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A Phenomenographic Analysis of Students' Experience of the Mohr Circle: A Case Study in Research-Led Engineering Education*

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The case study presented in this paper was motivated by a teacher's desire to understand better the conceptual difficulties his students were experiencing with the topic of the Mohr Circle in a materials engineering course. To gain such an understanding, a phenomenographic study was undertaken based on student interviews. The findings of the study identified four categories of qualitatively different conceptions about the Mohr Circle that were evident among the students. These, together with a number of subcategories, provided a conceptual structure for modifying the way the course was taught and also highlighted particular aspects of the topic that required pedagogical attention. Although the study derives from a specific group of students and relates to a specific context, the insights reported in the paper may have relevance to other contexts and groupings of students. In particular, it demonstrates the utility of phenomenography as a useful methodology in research-led engineering education.

Keywords: engineering education; student-centred education; evidence-based education; stress analysis; failure analysis; phenomenography; Mohr Circle

1. Introduction

This paper is part of a series on research-led education, i.e. education that is informed and shaped by evidence derived from educational research [1]. As explained in the previous paper in this series [2], this approach to teaching has become increasingly recognized as an effective way to improve the quality and impact of one's teaching.

The case presented in this paper relates to a module in a materials engineering course on failure analysis that introduces the topic of the Mohr Circle, an important technique for analysing the structural integrity of engineering materials subjected to stresses. In the experience of the coordinator of this course, students generally found the technique to be somewhat abstract in nature and the topic difficult to master, in line with experience reported elsewhere [3, 4]. A study was therefore undertaken to investigate the difficulties students had in grasping the relevant concepts and the application of the Mohr Circle in stress analysis, and, on the basis of the findings, to make appropriate modifications to the way the module was taught. The paper describes the study, presents its findings, and discusses the utility of studies of this kind as a means of improving teaching and learning.

Before presenting the study itself, it is necessary to give a brief overview of the Mohr Circle and how it is used in stress analysis. This is necessary as background for readers not familiar with the topic and also for readers who are but wish to know the technical level at which the topic was treated in the course.

2. The Mohr Circle in stress analysis

Figure 1 illustrates the basic principles of how the Mohr Circle can be used to analyse two dimensional stress patterns in a body. It highlights the key variables and aspects of the procedure for drawing and using a Mohr Circle and serves as a convenient reference for the reader not familiar with the procedure. The figure is a representation of a body subjected to a system of stresses in two dimensions and was the generic system used in teaching the topic. The module did address more complex, triaxial systems but the majority of the module (and this study) focused primarily on two-dimensional systems.

The intended learning outcomes for the students with regard to the analysis of two dimensional stressing of bodies can be summarized as follows. Students were expected to be able to analyse a given engineering context and to identify the various stresses applied to a particular point in a body of a given shape in that context. From this they were expected to be able to develop a diagram of the kind shown in Fig. 1a and to identify the normal stresses, σ_x and σ_y, in the x and y directions respectively, and any shear stress, τ_{xy}, in the xy plane. Students needed to be aware that such a system will set up a pattern of stresses in the body, which may lead to plastic deformation if the stresses at any point in the

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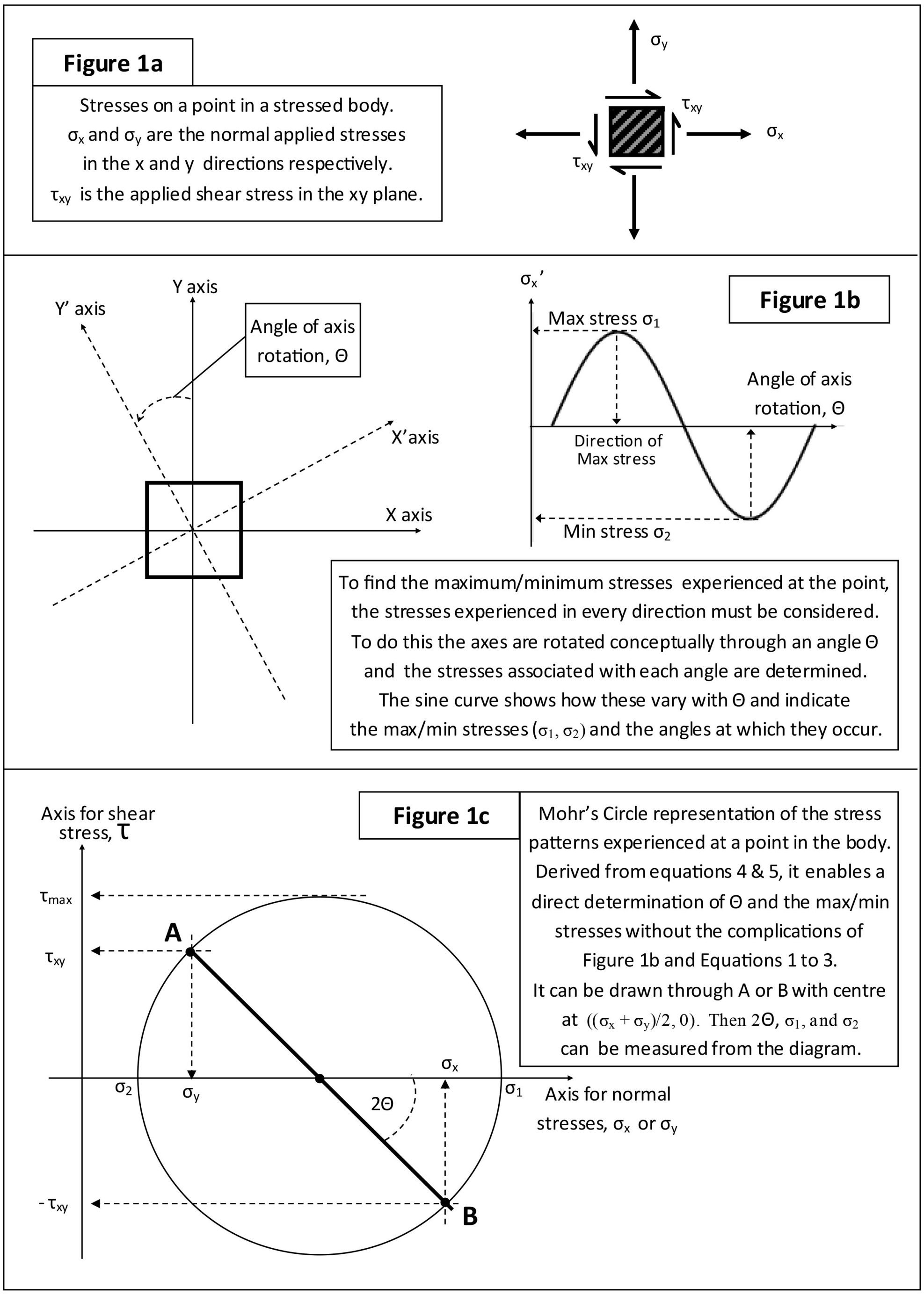

