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Engineering Mathematics in Context—Learning University Mathematics Through Problem Based Learning*

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A theory-based approach to scientific research has an inherent tendency to become secluded from the ongoing problems and discussions of the surrounding society. A problem-based approach to research immediately involves this context of problems and discussions from the outset. In this article, we argue that education in university engineering mathematics should take its outset in contextual problems in order to provide a foundation for the skills and capabilities engineers need in their future job settings, whether it be research or development activities.

Keywords: engineering mathematics; problem based learning; university mathematics education

1. Introduction: Mathematics—the core of modern science

Science is sometimes thought of as being trapped in an ivory tower, having little contact with the world outside university institutions. Scientists seem to work well in splendid isolation working on subjects of only indirect interest to the world outside science. But there are different approaches to scientific research and some of them are in fact capable of nurturing and securing the relation between research projects and the context they exist in—between research results and the outside world [1].

In discussions on university education, what good research *is* and how we best educate within a specific field of study are inseparable. The two must be considered in close relation to each other in order to secure a foundation for research-based education and in order to prepare students to possibly become researchers themselves. In the following we aim to keep this close connection in mind when considering the role of context in engineering science research when we focus on the teaching of mathematics.

In this paper, we will argue how mathematics in engineering science can be taught effectively as an integrated part of contextual real world scenarios. We will examine the relationship between context and mathematical content from both a research perspective and an educational perspective to support the argument that even university engineering education in the abstractions of mathematics can be conducted in a contextual setting.

Mathematics is a special element in Engineering Science. It is often considered the core of Science, as mathematics is the representational language used extensively in many sciences. Therefore—and because of its traditional status as absolute and timeless knowledge—mathematics is often thought of as context free and not in an essential way dependent on the scholars that do mathematics, the problems of the society they live in, the culture of which the mathematics is part of, or the sciences that mathematics plays a part in. Mathematics is pure, universal knowledge and it holds its own truth in the sense that mathematics is proven right through a very special procedure of verification—logical proofs.

A lot of this holds true—to some extend at least. There are especially good verification procedures in mathematics and several prominent figures in the philosophy of mathematics, like Plato and Wittgenstein, assert that we can be absolutely certain about the truth of mathematical knowledge despite the extreme differences that hide beneath the two scholars' thoughts. However, a strong argument can be made that mathematics is not universal knowledge for the entire galaxy, but in fact a highly contingent matter [2].

The standing debates on the certainty of mathematical knowledge and its apparent universality indirectly influence the discussions on the educational framework for teaching mathematics in engineering educations. If mathematics may be reasonably thought of as context free, absolute and universal knowledge, context does not enter into education in mathematics as a natural ingredient. However, if one can argue that real research, which uses mathematics, is conducted in close

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relation to contextual matters, the education in mathematics is put in a different perspective.

The question is therefore whether contextualised engineering education in mathematics is possible and why it is a strong alternative to the traditional learning setup of being introduced to the pure mathematical formalism. The research question we pursue in the following is therefore: Why should the learning of mathematics in university engineering educations be problem based?

In the first part of the article we will examine two different and general paradigms in relation to doing scientific research and consider the role played by mathematics in these paradigms. The first paradigm—the Euclidean theory-based approach—is oriented towards theory and devoid of contextual matters. The other paradigm—the problem-based approach—is oriented towards scientific research as immediately contextual.

In the second part of the article we examine the benefits of contextualising university mathematics education through examples from student projects, where mathematics was learned through a contextual setting.

Finally, in the third part of the article we will discuss some of the reasons for nurturing the connection between context and mathematics in university mathematics educations. We shall make an effort to show that even the most abstract of all ingredients in the engineering educations, mathematics, does not have to be taught as if it was a decontextualised formalism, and that there are no good reasons for educating future engineers—or in fact any university students in general—as if it did.

2. Theory-based science: The Euclidian method

One of the most successful paradigms for doing science is the so-called Euclidian method—or the Euclidian-Newtonian method [3, p. 172]. With this method scientists are able to represent physical objects and the motion of these objects in an abstract way. The tool for doing this is mathematics. The physical objects or their motion can now be transformed into digits and numbers, and with the aid of mathematical theories and formula it is possible to calculate and predict events in the future. This method is an invaluable achievement in the history of science and it is hardly possible to overestimate the importance of this achievement. Modern life as we know it would be quite different if not impossible without it.

