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The Art of Estimation*

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The ability to perform estimation is an asset, for example, when dealing with problems with limited or unavailable data. Engineering students, in general, are inadequately prepared to perform rough estimation. A possible reason for this shortcoming is that the emphasis in engineering curricula is on detailed analysis and not enough attention is given to developing students' estimation skills. This paper outlines the importance of estimation and lists attributes that are necessary to carry out reasonable estimation. Furthermore, it provides a list of activities and exercises, appropriate for use in the lower division courses, to engage students in estimation scenarios and to launch them on a path to developing this skill.

INTRODUCTION

ESTIMATION IS an integral activity of our daily lives—from determining how much cash to take along for an upcoming vacation to purchasing enough paint to add some color to our homes. Considering the latter, if our estimate is way off, we either end up with a lot of extra paint or need to make another trip to the paint shop to purchase more. Being able to estimate properly the various quantities that one encounters is a useful skill. People use experience and common sense and read labels and instructions to perform these estimations, which usually require simple arithmetic.

Engineers also make use of estimation in their practice to determine answers to ill-defined problems or when a detailed solution is not required. Estimation particularly plays an important role in the preliminary stages of the design process, when decisions have to be made based on incomplete or unavailable details/data, and selections must be made from a multitude of options. Furthermore, estimation is used to check the validity of answers based on back of the envelope type calculations. Engineering estimation (E^2) covers a large domain between guessing (instantly done and costing nothing) and detailed analysis (time consuming and expensive). Successful E^2 is performed based on knowledge of dimensions and units, basic laws of physics and modeling, the ability to relate and compare, and common sense. Like many other attributes, an engineer's ability to estimate is enhanced and strengthened by experience and gaining professional judgment.

CURRENT SITUATION

In the late 1990s, a comprehensive study was undertaken at MIT to ascertain mechanical engineering students' ability to perform simple engineering estimations. The estimation questions posed to students in the survey related to energy in a small battery, drag on a bicyclist, and power output of a small motor. (The study also included students from five other top-ranked mechanical engineering programs, as well as practicing mechanical engineers for comparison purposes.) The study culminated in a Ph.D. dissertation by Linder [1] and resulted in a number of publications (e.g. [2, 3]). The reader is encouraged to consult these references as they document details of the survey and findings with profound implications in engineering education. Two important conclusions of the study are quoted below.

> 'Students were found to have considerable difficulty making estimates for common engineering quantities, such as force and energy.'

> 'Students were also found to have difficulty applying basic engineering concepts in rough estimation situations even at the senior level.'

After reading the MIT study, the author became curious about how well University of the Pacific (Pacific) engineering students would perform E^2. He conducted a survey, borrowed from the MIT study, in the engineering Mentor III course in Spring 2003. Students in the two sections of this course were graduating seniors from all engineering majors. (At Pacific, the engineering degree is a five-year program including one year of co-op. Thus, there are two years at the senior level.)

Each question used in the estimation survey was given as an impromptu, five-minute in-class activity. There were two questions in the survey, but only one is reported here as the other resulted in the same findings. Students were told not to write their names on their responses and that the activity had no effect on their grades in the course.

The question was to estimate the energy content of a fresh 9-volt battery. Fifty-eight students participated and their numerical answers, which cover many orders of magnitude, are plotted in

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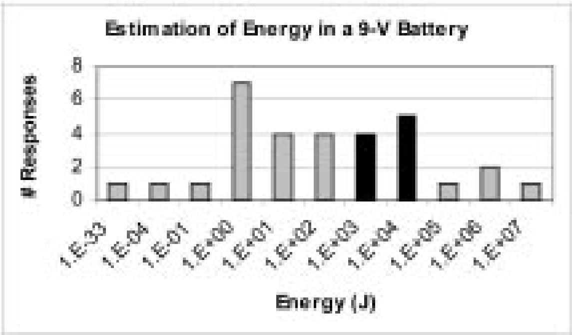

Fig. 1. Answers to estimation problem. Only those answers given in joules or equivalent are shown (31 out of 58 students).

Fig. 1. This, in addition to other results, as organized and tabulated in Appendix 1, corroborated the MIT findings that engineering students in general are not adequately prepared for even simple estimation problems and that they have problems with dimensions and units.

WHAT TO DO

To help alleviate the present situation, the author advocates a systematical approach to expose students to estimation scenarios. Students should be encouraged to practice estimation and be made aware of its importance through short exercises with everyday objects and experiences. The more we ask students to estimate, the better they are prepared to use this skill in follow-up courses and eventually in their careers. Estimation exercises should be included in all courses, and especially the lower division courses, where the engineering foundations are established. Examples of such courses are Introduction to Engineering and Statics, where dimensions, units, and basic engineering concepts are discussed and learned. Several estimation activities appropriate at the freshman level are presented below to illustrate examples of what can be done. These activities could be done in class or assigned as homework.