Fig. 1. The Mohr Circle representation of 2D stresses at a point in a body.

body were greater than the yield strength of that body. However, the exact nature of that pattern is not immediately obvious; the stresses in some directions within the body and the shear stress across some planes within the body will be greater than in other directions and across other planes.

Accordingly, students were expected to know that, in principle, attention had to be given to every possible direction and plane and to the stresses associated with those directions and planes at any point in that body. Further, they needed to be aware that this was best achieved, in a two dimensional system, by a conceptual rotation of the x-y axes as illustrated in Fig. 1b. Such a rotation would lead to a variation in the magnitude of the stresses associated with each particular orientation—i.e. σ_x', σ_y', and τ_{xy}'—as the axes were rotated through an angle Θ as illustrated by the sine curve in Fig. 1b. The magnitude of the stresses in the new directions can be determined in the normal way by resolving the resultant forces (from stresses σ_x, σ_y, and τ_{xy}) in the x' and y' directions and summing the components of each according to the new orientation to give the transformation equations (1) to (3). The equations become more complex for three-dimensional stress systems.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad (2)$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad (3)$$

The maximum and minimum stress experienced in the body at the point in question (i.e. at the intersection of the coordinate axes) occur at the orientation where the shear stress is zero; here the applied shear force components contribute only to the normal stresses. These stresses, respectively σ_1 and σ_2 in Fig. 1, as well as the angle Θ of the direction and plane associated with these maxima and minima can then be determined from the transformation equations—Equations (1) to (3). Students needed also to be aware that the transformations needed to develop the sine curve and determine Θ and the maximum/minimum stresses are complex and that the needed information can be obtained more easily and directly by means of a graphical representation known as the Mohr Circle—Fig. 1c. This is derived by squaring and adding Equations 1 and 3 to obtain Equation 4 which is the equation of a circle with its centre at (σ_{avg}, 0) and radius R as given in Equations 5.

$$\tau_{x'y'}{}^2 + (\sigma_{x'} - \sigma_{avg})^2 = R^2 \quad (4)$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \; and \; R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2 \quad (5)$$

The students needed to know that the circle can be constructed knowing σ_{avg} and R or by plotting the circle centre at coordinates ((σ_x + σ_y)/2, 0) and points A or B at coordinates (σ_y, τ_{xy}) or (σ_x, $-\tau_{xy}$) and then drawing the circle through A or B. Further, they needed to be able to determine from the diagram the relevant angle Θ and the maximum normal and shear stresses, i.e. σ_1 and τ_{max} respectively in Fig. 1c.

3. The study

3.1 Context of the study

The context of the study was a semester long materials failure analysis course which was part of the third year programme towards a degree in metallurgical engineering offered by a South African university. Two to three lectures and one afternoon tutorial (sometimes two) in this course were devoted to the topic of the Mohr Circle. The number of students typically registered for the course each year is between 20 and 40.

In the 4 years of teaching the course, the course coordinator had found that many of students had struggled to master the topic. The nature of their confusions and the reasons for their difficulties with the topic were not clear. Accordingly, the research reported in this paper was undertaken as a first step in gaining a better understanding of the nature of the students' conceptual and practical difficulties with the topic so that appropriate modifications could be made to the way it was taught. To guide the study, these objectives were expressed in terms of the following research questions.

(1) What are the qualitatively different ways in which students taking this course (a) understand the Mohr Circle representation of the stresses in failure analysis, and (b) apply that understanding?
(2) What pedagogical modifications to the course do the answers to these questions suggest?

The methodology selected to address these questions was phenomenography. The rationale behind this choice and an overview of the methodology follows.

3.2 An Overview of phenomenography

Phenomenography is a methodology widely used in educational research to investigate the qualitatively different ways in which students relate to or experience a particular phenomenon. It was developed over the period from the 1970's into the 1990's [5–7] but recent critical reviews [8, 9] show that "it still appears to have much to offer to higher education research" [9, p. 319]. It's usage within engineering education research is not as extensive as in other fields but it is receiving growing attention [10–13].

The foundational premise behind a phenomenographic study is that people respond to situations or to phenomena according to how they experience or conceive them [7]. Therefore, in order to gain useful insights into how they might act in a given situation requires that attention should be focused on the relation between the people and the relevant phenomenon and not on the phenomenon itself. This can be illustrated by considering the most widely known result of phenomenographic research, namely the recognition of two distinctly different approaches to learning—a 'surface' and a 'deep' approach. In the original work [5], students were asked to read an academic article in preparation for an interview in which they would be asked questions on the article. Analysis of the interview transcripts focused on how the students had related to the phenomenon of learning rather than on what they had learned or what the text was about. It was apparent that some students had approached the task of learning with the intention of accumulating and memorizing information that seemed relevant while some had sought more to understand the content of the text. The former approach was labelled a 'surface approach' and was found to be associated with lower levels of academic achievement while the latter approach, termed a 'deep approach' to learning, was found to be associated with higher levels of achievement. The value of the finding and its pedagogical utility lay in understanding the critically different ways in which students engaged with the learning task and how the different ways of engagement led or could lead to different learning outcomes. The pedagogical implications of this finding are obvious—teaching should be geared as much as possible to get students to adopt deep approaches to learning. What this means and how it might be achieved has been the subject of a considerable amount of subsequent research [14–17].