The problem is, however, that the Euclidian method in many respects has established itself, or has been established, as the only legitimate scientific method—the only right way to do science—in many fields. That is, sciences with a strong emphasis on mathematical theorising. The achievements untold, this research tradition—paradigm—is problematic, though, if recognised as the only legitimate way of doing science.

The Euclidian method works well in respect to well-defined physical objects, but in other fields of science, not necessarily dealing with physical objects—psychology, anthropology, economics, and what we are interested in here, engineering, the method will face serious problems as it is not necessarily meaningful to turn any subject into mathematical structures, digits and numbers for subsequent calculations. Therefore, the method only has a limited use in many fields of science [4]. The paradigm and its related research methods require well-defined objects to do research on. Without the well-defined objects, it becomes difficult to conceptualise the phenomena of interest mathematically and, for example, phenomena such as life, change, values, nature, mind, and learning all tend to end up in a rather dubious state when treated with the Euclidian method—even bridges or planning procedures in organisations may have serious trouble fitting this model of doing research. The phenomena become objectified and all sorts of problems emerge, especially problems of validity. The reason for this is found in the idea of truth and knowledge associated with the Euclidian method.

The Euclidian method stems from the world of mathematics. It is, however, from a time when mathematics was not used for practical purposes, as we normally understand it today. Rather the Pythagorean-Platonic schools of mathematicians inspired the mathematical milieu in Egypt—and part of this milieu Euclid—to present mathematics as an axiomatic, timeless theoretical building of eternal knowledge that was induced with religious mysticism and numerology. This lore surrounding mathematics as a worldwide logic and, later on, its conception as the structure by which God had constructed the universe prevailed in most interpretations until far into the period of modern science, where for example Kepler could still be considered a full-blooded Pythagorean in almost every respect [2]. The scientific truths discovered within this research paradigm are considered to be absolute, as in the Euclidean system of geometry. No matter where we are, and at what time, the laws formulated within this conception of research in mathematical equations hold good. The scientific truth is an a priori truth and our true knowledge is in the singular.

The meaning of data, theory, and method in the Euclidian method is very specific. Data is collected from the outside world, but the main thing is theory.

Data, or exemplars, are often, if not neglected, then used as a mere validation of the theory, as the general theory itself says it all. With a good theory, we are able to predict almost anything and if data is used, then the theory may function as a calculating device that provides us with new insight.

With this emphasis on a theoretical building it is easy to see how science may potentially end up in the ivory tower, taught in seclusion from the rest of the world. The Euclidean conception of doing research most often ends up in a logical ordering of abstractions—unable to say much about the real world outside. When we privilege this type of research we no longer have any true knowledge about changeable things. The ways of knowing which Aristotle called 'techne' and 'phronesis' are not within reach of the Euclidian method. It only deals with the world in terms of epistemological knowledge gathered in abstract and general theories.

The Euclidean method that works well for us when applied to well-known and well-defined objects, finds itself in severe problems when confronted with new phenomena. Around a hundred and fifty years ago, when somebody did research on a phenomenon later known as electricity, no one knew how to describe and understand it. Only after years of intensive research, it was possible to name and understand this new phenomenon. Only later, it was possible to establish theories along the Euclidian guidelines that could explain this phenomenon, but the Euclidian method does not have a procedure for this development process. The Euclidean method does not include a procedure for scientific development and innovation.

The emphasis on theory also has serious consequences for the way in which education programmes are constructed. Theoretical knowledge prevails over technical and practical knowledge. Endless lessons of incomprehensible theory are poured into many engineering students; with them grasping only a glimpse of what role e.g. mathematics could play in their working life. The number of dropouts from this type of teaching says it all—a pedagogical model borrowed a long time ago, when repetition was the important factor in learning processes. This is Normal science teaching [5, 131], which unfortunately does not show students how to do science, but leaves them convinced that theory and science are living in an isolated world of their own, and are therefore often seen as uninteresting and very hard to comprehend by many engineering students. This way, the teaching of theories—and mathematics is the crown example of this—act as an initiation rite that young people has to go through—passing exams—before they are accepted by the community of practice that hosts engineering science in other organisations of society.