Activity No. 1

Ask students to make a list of familiar objects to serve as easy-to-remember estimation standards for various quantities. For example, a credit card is approximately 3.5 inches long; a burning match converts chemical energy to about 3 Btus of thermal energy; a hair dryer requires about 1 kW of electrical power and so on. Of course, body parts such as feet and cubit (distance from elbow to fingertips) can be used for length standards. This activity can be performed by groups and then tallied on the board for the entire class to benefit. With such a list of objects in their memory, students will have a better grasp of units and how big, or small, they are. As students advance through their engineering programs, they can refine and add to their list of estimation standards. A fascinating source for visual comparisons among various quantities of distance, area, weight, density, energy, temperature, etc. is a book titled *Comparisons* [4], published in 1980.

Activity No. 2

Take a piece of lumber, such as a length of 2 by 4, to the class and ask students to estimate its length, width, and thickness. Point out the difference between the nominal and exact size in the lumber industry. Considering the lecture room, ask students to estimate the number of cans of paint needed to paint the walls, i.e. estimate the surface area. Point out the standard ceiling height in a typical older residential building (room's ceiling height $\approx$8 ft in the USA).

Activity No. 3

Ask students to estimate the weight of body extremities such as an arm or leg. This can be done, for example, by looking at a photograph with super-imposed gridlines to estimate the percentage of the body of each body part. With two views (front and side), fairly good estimates can be made. Simple experiments with bathroom scales can be performed to confirm the estimate. This kind of information is useful to designers of rehabilitative devices.

Activity No. 4

Ask students to estimate the drag on a bicyclist traveling at 20 mph. This was also one of the estimation questions that were used in the MIT study. There are several methods by which drag can be estimated as documented in Linder's dissertation [1]. A related article about drag on cars appeared in *Scientific American* [5].

Activity No. 5

Ask students to estimate their own physical power. (Two excellent books that present and discuss different quantities (size, energy, etc.) in biological organisms are *On Size and Life* [6] and *Exploring Biomechanics* [7].) One simple way to do this is by climbing a flight of stairs. By estimating the person's weight (W), the stair height (H), and the time it took to climb (t), the change in potential energy ($PE = W \times H$), and then power ($P = PE/t$) can be determined. This power is roughly 0.3 hp for an average adult, but note that it would be difficult to sustain it for a long time. (For comparison, the pumping power of a human heart is about 0.002 hp.) Of course, unit conversion plays an important role here; e.g., to convert from lb_f.ft/s or N.m/s to hp. Bryan Allen, an avid bicyclist in top condition, sustained about 0.4 hp for near 3 hours as he pedaled and piloted the human-powered *Gossamer Albatross* in June 1979 to cross the English Channel. Point out the difference between burst and sustained amounts of power. Point out the different units of power used—hp for prime movers, kW for appliances, MW for power plants, etc. Also, point out the difference between kW and kW.h as some, even senior students, have difficulty with this.

Activity No. 6

Ask students to estimate home energy costs and compare these with their monthly statements from the utility company. Important lessons about kW.h, Btu, therm, cost, etc. can be shared via this exercise. Also, students will get a sense of the relative magnitudes of power requirements of various appliances. Utility companies might have useful information on this subject (e.g., see www.pge.com).

Activity No. 7

Almost everybody has used 9-V batteries in consumer products such as toys, smoke alarms, audio devices, etc. Yet, most people, including some engineering students, are clueless about the amount of energy stored in such a battery. The idea behind this activity is to conduct a simple experiment to allow students to measure this energy.

Experiment—Have different groups use fresh batteries with different brand names for comparison. Connect a 9-V battery to a light bulb (or other suitable loads such as a microprocessor cooling fan). Using an inexpensive multimeter, measure and record the battery voltage (V) and the bulb's resistance (R) every hour until the bulb is very dim. (Instrumenting the circuit with two multimeters to measure voltage across the bulb and current going through the bulb will yield more accurate results but the above procedure is quite adequate for our purpose.)

Using Ohm's law, students should be able to determine the power ($P = V^2/R$) at each measurement instant, and then calculate the energy during the entire period ($E = \Sigma P.\Delta t$). After finishing the experiment, students will have been exposed to a simple circuit and use of a multimeter, Ohm's law, units of energy and power, and perhaps learned some safety lessons. Additionally, they will hopefully remember the amount of energy stored in a 9-V battery for future reference. It is interesting to compare the cost of this energy with the PG&E (power company in California) residential rate, which is about 12 cents per kW.h or 0.003 cents per kJ. A 9-V battery contains about 10 kJ and costs $3, which results in 30 cents per kJ. A battery is indeed a very expensive, but convenient, source of electrical energy!