The intent of a phenomenographic study, therefore, is to establish a profile of the critically significant variation in the ways that students relate to, experience or conceive a particular topic or aspect of learning. That profile, usually termed the 'outcome space' of the study, will consist of a parsimonious set of qualitatively different categories that are logically related and, together, aim ''to describe the totality of variation in the pool of [the students'] experience'' [10, p. 199]. Parsimony is important here because otherwise the individual idiosyncrasies of a multitude of students are likely to overwhelm and obscure any pedagogically useful insights that might be forthcoming from the study.

To maximize the pedagogical usefulness of the 'outcome space', the critical factors that distinguish one category of variation from another must be clearly understood and articulated. Clarity here facilitates the subsequent use of the study findings in, for example, ''the design of educational learning objectives, pedagogical strategies, assessments, and evaluations'' [10, p. 199]. In particular, the labels given to each category should be striking and memorable and should capture the essentials of the distinguishing features of that category. In effect, they become a 'vocabulary' describing the essentials of the variation in question [18].

Examination of how the categories differ qualitatively invariably reveals a progression or hierarchy of some kind from less to more sophisticated ways of experiencing or conceiving the topic or phenomenon. Because this progression derives directly from student perceptions it can provide very useful information about how those students might progress from less to more sophisticated understandings of the topic; in essence it provides a conceptual scaffolding of a topic that is based on how students respond to it, and consequently suggests the kind of 'learning pathways' the students typically would need to follow in order to master that topic [18]. This will be discussed in more detail later in the paper.

The data needed for a phenomenographic study typically consists of student interview transcripts but may derive from other sources such as questionnaires or video recordings. A study typically involves three stages: the selection of students for an interview study; the development of the interview protocol and the subsequent interviewing of the students selected; and the phenomenographic analysis of the interview transcripts. Details about each of these stages, as carried out in the case study, follow.

3.3 Sampling method

The primary criterion in selecting students for a phenomenographic study is to strive for maximum variation so that as wide a range of ways of experiencing a topic is represented across the sample of students selected [19]. Typically, 8 to 15 students in a carefully selected sample is sufficient to provide an adequate representation [7]. In this study, the class size was 25 and 10 accepted the invitation to participate in the study by being interviewed. In order to maximise the variation among the students invited, consideration was given first to academic ability (as reflected by marks in a related course), and then to gender and ethnicity. The selected students were asked to volunteer for the study and the normal ethical procedures were followed before they signed the relevant consent forms.

3.4 Interview protocol

The interviews were conducted using a semi-structured interview protocol [20]. The protocol, developed and refined through two pilot interviews, was based on five sets of questions but allowed the interviewer to deviate from these when this was appropriate. The first question set enquired about how the interviewee related to the topic, and what he or she found interesting, difficult, confusing, helpful or important. The second and third sets addressed the issues of what a Mohr circle was, and the students' conceptions about stresses and their relationship to the Mohr Circle. The fourth and fifth sets solicited the interviewee's experience of learning the topic, their experience of the textbook and any suggestions they might have about modifications that would be helpful for learning the topic.

3.5 Analysis

The interviews were recorded, transcribed and the transcripts analysed using qualitative analysis software [21]. The analysis followed a typical phenomenographic procedure [6, 7, 22–24]. It began by reading the transcripts carefully to identify and extract phrases, conceptions, or experiences that were relevant to the research questions. Extracts with similar meanings were coded and identified with an appropriate short label. The list of codes was emergent in that it was built up as the transcripts were analysed and more extracts identified. Once all the transcripts had been analysed in this way the list of codes was refined by collapsing those codes that were conceptually similar, and, where necessary, redefining the codes as seemed appropriate. During this refining process, reference was continually made back to the original transcripts to ensure that the emergent list of codes remained faithful to the original meanings expressed by the students. This was a highly iterative process with several generations of code lists being formulated before a refined list emerged that was considered to express best the qualitative variation in the students' conceptions. This list then constituted the 'outcome space' of the phenomenographic analysis, i.e. the qualitatively different categories of student conceptions regarding the Mohr Circle.

The final stage in the analysis was to examine the structure of the outcome space, and to establish as clearly as possible the ways in which the categories were qualitatively different, and whether or not subtle nuances—subcategories—existed within the categories. Particular attention was paid to whether or not any meaningful progressions were evident from one category or sub-category to another. As part of this process, a careful re-evaluation and labelling of the categories (and subcategories) was undertaken so that the names assigned to each were as memorable and as accurately descriptive of the category or sub-category as possible.

4. Research findings

4.1 The phenomenographic outcome space

The phenomenographic study identified four qualitatively different conceptions of the Mohr Circle that were evident in the group of students interviewed. In summary, these were that the Mohr Circle is a topic, a procedure, a tool, and a visualization. Each conception, along with its associated subcategories and nuances, is described briefly in Table 1 and is explained with illustrations thereafter. The illustrations are extracts from the student interviews as indicated by [Student (or Transcript) X] where X is a number representing one of the students interviewed.

1: *The Mohr Circle is a* ***Topic*** *to be studied and learned.* This conception sees the Mohr Circle merely from the perspective of a component of a course that requires the student's attention in order to pass the course. While this is an obvious fact that the students were aware of, it was evident that some

Table 1. Variation in the students' conceptions of the Mohr Circle
Labels for categories and sub-catorgies are highlighted in bold itallics

	Conception: The Mohr Circle is . . .	Sub categories of the Conception
1	***A Topic***: The Mohr Circle is just a topic to be studied and learned	
2	***A Procedure***: The Mohr Circle is a procedure for analysing stresses in a body	2a) It's a procedure for ***Simplifying*** a stress analysis 2b) It's a procedure for ***Replacing*** with a graphical representation the equations that describe stresses - - - - - - - (*conceptual transition*) - - - - - - - - 2c) It's a procedure for ***Analysing*** the stresses in a body to determine . . . 2c1) the maximum/minimum stresses in the body 2c2) and their orientation 2c3) and the location of the principal planes in the body and the maximum/minimum stresses in those planes
3	***A Tool***: The Mohr Circle is a tool for analysing the structural integrity of a body	3a) It's a tool for ***Modelling*** how a body responds to stresses 3b) It's a tool for ***Examining*** the stress patterns at specific points in or on a body subjected to stresses 3c) It's a tool for ***Evaluating*** the structural integrity of a body subjected to stresses 3d) It's a tool that is useful when ***Designing*** mechanical components or when ***Troubleshooting*** their failure
4	***A Visualization***: The Mohr Circle enables a graphical visualization of the patterns and significance of stresses in a body	

of them related to the Mohr Circle merely as a topic to be 'endured' or 'struggled through' as the following interview extract illustrates.