3. Problem-based science: Bringing innovation and meaning into the context

There are other ways of doing science. We could ask what happens in what Kuhn called a pre-paradigmatic phase as opposed to the normal situation in a scientific milieu, which highly resembles the Euclidean approach as we refer to it here [6]. The theories and methods of the Euclidian method did not turn up out of nowhere. The stories of Galileo and Newton are well known. It took time and a lot of effort—research—in order to establish their systems and it is this establishment process that is of interest to us. It is what we may call the knowledge production or simply: research. In Kuhn's so-called pre-paradigmatic phase, we are confronted with unknown phenomena and we do not have any established rules and procedures that may aid our understanding of the unknown phenomena.

Unknown phenomena make us wonder, and starting with wondering, curiosity and the actual problems of that unknown phenomena instead of theory, makes it clear that it is possible to escape some of the shortcomings of the Euclidian model. The unknown phenomena pose a problem, which Kuhn called anomalies. The findings did not fit our knowledge (theories). Starting out with problems, it becomes possible to connect the research to the outside world and escape the ivory tower. The central alternative to a theoretical explanation of a phenomenon is therefore not other theories, but methods that are able to solve problems scientifically. By the aid of the researcher's curiosity it becomes possible to wonder about the phenomena we encounter in the world and investigate the phenomena we wonder about. We seek meaning in this world; we like to name things and think about these names. This search for and thinking about meaning should be our main interest and with curiosity and wondering as our starting point—not theories—we are able to ask fundamental questions about the phenomena of interest. Problem statements become much more important than theory, at least to start with.

To research something and find out what it is all about is a conceptualisation process. During this process, the research could be structured in the following manner: We encounter new phenomena, as was the case of electricity above; we ask questions and start searching for explanations and ways of understanding the phenomena. This is done in a process of trial and error. Experiments, discussions, arguments, attempts to formulate theory are all part of the process. Some of the attempts are successful, some turn out to be less fruitful. The point is that this may take some time and the process is not necessarily a smooth and easy ride, but characterised by conflict, disappointments, failure, and frustration (see e.g. [7]). However, succeeding with a scientific project can be very rewarding. It could include the ability to describe and understand unknown phenomena in a new theory, in a new language, and being able to make the unknown known. This process is also a conceptualising process [3]. A process through which the phenomenon is made accessible through language. Again, we may turn to the example of electricity; at first no one really knew what it was, but through years of research, concepts such as Ohm, Watt, etc. became accepted and were used by other scientists and, today, we have a well-defined set of concepts and theories enabling us to describe the phenomenon of electricity.

Truth in this conception of scientific research is found in the ability of new concepts and theories to solve our problems—or develop our visions—and answer our curiosity and wondering. A successful and interesting research project is not just true when some rules or theories of science have been obeyed—but when new rules have been established, e.g., when a new connection is made, or even when a new field of research is introduced (like that of nanoscience when it first came into being). Lyotard has termed this 'new moves in the language games of science' and it portrays a science that attempts to consistently reinterpret and re-conceptualise the world instead of building everlasting theories within a traditional discipline of science [8].

The problem based methodology is not only about developing new general and universal theories, but could also be concerned with more mundane problems. Problems faced by real living people in the social settings they live in. In their working life, at home, or anywhere else they encounter other people or technologies. Such occasions may all initiate new and interesting research questions. How can a specific city handle its traffic problems in the inner city? How can people concerned with ecological issues adjust their lifestyles? What are the risks of contracting bird flu if bird flu mutates into a human-to-human spread scenario?

The point here is that research can be immediately connected to the practices of people through research questions—questions we cannot ask from inside a world of absolute theories. Is the classical theory preserving research then necessary at all? Is it a total failure to spend one's time on old theories developed for the purpose of being general if not universal? The answer to this is for another paper but there is some ambivalence. Crystallised theories are condensed reports to the present about the research of the past. It is therefore often beneficial to know something about some of them but probably only if you actually need them in relation to the specific problem you are working on. They have the role of experience in relation to new problems, but in almost any case, they will not be able to solve exactly the problem at hand because they are not applicable to the problem—only constructed theoretical problems in a purely theoretical framework are directly solved by existing theoretical constructs. To solve real world problems, new moves in the language games of science are needed.