Activity No. 8

Ask students to estimate the power of a commercial jetliner. Students need some basic data to properly respond, and such data for both commercial and military aircrafts can be found at www.boeing.com. For example, specifications for the Boeing 747–400ER, the largest in the 700 series jetliners in production, are: $W_{max} = 910{,}000\,lb_f$, $V_{cruise} = 567$ mph at 35,000 ft, engine thrust $=$ $63{,}300\,lb_f$. Therefore, for this aircraft, the power to takeoff and reach the cruising altitude in about 10 minutes is roughly 70 MW. In comparison, the power of a Boeing 717, the smallest in production, turns out to be about 8 MW. It is interesting to note that the length of a 717 is roughly one half that of a 747 as seen in Fig. 2, resulting in a volume (read weight) ratio of about one eighth, which is roughly the same as the power ratio. This exercise will allow students to appreciate the enormous power exhibited in jetliners and get a sense for how big a MW is. It is also educational to estimate the energy content of a jetliner due to its fuel. At 130×10^6 J/gal, a gallon of jet fuel is roughly equivalent to 40,000 burning matches or 50 lb of TNT. With tens of thousands of gallons of fuel onboard, a jetliner becomes a deadly weapon—the destructive force that brought down the Towers in the WTC tragedy on September 11 [8]. An excellent source containing estimates related to various aspects of flying is a book by Henk Tennekes [9].

Fig. 2. Accurate scaled models of the Boeing 717 and 747.

Activity No. 9

As a follow-up exercise, ask students to report on output power of different kinds of motors such as a humming bird, human heart, small DC hobby motor, car engine, power plant, rocket, etc. Power is an important quantity in all branches of engineering and, with this kind of exercise, students will be better able to grasp the relative magnitudes of power among somewhat familiar objects. Figure 3 represents a graphical example of the outcome of this activity, which clearly depicts order of magnitudes among familiar objects and living systems. Appendix 2 provides the information used to estimate the quantities shown in Fig. 3.

Activity No. 10

Assign Fermi type problems either during lectures or as homework problems. Enrico Fermi, the famous nuclear physicist of the mid-twentieth century, used to surprise his physics students by asking them questions such as: 'How many piano tuners are in Chicago?' These kinds of questions require students to use very simple arithmetic and relations to find answers to seemingly impossible problems, and to gain some insight. To see the

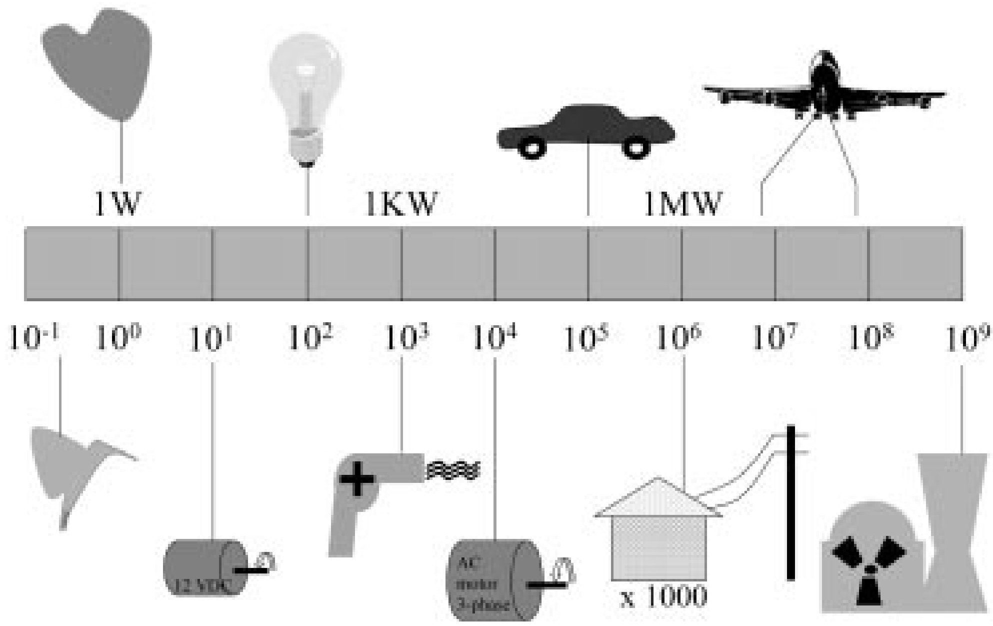

Fig. 3. Ten orders of magnitude among power outputs of familiar biological and engineering systems.

answer to this and some dozen other such questions, refer to the NASA web site [10]. Two examples of Fermi type problems that the author has assigned to students are given below.

1. Estimate the amount of paper needed to print the university newspaper on an annual basis. Determine the answer in terms of boxes of standard letter size paper.
2. Estimate the number of books in the university library. Estimate their weight.