> It's just that part of the course where you just have to get through it, hope it's not in the test. I don't understand it so I can't relate them [i.e. Mohr Circles] to my work. [Student 7]

2: *The Mohr Circle is a* ***Procedure*** *for analysing stresses*. This conception recognizes the Mohr Circle as a procedure for analysing stresses in a body. As indicated in the table, the study identified three sub categories of this conception. The first sees the Mohr Circle as a procedure for ***simplifying*** a stress analysis. Student 8, for example, put it this way: "[Mohr Circles] are simplifying the way we look at triaxial systems [. . .] So I think they are very good in terms of simplification".

The second sub category sees the Mohr Circle as a procedure that uses a graphical representation of the stresses in a body to replace the equations which describe those stresses; i.e. ***replacing*** the equations with a graphical representation. This conception was illustrated by Student 1 as follows: "The equations do the same thing as the Mohr's Circle, but for me I saw the Mohr's Circle as easier to grasp because [with] the equations you basically put in the values and calculate different intermediate values for the next stage until you get the maximum shear stress. But for the Mohr's Circle it's just a graphical way of doing it whereby you have your axis and you plot, you draw a circle and thereby find your shear stress".

Unlike with the two previous sub categories which have to do with perceptions about what the procedure does in general, the next sub category focuses more on the procedural details for ***analysing*** the stress patterns in a body. The following transcript extract, despite its confusions, illustrates the point.

> In one example, we've done we were given σ_X and shear stress σ_Y [sic] where we have to somehow do a circle from what we're given and then find the average. From the values that we're given we have to draw a circle. From then you're going to see the minimum shear stress and maximum shear stress, which is σ_3 and σ_Y [sic]. [Student 7]

The study showed that among the students there was a range of conceptions about what the Mohr Circle, conceived as a procedure for analysing the stresses, involved and about the kind of calculations which it enabled. The most basic of these is that it enables the calculation of the maximum and minimum stresses in the body. This is illustrated by the previous quote and was evident in most of the transcripts. However, it was clear that among the students a conception existed that this was *all* that the Mohr Circle procedure enabled; the fact that it also enables the determination of the orientation of the maximum/minimum stresses and that these were associated with 'principal planes'—i.e. the planes in which the maximum/minimum stresses occur—is overlooked. The following extract illustrates this.

> All I know is that you draw it [the Mohr Circle] from what we're given and then somewhere here it's the minimum shear stress and somewhere here is the maximum shear stress. [Student 7]

More sophisticated conceptions of the Mohr Circle as a procedure were also evident and were characterized by awareness of the aspects missing from the previous conception. First, there is the awareness about the importance of the orientation of the stresses when determining the maximum/minimum. Student 2 expressed this very clearly as follows.

> You can calculate the stresses at different orientations of your element [in the body]. If you have a certain orientation and you have normal stresses and shear stresses, then if you want to calculate the stresses at a certain angle then you can use Mohr's Circles. [Student 2]

Second, there was the awareness that the maximum stress lay in the 'principal plane' oriented at the angle Θ to the coordinate axes and that failure, if it occurred, would occur in that plane. However, only one transcript hinted at this awareness. This suggests, therefore, that three qualitatively different conceptions of the Mohr Circle as a procedure for analysing stresses were evident among the student transcripts. As indicated in Table 1, these form a hierarchy of increasing conceptual sophistication and completeness: i.e. from the most basic conception that the procedure enables the determination of the maximum/minimum stresses; to awareness that it also enables the determination of their orientation; to awareness that the maximum/minimum stress lies in the principal planes in these determinations.

The final point to note with regard to perceptions about the Mohr Circle as a procedure is to emphasize the qualitative distinction between the first two subcategories (2a and 2b, ***simplifying*** and ***replacing***), and the third subcategory ***analysing*** (subcategory 2c). On the one hand, ***simplifying*** and ***replacing*** are perceptions about what the Mohr Circle procedure does in general, while, on the other hand, ***analysing*** involves perceptions about procedural details; namely, the procedural steps involved and the specific calculations which the Mohr Circle enables. This distinction is highlighted by the broken line in Table 1.

It is evident, therefore, that there is a progression in the sophistication and completeness of the conceptions of the Mohr Circle as a procedure: first it simplifies a stress analysis; second it does so by replacing equations with a graphical procedure; and third, there are the conceptions about the procedural details which are themselves nuanced at three levels of sophistication.

3: *The Mohr Circle is a* ***Tool*** *for analysing the structural integrity of a body.* This conception goes beyond seeing the Mohr Circle merely as a procedure in that it also sees the purpose behind the procedure, i.e. as a tool for analysing the stress patterns in a body, and thereby investigating its structural integrity when it is subjected to stresses. This is evident in the following extract where the student is clearly aware of the Mohr Circle 'procedure' for analysing the stresses (or, to paraphrase the student, 'for describing what's happening') but sees this analysis as a tool that is "useable", i.e. the procedure has a clear and useful purpose.

> It [the Mohr Circle] looks more complicated than it should be because it's a tool. The first instances of meeting it you don't understand what it is or what you should use it for and it does look very scary and very complex. After understanding it, I won't say it's easy but it's useable. [. . .] They [the stresses] are there, they do that, they show what the things are. I see them [sic] as a tool, not [so much just] as a description of what's happening. [Student 6]

As indicated in Table 1, the conception of the Mohr Circle as a tool is nuanced, and four sub categories of this conception are evident. The first, sub category 3a, sees it as a tool for ***modelling*** how a body responds to stresses. Student 5, for example, put it this way: "I think Mohr's Circle is like a mathematical model for stresses". This conception constitutes an advance on the previous conception in that a model implies purpose—something is modelled for a reason—whereas a procedure is fundamentally just a series of manipulations. However, the purpose implied is rather general; i.e. to model how a body responds to stresses. As Student 3 put it, "I understand that they [i.e. Mohr Circles] are used to analyze stresses of materials e.g. when forces are applied to a component, how will it react [i.e. how will it respond to those stresses]".