How can we understand this problem-based approach in research with respect to university educations? The model known as Problem-based learning (or just PBL) is one of the educational frameworks that take seriously the way research, scientific development and innovation are actually lived out in practice [9]. The meaning of data, theory and method in the problem-based method enters into a complex mix of iterative processes of conceptualisations that entail reformulations of research problems and new connections between different fields of study, even cross-disciplinary connections. In the following we present examples of the PBL model in action and consider how it brings about an entirely different approach to learning engineering mathematics than a Euclidean theory-based approach.

4. Educating mathematics through context—Examples

At Aalborg University, we have for some decades based our engineering educations on problem-based learning. The problem-based learning takes place in project groups of usually with 3 to 5 group members. Each semester each group of students has to produce a project report of about 70–90 pages, which means that they are trained in all kinds of teamwork skills, communication skills, the structuring of large research texts, etc. in order to actually constructing the project report in cooperation. Here however we shall focus on their opportunity to work actively with mathematics in relation to more than just their mathematics course textbooks. We consider the situation for the science and engineering students in their first year. Now, let us first address an example of what the education integrating mathematics for engineering students can look like in the Aalborg model of a PBL-setting.

During their first year of studies the engineering students have to choose a particular contextual problem setting, which has the potential for using some of the mathematical theory they are supposed to learn. This procedure for choosing a contextual setting is done in cooperation with a group of supervisors. Often a catalogue of possible problems to work on is produced upfront by the supervisor group—for example relating to these researchers' special interests or current research projects—but in the end students choose a project to work on and in most cases, have the freedom to choose any relevant scenario.

This setup provides the opportunity to relate engineering problems using mathematics with the problems of the real world from the outset, and often this process is non-linear, frustrating, evolves around cross-disciplinary issues, etc. It is in obvious contrast to the traditional way of learning mathematics. Traditionally students work exclusively with the reliability of mathematics and, at least to some extent, copy from textbooks, replicating proofs of a given statement or goes through small problem solving assignments. In the first and second semester, the science and engineering students are taking this type of traditional mathematics courses in linear algebra and calculus and are thereby taught an app. 20 ECTS amount of mathematical vocabulary entering into the problem based project work. It is possible to learn a lot about the mathematical syntax from such courses, but in general one will only develop certain limited mathematical skills and absolutely no skills in relation to mathematics in cross-disciplinary contextual scenarios.

Let us now look at some examples of how mathematics is taught through a problem-based approach for the first year engineering students in their project work which runs simultaneously to (among more courses) the mathematics course of the semester. What could this so-called context that includes mathematics consist of and what are the mathematics in it? The examples stem from the problem-based projects for science and engineering students first year of studies at Aalborg University.

4.1 A project example: Cleaning your digital pictures

From a variety of project themes presented by the group of supervisors, a group of students chose to look into the use of mathematics with regard to the red-eye removal, scaling of pictures, adjustment of skewed horizons and more everyday faults when taking amateur pictures. The group discussed in a joint process with their supervisors how to proceed with this initial problem setting. The idea for a project revolved around being innovative in relation to producing an online software solution that could be of ease to customers for editing their pictures.

From this outset, several possible trajectories for the project were discussed. What was the existing web-based or software based solutions for handling these types of editing? What are the actual problems that people really want to have fixed in their photos? Which role could the mathematical focus—linear algebra—in this semester of the students' education be integrated in the project? Would an implementation in the form of programming of the mathematical tools be part of the project? These questions and many more were a first encounter of the possibilities of what a PBL-project within this area could entail.

After some meetings with the supervisors and deciding on the ambitions and interests of the group a focus was made towards developing a functioning software prototype that could be used for the optimisation of digital images.

The project as a whole included sections about the state of the art and existing solutions, sections about linear algebra as a description of the use of selected elements of the theory for exactly this or that implementation in the problems fixed by the software. Finally, it included a section about the programming language and the implementation of the code for fixing the issues with pictures in the software.

The mathematics involved in the project related to different types of reconfigurations of images. In relation to 'red eye removal' this entailed the ability to detect edges in pictures. This again can be tackled by using convolution and students in their project report presents how to develop an 'edge map' of a picture by using mathematical elements from both calculus and linear algebra. Finding 'red eyes' then require the ability to find circles in this 'edge map' and here students explored the Circular Hough Transform technique. Finally, this part of the project ended in developing algorithms for the software implementation.

