tau}$$

Figure 14 shows how this new way of interactive control education provides practical insights into control systems fundamentals [17]. It is a dynamic picture in the sense mentioned earlier, and when
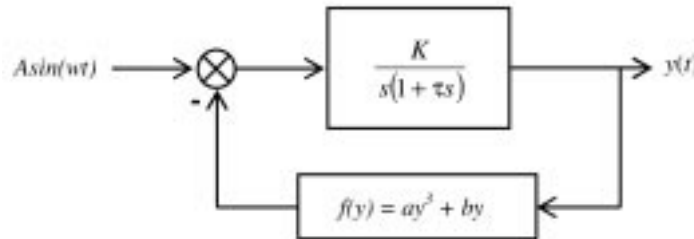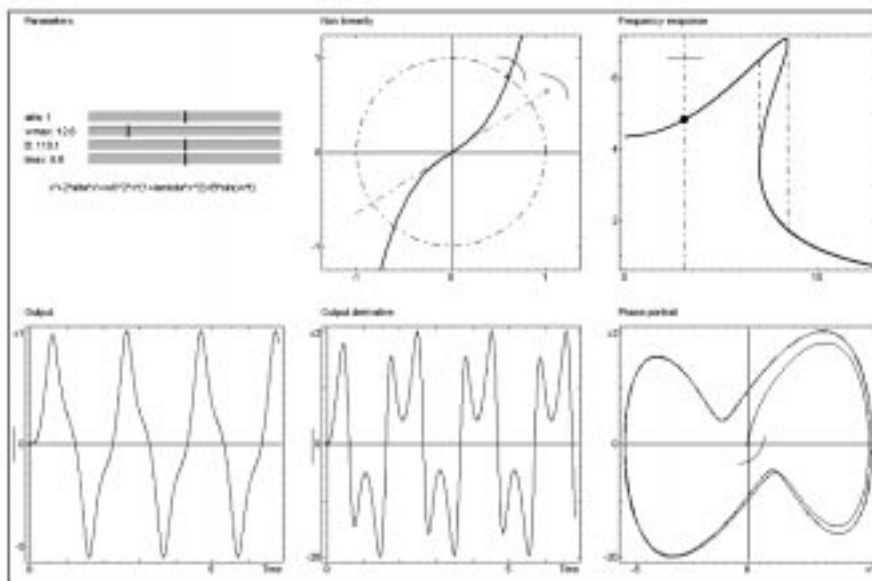

Fig. 13. Nonlinear system.

Fig. 14. An interactive tool for the study of the jump-resonance phenomenon.

the student manipulates some active element in the figure the new result is automatically produced.

In order to understand the jump resonance phenomenon the student can modify interactively the following parameters:

- The parameters α, B
- The non-linearity parameters a and b.
- The initial condition
- The frequency

CONCLUSIONS

One major issue has been addressed when developing these interactive examples: to maximize clarity and simplicity focusing each example on a single core concept. The objective is to design interactive tools that encourage our control engineering students to think and interact with the examples differently from the way they do in the case of textbooks or lecture presentations. Because the examples can be used in an unsupervised environment, simplicity and clarity are key features.

It is interesting to observe that new communication and information technology in education has always met with a certain resistance from the academy. This situation seems to be changing lately with a general enthusiasm about the Web, however difficult it may be to predict the future course of efforts such as ours to develop 'interactive tools' [18]. Issues of licensing and commercialization are complex. Our plan is simply to continue exploration of concrete educational issues by focusing on interactive modules for concepts in control systems [9, 19–23].

The scenario for control education is changing and we must adapt to this changes [24–27]. Information technology opens a whole new world of real opportunities. Computers show a great potential to enhance student achievement, but only if they are used appropriately as part of a coherent education approach. Computers do not change in the way books or labs do: they allow us to go deeper and faster.