The next sub category (3b) is less general in that it sees the Mohr Circle as a tool for ***examining*** stresses *at a point* in a body. The implication here is that stress patterns are frequently different at different points in a body and that a convenient tool for examining these patterns at any point is helpful. This conception is illustrated by the following transcript extracts.

> [The purpose of the Mohr Circle is] to actually describe the types of stresses that are present at a particular point in the component and how they are inter-related or affect each other at that point. [Student 5]

> Within an object there are different points and you put stress. Different points experience that stress [differently]. Mohr's Circle takes that point and it interprets what's happening there. [Student 6]

Seeing the Mohr Circle as a tool for examining stresses at particular points in a body is an advance on the previous conception in two respects. Firstly, the awareness that the analysis is conducted explicitly at and is related to a particular point in a body is missing in all the previous conceptions. Secondly, it embodies more explicitly the purpose of a stress analysis, namely that in order to understand how a body responds to stresses it is necessary to examine the stress patterns that occur at many points within that body.

The next sub category (3c) embodies a still more complete conception of the purpose of the Mohr Circle, namely as a tool for ***evaluating*** the structural integrity of a body subjected to stresses; i.e. "will it stay intact or fail" [Student 3]. The advance in this conception is the awareness that a body will deform or fail if the maximum stress experienced anywhere in the body is greater than the yield strength of that material. The evaluation brings into the analysis the properties of the material, i.e. its yield strength. Although a little conceptually and terminologically confused, the following extract illustrates the point.

> You get the maximum stress and you come back and plastic deformation happens. [Paraphrasing and correcting: if the stress experienced exceeds the yield strength, plastic deformation occurs.] I just understand the responses of a material to those [stresses]. So if you have maximum stress it would plastically deform afterwards and with minimum stress it goes back to its original shape. [Paraphrasing and correcting terminology: if the stress experienced is greater than the yield strength it would plastically deform and if less it goes back to its original shape when or if the stress is released]. [Student 7]

In summary, this third sub category sees the Mohr Circle as a tool for evaluating the structural integrity of a body by enabling (1) a determination of the magnitude of the stress patterns that will occur at specific points in a body when it is stressed, and then (2) evaluating whether or not the material will fail or deform by determining whether or not the maximum stress experienced at one or more points exceeds the material's yield strength. In addition, (3) it enables the determination of the orientation of the principal planes, i.e. the planes associated with the maximum/minimum stresses, and an indication of the direction in which failure or deformation will occur.

The fourth sub category (3d) of the conception of the Mohr Circle as a tool takes the awareness of the purpose of the Mohr Circle one step further. Whereas the previous conception focused particularly on its utility in evaluating the structural integrity of a body when stressed, this sub category focuses on the purpose of such an evaluation, namely as a tool that is useful when ***designing*** a mechanical component or when ***troubleshooting*** the failure of a component. In the former case, the evaluation is conducted to ascertain whether or not a particular design is viable in that it is unlikely to fail under the stresses it is likely to encounter in operation. In the latter case, when a component has failed, the evaluation is conducted to assist in understanding why: was it because of a design flaw, or the result of being over-stressed in operation, or some other reason? The conception of the Mohr Circle as a tool when designing components or troubleshooting component failure is illustrated in the following transcript extract.

> I think Mohr's Circles are more applicable when analyzing a situation—maybe a finite element analysis of things such as pressure vessels, trying to see how much force in terms of shear stresses that component is subjected to, the design limit, or even the initial design whereby you want to determine where [sic] you don't know the operational stresses. So you would be trying to find the maximum theoretical use factor to find the design operational stresses. [Student 1]

4: *The Mohr Circle is a* ***Visualization****, a graphical visualization of stress patterns in a body.* This conception of the Mohr Circle is considered a separate category from the other conceptions because it has to do with the visualization of stress patterns rather than the performance of the relevant calculations or analyses. The conception embodies the awareness of the Mohr Circle as a tool for analysing the structural integrity of a body subjected to stresses: i.e. awareness of how it provides a means of 'simplifying' the analysis of the stress patterns; of which aspects of those patterns are key, and of the significance of these key aspects in relation to the structural integrity of that body when stressed either in a design or a troubleshooting context. However, as the following extract illustrates, the conception also embodies the awareness of the Mohr Circle as a tool that can facilitate an actual visualization of those aspects of the stress patterns in a body that are critical to its structural integrity.

> "They [Mohr Circles] are simplifying the way we look at triaxial systems e.g. the three dimension ones. I do have difficulty grasping these systems the way they are, but with the Mohr's Circle I can really see where the maximum shear is. I can actually see where the X and Y directions are. So I think they [Mohr Circles] are very good in terms of simplification". [Student 8]

4.2 A conceptual scaffolding

When the categories just described are analysed for structure, it is evident not only that there is a progression among the categories and subcategories of variation, but also that the progression has three aspects: a general aspect; an aspect relating to usage; and an aspect relating to visualization. As summarized in Table 2, the general aspect relates to what Mohr Circles do: they simplify a stress analysis; they replace the transformation equations with a graphical procedure; they enable an analysis of stresses that enables an evaluation of the structural integrity of a body that is useful for design and troubleshooting purposes. With regard to usage, the progression in Table 2 spells out the procedural and application steps from the most basic to the most sophisticated. With regard to visualization, the progression follows the procedural steps but relates to the visualizations which these enable. (Note that Table 2 has dropped the first conception of the Mohr Circle as a topic to be studied because it is obvious and not very helpful.)

The conceptual breakdown presented in Table 2 details how the different student conceptions build on and relate to one another; each category or

Table 2. A conceptual scaffolding: Three progressions in students' conceptualizations of the Mohr Circle
(The original labels given to the conceptions are highlighted in bold.)