In addition to this 'red eye removal' procedure the students also discussed manipulating images in the form of rotation, noise removal, cropping and resizing as well as contrast and brightness adjustments. Not all parts of the project were equally deeply dealt with mathematically but all themes were touched upon to a reasonable degree of understanding and some topics were then explored in more depth in the report.

The content of the project was clearly in alignment with the basic mathematics course in calculus and linear algebra that students were attending in parallel. But not in the sense that they had seen the equations or techniques that they needed in their specific work on image manipulation but rather as a continuation of the course and a deeper insight into particular subparts of the general mathematical vocabulary in linear algebra and calculus.

To reflect on the learning process students were experiencing in this project let us address a few issues. First of all, it is important to remember that these were first year students. They were not close to finishing their masters or the like and in that respect the entrepreneurship and drive of this group of students was quite impressive. This is obviously not always the case to observe as a supervisor but on the other hand it happens quite frequently that the supervisor(s) has to keep the ambitions of the groups within the world of semester possibilities.

Secondly, one should mention that even tough it could sound like an exemplary elite group with extensive programming skills etc. this was not the case. The final project ended up satisfactory but there were many things that could have been improved, outlined in more detail and better coordinated etc. So, it is not an example of best practice but rather an example where we try and illustrate the kind of project students can work on.

Thirdly, the students worked with an interdisciplinary approach. They developed the foundation of a particular product—a program for fixing pictures—and at the same time drew upon several types of sciences in their approach. Mathematical theory in the form of linear algebra, programming of the actual implementation of the mathematical code together with all the turmoil of a programming environment and the ability to discuss and judge their product against competitors in the field. The learning of mathematics for these students was intertwined in a real context where the needs of people in general relating to the editing of images played a crucial part. This is what most real life scenarios involving mathematics look like. They come in a complexity that resembles the stock market and includes dimensions that involve several traditional university faculties—in the above example involving the Faculty of Humanities, the Faculty of Social Science, and the Faculty of Engineering and Science. By knowing just a little bit about what types of knowledge different types of sciences are able to handle, the engineering student becomes more aware of his or her own skills and abilities as a particular kind of scientist.

4.2 Other examples

We will give a few other brief examples to show other types of projects possible to use in the PBL-approach. In reality engineering students have worked on thousands of diverse projects through the years and they have the quality of being unique as they are each one of them connected to a particular context. Even for projects that are within the same area of research and context in the same semester the outcome can be quite different because the research focus formulated in a problem statement will vary. The two examples we outline below have been discussed in more detail in [10].

A group of students have been working on the DNA-micro-array technology, which is a technique for producing data about a person's DNA. In medical research projects hundred of thousands of DNA-substring data is compared for large groups of patients in order to determine correlation among certain gene expressions and certain illnesses etc. To handle all this data mathematics is the tool to use in this cross-disciplinary research area where doctors, engineers, biology experts etc. develop the technology complex. Students in this project experienced how doctors were lost to the complexity of the mathematical algorithms and theories involved—they were asked questions about how the whole thing works! In addition, they experienced the need to know more about what a gene expression really is and that the outcome of this more biological issue would have a direct impact on the later use of the technology and its entire construction. And a final outcome to address here was the enormous multiplicity in approaches to work on the massive sample data from a mathematical perspective—which measure of distance between two expressions was the right one? And what type of cluster analysis would really be the best in this scenario and why? The students explored the differences between measuring distances in n-dimensional spaces by using either the Euclidean approach or the Standard-Euclidean approach as well as using Pearson correlation (which is not really a measure but can be practical to use for biological reasons in the context) and so on. Issues of methodology in using mathematics entered the equation as the choice of measure will have an influence on the resulting cluster analysis.

A last example of a student group project fixed on the context of the google search engine. A group of students have worked on the quite invisible mathematics hiding beneath the surface of Google's search engine—the Page Rank Algorithm. The page rank system could probably be termed one of the most everyday technologies used as it comes into operation every time we google for something on the Internet. The ordering of relevance of webpages for the search is the result in each case and this process involves a load of linear algebra in the form of massive matrices that list how the Internet is interconnected webpage to webpage.