REFERENCES

1. B. S. Heck (ed.), Special report: future directions in control education, *IEEE Control Systems Magazine*, **19**(5), 1999, pp. 35–58.
2. R. C. Garcia and B. S. Heck, Enhancing classical controls education via interactive GUI design, *IEEE Control Systems Magazine*, **19**(3), 1999, pp. 77–82.
3. D. Poulis and A. Pouliezos, Computer assisted learning for automatic control, *IFAC Symp. Advances in Control Education*, Istanbul, Turkey, 1997, pp. 181–184.
4. K. Szafnicki and F. Michau, New educational technologies applied to control education-example of resources sharing between engineering schools, *IFAC/IEEE Symp. Advances in Control Education,* Gold Coast, Australia, 2000.
5. B. Wittenmark, H. Häglund and M. Johansson, Dynamic pictures and interactive learning, *IEEE Control Systems Magazine*, **18**(3), 1998, pp. 26–32,
6. M. Johansson, M. Gäfvert and K. J. Åström, Interactive tools for education in automatic control, *IEEE Control Systems Magazine*, **18**(3), 1998, pp. 33–40.
7. Y. Piguet, *SysQuake: User Manual,* Calerga, (1999).
8. Y. Piguet, U. Holmberg and R. Longchamp, Instantaneous performance visualization for graphical control design methods, *14th IFAC World Congress*, Beijing, China, 1999.
9. S. Dormido, F. Gordillo, S. Dormido-Canto and J. Aracil, An interactive tool for introductory nonlinear control systems education, *15th IFAC World Congress,* 2002, Barcelona, Spain.
10. S. G. Crutchfield and W. J. Rugh, Interactive learning for signal, systems, and control, *IEEE Control Systems Magazine*, **18**(4), 1998, pp. 88–91.
11. K. J. Åström, The future of control, *Modeling, Identification and Control*, **15**(3), 1994, pp. 127–134.
12. K. J. Åström, Limitations on control system performance, *European Journal of Control*, **6**, 2000, pp. 2–20.
13. K. J. Åström and B. Wittenmark, *Computer Controlled Systems*, Prentice-Hall 3rd edition (1997).
14. G. C. Goodwin, S. F. Graebe and M. E. Salgado, *Control System Design*, Prentice-Hall (2001).
15. S. H. Strogatz, *Nonlinear Dynamics and Chaos: with applications to physics, biology, chemistry and engineering*, Addison Wesley (1995).
16. C. Mira, *Cours de Systemes Asservis non linéaires*, Dunod université (1969).
17. S. Dormido, Control learning: present and future. plenary lecture, *15th Triennial World Congress of IFAC,* Barcelona, Spain, 2002.
18. P. Antsaklis, T. Basar, R. DeCarlo, N. Harris, M. Spong and S. Yurkovich, Report on the NSF/CSS WorkShop on new directions in control engineering education, *IEEE Control Systems Magazine*, **19**(5), October 1999, pp. 53–58.
19. S. Dormido, M. Berenguel, S. Dormido-Canto and F. Rodríguez, Interactive learning of introductory constrained generalized predictive control, *IFAC Symp. Advances in Control Education*, 2003, Oulu, Finland.
20. J. Salt, P. Albertos, S. Dormido and A. Cuenca, An interactive simulation tool for the study of multirate sampled data systems, *IFAC Symp. Advances in Control Education*, 2003, Oulu, Finland.
21. J. Sánchez, F. Morilla, S. Dormido, J. Aranda and P. Ruipérez, Conceptual learning of control by Java-based simulations, *IFAC/IEEE Symp. Advances in Control Education*, 2000, Gold Coast, Australia

22. J. Sánchez, F. Morilla, S. Dormido, J. Aranda and P. Ruipérez, Virtual control lab using Java and Matlab: a qualitative approach, *IEEE Control Systems Magazine*, **22**(2), 2002, pp. 8–20.
23. N. Tan, D. P. Atherton and S. Dormido, Systems with variable parameters; Classical control extensions for undergraduates, *IFAC Symposium on Advances in Control Education*, 2003, Oulu, Finland.
24. N. A. Kheir, K. J. Åström, D. Auslander, K. C. Cheok, G. F. Franklin, M. Masten and M. Rabins, Control system engineering education, *Automatica*, **32**(2), 1996, pp. 147–166.
25. C. C. Bissell, Control education: Time for radical change? *IEEE Control Systems Magazine*, **19**(5) 1999, pp. 44–49.
26. D. S. Bernstein, Enhancing undergraduate control education, *IEEE Control Systems Magazine*, **19**(5), 1999, pp. 40–43.
27. Dorato, P., Undergraduate control education in the US, *IEEE Control Systems Magazine*, **19**(5), 1999, pp. 38–39.

S. Dormido received his Physics degree from Madrid Complutense University (1968) and his Ph.D. from Country Vasc University (1971). In 1981, he was appointed Full Professor of Control Engineering at UNED Faculty of Sciences. He has supervised 25 Ph.D. Thesis and co-authoring more than 200 conference papers and 150 journal papers. Since 2002 is President of the Spanish Association of Automatic Control, CEA-IFAC. His scientific activity includes various topics from the control engineering field: computer control of industrial processes, model-based predictive control, robust control, modeling and simulation of hybrid systems and control education with special emphasis on remotes and virtual labs.

S. Dormido-Canto received his MS degree in electronic engineering in 1994 from the Universidad Pontificia de Comillas University (ICAI) and his Ph.D. degree in physics from the UNED (Universidad Nacional de Educación a Distancia) in 2001. He joined at UNED Department of Computer Sciences and Automatic Control as an Assistant Professor in 1994. His current research and teaching activities are related with the analysis and design of control systems via intranet or internet, high performance interconnection networks for cluster of workstations and optimal control.

R. Dormido received her Physics degree from Madrid Complutense University (1994) and her Ph.D. degree with a thesis on 'Robust Control' from UNED (Universidad Nacional de Educación a Distancia) in 2001. Since 1994 she has been working at UNED Department of Computer Sciences and Automatic Control as an Assistant Professor. Her current interests are the robust control and modelling and simulation of continuous processes.

J. Sanchez received his Computer Sciences degree in 1994, from Madrid Polytechnic University and his Ph.D. in Sciences from UNED (Universidad Nacional de Educación a Distancia) in 2001. Since 1993, he has been working at UNED Department of Computer Sciences and Automatic Control as an Assistant Professor. His current research interests are the design of new systems for control education, virtual labs, telepresence, multimedia, and the use of the Internet in education.

N. Duro received her Physics degree from Madrid Complutense University (1995) and her Ph.D. degree in Sciences from UNED (Universidad Nacional de Educación a Distancia) in 2002. Since 1997, she has been working at UNED Department of Computer Sciences and Automatic Control as an Assistant Professor. Her current interests are control process, modelling and simulation of continuous processes and the design of new systems for control education.