A GENERAL PROGRESSION The general nature of what Mohr Circles do. They . . .	**PROGRESSION RELATED TO USAGE** The Mohr Circle is a **Procedure** used to determine . . .	**PROGRESSION IN VISUALIZATION** The Mohr Circle enables a **visualization** of . . .
. . . **simplify** stress analyses		
. . . **replace** stress-equations		
. . . **analyse** stresses	. . . **maximum/minimum stresses**	. . . relative magnitude of max/min stresses
	. . . **orientation** of these stresses	. . . orientation of these stresses
	. . . **principal planes**	. . . principal planes
Mohr Circles facilitate . . .	The Mohr Circle is a **Tool** for . . .	
	Modelling how a body responds to stresses	
	Examining stress patterns at points	
. . . analysis of structural integrity	**Evaluating** structural integrity	
. . . the **designing** of components or **troubleshooting** their failure	**Designing** of components or **troubleshooting** their failure	

subcategory being a more sophisticated conception of the Mohr Circle than the conceptions above it or to the left of it in the table. This breakdown therefore constitutes a conceptual scaffolding that suggests how students might progress to more sophisticated and complete understandings of the topic; it suggests 'learning pathways' that students could or need to pursue in order to master the topic and develop the associated competencies.

4.3 Distribution of the conceptions

It is sometimes useful in a phenomenographic analysis to estimate how the conceptions that have been identified are distributed among the students. This needs to be done with caution for two reasons. Firstly, there is no one to one mapping between students and the categories itemised in the tables; any individual student may possess several if not all of the different conceptions that have been identified. Secondly, the interviews were not intended to provide quantitative data so any quantitative information derived from them must be treated only as tentative estimates that would require confirmation and should be interpreted with caution. Table 3 presents the distribution of the various conceptions as found in the interviews.

The first point that emerges from Table 3 is the relatively low occurrence of the first three conceptions, 1, 2a and 2b. These conceptions are general in nature; they are more about what the Mohr Circle does in general than about the specifics of its use. In addition, they are the least sophisticated and most obvious conceptions regarding the Mohr Circle and as such it is perhaps not surprising that only a few of the students paid any attention to them in their interviews. However, it is perhaps a cause for concern that only 40% of the students interviewed drew attention to conceptions 2b—that the Mohr Circle was a graphical representation replacing the transformation equations that describe the stresses from different orientations.

With regard to the conceptions that relate to the specifics of the use of the Mohr Circle (conceptions 2c to 3d), there appear to be a number of shortcomings among the students. While there appears to be a solid appreciation that the Mohr Circle enables a modelling of the response of a body to stresses, the calculation of the maximum/minimum stresses, and an evaluation of the structural integrity of a body under stress (conceptions 3a, 2c and 3c respectively), the awareness of the importance of the orientation of the maximum/minimum stresses (conception 2c, second nuance) and the fact that the stress analysis needs to be conducted at different points (conception 3b) appears to be weak. It is also a concern that few of the students expressed an awareness that the evaluation of the structural integrity of a body provided information that was important for design and troubleshooting contexts, and that this was the overall objective of conducting such an evaluation. Also, a concern is that very few students made mention of the concept of principal planes and their role in the understanding of stress patterns in a body.

The low occurrence of the last conception (4 in the table)—that the Mohr Circle enables an actual visualization of critical aspects of stress patterns in a body—is not considered to be problematic from a pedagogical point of view. This is a high level conception or experience of what the Mohr Circle enables and as such it is not surprising that only one of the students mentioned it. What is critical pedagogically is that students develop a solid awareness of conceptions 2c to 3d and master the associated calculations and manipulations; these are the key aspects essential to mastery of the topic.

5. Discussion

The study presented in this paper is interesting at several levels. In the first place, it has generated specific pedagogical insights relevant to the teaching

Table 3. Occurrence of the Conceptions among the Student Transcripts

Conception	**Sub categories of the conception**	**Nuances**	**% of transcripts that implied or described the conception**
The Mohr Circle is . . .			
1) **A Topic**			20%
2) **A Procedure** for	2a) **Simplifying** the analysis		20%
	2b) **Replacing** equations		40%
	2c) **Analysing** stress patterns with respect to . . .	Max/Min Stresses	70%
		Their Orientation	40%
		Principal Planes	10%
3) **A Tool** for	3a) **Modelling** a body's response to stresses		60%
	3b) **Examining** the response at different points		20%
	3c) **Evaluating** the structural integrity of a body		70%
	3d) **Designing** a component or **Troubleshooting** failure		20%
4) **A Visualization**			10%

of the Mohr Circle in a particular context. It has highlighted where and how conceptions among a particular group of students diverged from the desired learning outcomes and, therefore, where pedagogical reflection and modification is indicated in that context. So, for example, the study suggested that more attention should be paid to the significance of the orientation of the maximum/minimum stresses, and that a stress analysis is typically conducted at multiple points. These observations suggest pedagogical modifications such as augmenting tutorial questions by requiring multi-point analyses and the determination of the orientation of the planes in which failure would occur. The study findings also directed pedagogical attention towards facilitating a deeper appreciation of the purpose of the stress analyses which the Mohr Circle facilitates (i.e. their utility in component design and failure analysis), as well as of the theoretical underpinning of these analyses (e.g. the association between the transformation equations and the Mohr Circle representation of the stresses; and the relevance of the principal planes). Tutorial questions and assignments could be shaped accordingly, and these issues could be emphasized or reinforced at appropriate points during formal input, in feedback to students, and in informal discussions with them. With regard to different teaching contexts, the study findings may point to the need for similar modifications or may spark alternative ideas. Although the study findings and the insights emanating from them are pertinent to a particular context, it is likely that they will be of interest to others teaching the topic in different contexts.

It is interesting to note that the literature offers only limited help with regard to useful pedagogical insights related to the teaching of Mohr's Circle. Only two relevant references were found [3, 4], a paucity that reaffirms the need for teachers to research teaching and learning in the topics they teach. Both references emphasize the conceptual difficulty associated with the topic. In addition, the first suggests a number of rules for consistent assignment of positive and negative stresses and presents a method for constructing the Mohr Circle that may or may not differ in emphasis from the method a teacher normally employs. The second reference presents a concept map of "Mohr's Circle-related concepts" (p. 4) compiled from both student interviews and from "combing through undergraduate textbooks" (p. 5). It also suggests that a poor grasp of the nature of stresses may be a significant reason why students struggle with the Mohr's Circle. However, apart from the latter observation, neither reference offered any insights that were particularly informative to the context of the study, though they do offer different perspectives that a teacher may find helpful.