Overall, students should be given ample opportunities to develop E^2 ability by systematically going through basic dimensions and units and working up on physical laws—applying their knowledge to common things and experiences. The above activities, as well as others available in other references (e.g. [11, 12]), will help students in this endeavor. As implicitly indicated earlier, there is much more to E^2 than units and dimensions. However, units and dimensions and simple relationships (e.g. $P = E/t$) form the foundation. Once students are equipped with this, they are on the road to making estimations in more complex problems and situations.

Work done at the University of the Pacific

In collaboration with colleagues, the author has integrated some of the above activities in the Introduction to Engineering course, which is a required course for all engineering students. Specifically, one formal lecture was delivered on estimation, followed by a homework set of six problems and mini-projects, similar to the set described in this paper. One laboratory session was also dedicated to the height estimation, followed by crude measurement, of a tall building on campus. Students actively participated in these exercises. We are hoping to continue to strengthen our coverage of estimation throughout the curricula with the goal of equipping our students with this important skill.

CONCLUSIONS

Based on recent survey results, it appears that engineering students are not adequately prepared to make reasonable estimations of engineering quantities. Current engineering education practice places too much emphasis on detailed analysis. For students to learn to estimate, they have to practice by doing it. Ten activities were suggested to help students in developing their estimation skills. Much more needs to be done in rectifying the current deficiency in students' ability to estimate.

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APPENDIX 1

Estimation survey—Pacific's Engineering Mentor III Classes—March 2003

Survey question: Estimate the energy stored in a new 9-V battery. Write your estimation process (5 minutes).

Answer: 7000–17,000 J for a small (transistor radio) 9-V battery, depending on the brand and type.

Survey participants:

Majors	No. of responses	Comments
CpE	16	Students were asked to identify their majors but not give their names.
EE	15	
Eph	6	
CE	10	
ME	6	
EM	3	
Unknown	2	
Total Students	58	

Survey results: Organized in three sections: (1) Concepts used, (2) Range of answers, (3) Units reported.

1. Concepts used	No. of responses	Comments
Ohm's law & $E = P \times t$	26	Some used these equations incorrectly.
Wild guess	4	Wrote a number without any explanation.
Others	24	Used various incorrect relations and explanations.
No idea	2	Actually wrote they had no idea!
Need more information	2	One student correctly indicated size is important but did not make any estimate.

2. Range of answers, J	No. of responses	Comments
1.E-33 – 1.E-32	1	Answers ranged from 5.9E-33 to 3.4E+7 J.
1.E-4 – 1.E-3	1	
1.E-1 – 1.E0	1	Only answers reported in J, or its equivalent, are presented here. Several students indicated the correct unit but did not provide a numerical answer.
1.E0 – 1.E1	7	
1.E1 – 1.E2	4	
1.E2 – 1.E3	4	
1.E3 – 1.E4	4	Nine students reported their answers in the acceptable range, 1E3 – 1E5 J.
1.E4 – 1.E5	5	
1.E5 – 1.E6	1	
1.E6 – 1.E7	2	
1.E7 – 1.E8	1	

3. Units reported	No. of responses	Comments
Joule	36	Including several who wrote W.h for the unit.
Incorrect units	14	Including 7 who wrote W for the unit.
None	8	

APPENDIX 2

Information used to obtain estimated quantities shown in Fig. 3

a. Hovering large humming bird: James Lighthill, a brilliant fluid dynamics expert, estimated the power output for hovering hummingbirds to be about 13 W per kg of body mass based on momentum theory. A large humming bird is about 20 g.
b. Human heart: Pumps about 5 liters of blood per minute (1/min) at about 100 mm of mercury (mmHg) in pressure rise. Proper conversion factors have to be considered to perform calculation to obtain power.
c. Hobby DC motor: Consult a hobby electronics catalog, for example, www.jameco.com. Look at the DC motors and find out their power rating; e.g., one is listed at 12 V and drawing 0.75 Amps.
d. Light bulb: This is obvious!
e. Hair dryer: Look at the name plate of a typical household hair dryer.
f. 3-phase motor: Consult equipment catalog, for example, www.grainger.com. Look at medium-size 3-phase motors and find out their power.
g. Car engine power: Drive your car from start to a certain speed and time it. Using simple relations ($a = \Delta V/\Delta t$, $F = m . a$, $P = F . V$), determined the power, neglecting drag, etc. Or, perform calculation based on typical values: car mass = 2,000 lb_m ($\approx$ 1,000 kg), reaching 60 mph ($\approx$ 30 m/s) in about 10 seconds.
h. Electrical power for 1000 homes: If your household uses 1 kW of electric power *all* the time, your monthly (720 hours) electric bill will show 720 kW.h.
i. Jet airliner power: This was shown in Activity No. 8.
j. Nuclear power plant output: Consult text books on thermodynamics or electric power generation or visit PG&E web site (they might have info on their nuclear power plants).

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