This was the context for a group of students who worked on actually understanding the mathematics involved and dealing with the ethical problems, the financial issues, and the technical challenges related to performing this type of ordering of relevance for the Internet searcher and community.

5. Conclusion and discussions: Context does matter

Is engineering mathematics abstract theories that come to life only as chalk on a blackboard? Or is it something more? Something that may be of immediate interest and use in people's everyday life? The outset we choose will also affect the way we teach mathematics to engineering, mathematics and science students at universities and, eventually, by way of tradition also at numerous other levels of education.

If we try and recapitulate the learning obtained by students in the PBL educational setting for engineering students, we find that several of the problems a traditional Euclidean way of thinking about research and education fosters are countered by this kind of approach.

In the examples above, students were acquainted with mathematical tools (here mainly linear algebra but it could obviously be other types of mathematics) only through an interdisciplinary contextual problem analysis. They studied phenomena where mathematics is a central part of the engineers' toolbox but at the same time a component that is intertwined with other types of knowledge related to the context that could highly relevant for using engineering mathematics appropriately.

The examples used here are of course just a few examples to give an idea about the PBL approach to working with engineering mathematics. A myriad of other contextual settings are possible and infinitely many more could be developed. Mathematics, just like any science, is involved in uncountable types of interdisciplinary research, political debates, philosophical problems, visions for the future of a city or the planetary environment, and so on and so forth. Contextualising education in the fields of science and mathematics will not remove attention from the fields of study in question. Rather, it seems that it would be a tool for moving science and mathematics in the direction of the human sphere as opposed to a doubtful timeless sphere of crystalized and abstract theoretical knowledge.

What does all this suggest for university education in engineering and science? Introducing context and PBL the problem of validity becomes a central theme for students from the very onset of the mathematical training. Thereby the engineering students become skilled in contextualising mathematics and, consequently, in utilising it in a complicated setting. Students also learn that mathematics is much more than just chalk on a blackboard—they learn that mathematics is highly relevant, even indispensable, to their lives and general reasoning about the world, an integral part of modern society and modern science especially. They also learn that math can be interesting and worthwhile working with in cooperation with other people, for five years at university and possibly for the rest of their life.

In a PBL setting, engineering students work with the problems, visions and ideas of reality instead of having their mathematical skills introduced as purely abstract theories that make it very difficult, at a later stage, to work with mathematics in a practical setting. Thereby context becomes part of the perspective in engineering education, not at the expense of mathematical skills but in addition to them. For students learning mathematics the setup may be paralleled to the processes that are taking place when learning a new language; students learn that it is important to get out and use the language in practice where all the complications begin because the grammatical theory of reciting mathematics will only take them so far.

References

1. L. Nørreklit, *Problemorienteret forskningspraksis og den reale virkeligheds konstitution (Problem oriented research practice and the real reality's constitution)*, Aalborg University, Aalborg, 1978.
2. O. Ravn and O. Skovsmose, *Philosophy of Mathematics—Connecting Humans and Equations*, Springer, Dordrecht, 2017.
3. L. B. Henriksen et.al. *Dimensions of Change—Conceptualising Reality in Organisational Research*. CBS Press. Copenhagen, 2004.
4. O. Skovsmose, K. Yasukawa and O. Ravn, Scripting the World in Mathematics and its Ethical Implications in *Philosophy of Mathematics Education Journal*, Nr. 26, 12.2011.
5. T. Børsen Hansen, Scientific Bildung for the Post-Normal Epoch. In J. Christensen, A. Kolmos and L. B. Henriksen Eds. *Engineering Science, Skills and Bildung*, Aalborg University Press, Aalborg, 2004.
6. T. S. Kuhn, *The structure of scientific revolutions*, University of Chicago Press, Chicago, 1971.
7. A. Pickering, *The Mangle of Practice: Time, Agency, and Science,* University of Chicago Press, Chicago, 1995.
8. J.-F. Lyotard, *The Post-modern Condition: A Report on Knowledge*, Manchester University Press, Manchester, 1979.
9. A. Kolmos et al. Eds., *The Aalborg PBL Model—Progress, Diversity and Challenges*, Aalborg University Press, Aalborg, 2004.
10. O. Ravn, Closing the gap between formalism and application—PBL and mathematical skills in engineering, *Teaching Mathematics and its applications*, **27**(3), 2008.

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