The second level at which the case study is interesting has to do with the outcome space of the phenomenographic study. The pedagogical implications and insights from the study that have been mentioned so far are somewhat ad hoc in nature; they are of the kind that would be forthcoming from almost any serious engagement with students about their experience of learning the topic. However, the phenomenographic analysis has produced a carefully and rigorously developed outcome space that is intended to provide as comprehensive a description as possible of the qualitatively significant variation in students' conceptions and experiences of the topic of the Mohr's Circle as taught in the current context. This has very significant implications for teaching that go beyond the somewhat ad hoc insights and implications mentioned earlier in the discussion. In effect, the categories of description that define the outcome space, along with their hierarchical structure and distinctive features, constitute a researched 'conceptual scaffolding' of the topic as taught, and provide an indication of the kind of 'learning pathways' that students may or should follow in order to master the topic. To facilitate an appreciation of these points and their significance, the more formal presentation of the outcome space presented in Table 2 is reworked into a graphical form in Fig. 2.

The conceptual scaffolding depicted in Fig. 2 has multiple utility. As a concept map it can serve as a useful overview of the topic for students in course outline documentation. It provides a reference that is useful for the articulation of learning outcomes and the design of assessments by highlighting the stages in conceptual development associated with mastery of the topic. It can inform a review of the course structure to gauge the extent to which it aligns with researched evidence about how the students being taught typically relate to the topic. This can inform efforts to redesign or modify the course or aspects of it. In this regard, the outcome space depicted in the figure provides both a conceptual progression and multiple perspectives on the topic. The conceptual progression in how the Mohr Circle is used suggests a generic 'learning pathway' from the more basic to the more sophisticated aspects of mastery of the topic. Significantly, this 'pathway' is derived from how students see and experience the topic rather than from textbooks or teacher intuition or their experience. The multiple perspectives in the outcome space consist of a general, 'big picture' perspective (i.e. what the Mohr Circle does); a perspective that focuses on how it is used; and a visual perspective. This can inform pedagogical redesign or modification of the

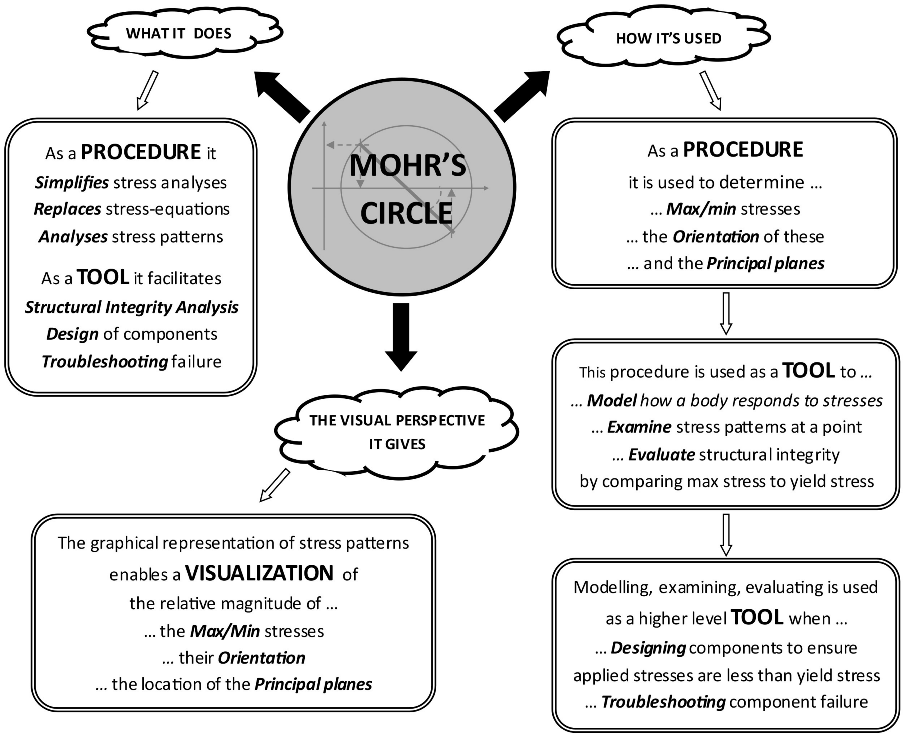

Fig. 2. A graphical presentation of the outcome space of the phenomenographic analysis.

course in a different way in that the different perspectives can be interwoven to enrich the teaching and learning of the topic. It is well known that such interweaving of multiple perspectives can generate a synergy that enhances student learning (see for example [25]).

There is a further and more sophisticated usage of the outcome space from a phenomenographic study that is worth noting. This derives from what Pang [26] refers to as 'the second face of phenomenography'. The outcome space from a phenomenographic study is essentially descriptive, but it can also be used to inform theoretical considerations about how teaching and lesson plans might be organized so that students experience critical aspects of the topic in different ways. The premise here is that "without such a pattern of experienced variation there can be no discernment, and without discernment there can be no learning" [27, pp. 102–103]. So, for example, Fig. 2 highlights three aspects that are critical for mastery of how the Mohr Circle is used: its use as a procedure, as a tool for stress analysis, and as a higher level tool for component design and failure analysis. Suppose the problems set in class and in tutorials present the students with variations only at the procedural level by, for example, only varying the numerical values and orientations of stresses and the shapes of the stressed bodies. The experienced variation of students would be at the procedural level and they should develop competency at that level. However, they would not be experiencing variation at the level of the Mohr Circle as a tool and so may not develop the desired competency at that level. In order to shift their focus in that direction, the set problems should, according to the outcome space, provide varying experiences of modelling how a body responds to stresses, of examining stress patterns at different points, and of examining if, how and where stresses exceed the yield stress. Just how the varied experience is engineered most effectively for student learning at this level would require creative thought and analysis and cycles of appropriate research and development. The outcome space from the phenomenographic analysis may not inform these further efforts directly beyond pointing out the progression from modelling to examining to evaluating that is associated with the usage of the Mohr Circle at this level. Research in the area of this 'second face' of phenomenography suggests that if effective experienced variation and the associated shifts in student attention can be engineered it can result in significant improvement in students' learning of difficult engineering topics (see, for example, [27, 28]).

6. Conclusion

The case presented in this paper has demonstrated how the difficulties students have in learning a difficult topic and teachers have in teaching it can be 'opened up' through appropriate educational research. It illustrates the use of a particular research methodology, phenomenography, what it involves, what kind of findings it can generate, and how useful these can be pedagogically. It shows how the methodology is able to open up a teacher's awareness to the conceptual variations that typically exist among the students they teach and how such awareness can be used to design and implement appropriate pedagogical interventions and refinements.

References

1. K. Valter and G. Akerlind, Introducing students to ways of thinking and acting like a researcher: A case study of research-led education in the sciences, *International Journal of Teaching and Learning in Higher Education*, **22**, 2010, pp. 89–97.
2. L. C. Woollacott and L. van Dyk, Case studies in research-led engineering education. I: analysing errors students made in a reactor design test, *International Journal of Engineering Education*, **33**(2A), 2017, pp. 662–671.
3. S. Karunamoorthy, A rule based method to construct the Mohr's Circle for plane stress, *ASME 2014 International Mechanical Engineering Congress and Exposition (IMECE2014)*, 2014 Montreal, Canada, 2012, pp. 1–6.
4. K. M. Quinlan, S. Male, J. Fill, Z. Jaffer, A. Stamboulis and C. Baillie, Understanding thresholds in first year engineering: digging beneath Mohr's Circle, *4th International Symposium for Engineering Education*, University of Sheffield, UK, 2012, pp. 1–8.
5. F. Marton and R. Saljo, On qualitative differences in learning: I. Outcome and process, *British Journal of Educational Psychology*, **46**, 1976, pp. 4–11.
6. F. Marton, Phenomenography—describing conceptions of the world around us, *Instructional Science*, **10,** 1981, pp. 177–200.
7. F. Marton and S. Booth, *Learning and Awareness*, Lawrence Erlbaum Associates, Mahwah, New Jersey, 1997.
8. S. K. Cibangu and M. Hepworth, The uses of phenomenology and phenomenography: A critical review, *Library & Information Science Research*, **38**, 2016, pp. 148–160.
9. M. Tight, Phenomenography: the development and application of an innovative research design in higher education research, *International Journal of Social Research Methodology*, **19**, 2016, pp. 319–338.
10. J. M. Case and G. Light, Emerging methodologies in engineering education research, *Journal of Engineering Education*, **100**, 2011, pp. 186–210.
11. N. C. Magpili and J. Vasuthanasub, Providing new research methodologies for engineering research: Qualitative studies and mixed methods research, *Annual International Conference of the American Society for Engineering Management 2012, ASEM 2012*, 2012, pp. 723–729.
12. E. Dringenberg, J. A. Mendoza-Garcia, M. Tafur, N. D. Fila and M. Hsu, Using phenomenography: Reflections on key considerations for making methodological decisions, *Conference Proceedings, 122nd ASEE Annual Conference and Exposition*, 2015, paper 11839.
13. A. T. Cummings, W. C. Oakes and C. B. Zoltowski, Phenomenography: A qualitative research method to inform and improve the traditional aerospace engineering discipline, *123rd ASEE Annual Conference and Exposition*, 2016, paper 14755.
14. J. Case and D. Marshall, Approaches to learning. *In:* M. Tight, K. H. Mok, J. Huisman and C. Morphew (eds.) *The Routledge International Handbook of Higher Education*, London and New York: Routledge Falmer, 2009.
15. M. Prosser and K. Trigwell, *Understanding learning and teaching: the experience in higher education*, Open University press, 1999.
16. N. J. Entwistle, *Teaching for understanding at university: deep approaches and distinctive ways of thinking*, Basingstoke, Palgrave MacMillan, 2009.
17. K. Trigwell, Promoting Effective Student Learning in Higher Education, *In:* P. Peterson, B. Baker, and B. McGaw, (eds.) *International Encyclopedia of Education,* Oxford: Elsevier, 2010.
18. L. C. Woollacott, *On the learning practices of first year chemical and metallurgical engineering students at Wits: A phenomenographic study,* Ph.D., University of the Witwatersrand, 2013.
19. P. Green, A rigorous journey into phenomenography: from a naturalistic enquirer standpoint. *In:* J. Bowden and G. Green, (eds.) *Doing Developmental Phenomenography,* RMIT University Press, Melbourne, 2005.
20. L. Cohen, L. Manion and K. Morrison, *Research Methods in Education*, RoutledgeFalmer, London, 2007.
21. Atlas.ti7, Version 7.5.9, © 1993–2016 by ATLAS.ti GmbH, Berlin, atlast1.com
22. G. S. Akerlind, Learning about phenomenography: interviewing, data analysis and the qualitative research paradigm. *In:* J. Bowden and G. Green, (eds.) *Doing Developmental Phenomenography,* RMIT University Press, Melbourne, 2005.
23. G. S. Akerlind, Variation and commonality in phenomenographic research methods. *Higher education research and development*, **24**, 2005, pp. 321–334.
24. B. Collier-Reed, A. Ingerman and A. Berglund, Reflections on trustworthiness in phenomenographic research: Recognising purpose, context and change in the process of research, *Education as change*, **13**, 2009, pp. 339–355.
25. R. M. Felder and L. K. Silverman, Learning and Teaching Styles in Engineering Education. *Engineering Education*, **78**, 1988, pp. 674–681.
26. M. F. Pang, Two faces of variation: On continuity in the phenomenographic movement, *Scandinavian Journal of Educational Research*, **47**, 2003, pp. 145–156.
27. D. Fraser, S. Allison, H. Coombes, J. M. Case and C. Linder, Using variation to enhance learning in engineering, *International Journal of Engineering Education*, **24**, 2006, pp. 102–108.
28. D. Fraser and C. Linder, Teaching in higher education through the use of variation: examples from distillation, physics and process dynamics, *European Journal of Engineering Education*, **34**, 2009, pp. 369–381.

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