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Laboratory Installation for Simulating Groundwater Flow in Saturated Porous Media in Steady-State and Transient Conditions*

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A laboratory installation designed for simulating groundwater flow in homogeneous and isotropic unconfined aquifers between two rivers or parallel channels is described in this paper. The laboratory installation allows to visualize and to study the behaviour of the phreatic surface, to determine the saturated hydraulic conductivity, and to compare experimental data with analytical and numerical solutions of partial differential equation describing the groundwater flow in steady-state and transient conditions. The installation is simple to use and has been proven to be robust and useful in improving scientific knowledge on modelling principles of groundwater flow in saturated porous media.

Keywords: groundwater modelling; porous media; laboratory installation; educational tool

1. Introduction

In 1856, after numerous laboratory tests investigating the movement of water through a sand sample, a French civil engineer named Henry Darcy concluded that the rate of flux per unit time (Q) is directly proportional to the cross-sectional area (A) of the sample, to the loss of hydraulic head (Δh) between two points of measurement and inversely proportional to the distance (l) between these two points:

$$Q=kA\frac{\Delta h}{l} \tag{1}$$

where k is the constant of proportionally called the saturated hydraulic conductivity with the dimensions of length/time, or velocity.

Eq. (1) is known as Darcy's law and it was combined later with the principles of continuity of mass and conservation of energy in partial differential equations to derive the main equations describing the groundwater flow in aquifers. There are numerous analytical and numerical solutions of the physical equations that simulate the water flow through porous media considering different hypothesis (e.g., confined or unconfined aquifers, steady-state or transient conditions, homogenous and isotropic or heterogeneous and anisotropic porous media) described in several textbooks [1–4].

Today, educational software packages are available which are specially designed for teaching and learning of fundamental concepts of groundwater flow [5–7]. But even with the advent of computers and sophisticated technological tools that has meant more emphasis in numerical modelling, Saidani and Shibani [8] argued that physical modelling is still a powerful aid tool to a better understanding in engineering education. As discussed by Crouch [9] and Behnejad [10], engineering students usually show greater interest in topics which are demonstrated physically rather than those that are explained only by passive lecture; and small scale models are an effective way to visualize and study fundamental concepts. Physical models can be also inexpensive compared to modern numerical techniques [11]. Recent learning approaches proposed to teach fluid mechanics in engineering foresee the adoption of various learning materials including laboratory work with hands-on experience [12, 13]. As previously stated by Wartman [14], physical modelling is a powerful simulation technique that can be applicable to a wide range of subjects. Laboratory instruction has played traditionally an important role in engineering education as it permits students to develop skills in experimentation, data interpretation and synthesis, communication, and teamwork. When strategically integrated into courses programs, interactive physical modelling can improve learning and motivate students [14]. As highlighted by Miller et al. [15], a conceptual understanding beforehand is necessary for a correct observation of the demonstrations and for a most effective promotion of students learning, as also as for successful hands-on experiences.

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The authors describe an experimental laboratory installation which allows the simulation of groundwater flow in homogeneous and isotropic unconfined aquifers between two rivers or parallel channels (Fig. 1). The laboratory test designed is aimed to improve students' knowledge in hydrogeology beyond visualization of theoretical concepts, discussion of the basic processes and determination of characteristic parameters in aquifers (e.g., saturated hydraulic conductivity) as in [16-17]. Specifically, the physical model presented here allows students to verify from results of hands-on experiences the validity of the equations derived for describing the groundwater flow in an equivalent porous medium. The laboratory installation was developed and built for educational purposes at the Laboratory of Hydraulics, Water Resources and Environment of the Department of Civil Engineering of the Faculty of Sciences and Technology of the University of Coimbra (Portugal) [18], and was used by more than 2000 students from undergraduate, graduate and PhD hydrology courses.

2. Governing equations

Consider a homogeneous and isotropic saturated porous medium, where incompressible water is bounded by a horizontal impermeable base layer and the water table is at atmospheric pressure. Assuming the hypothesis known as the Dupuit assumptions [1, 4] which state that the hydraulic gradient is equal to the slope of the water table, and for small watertable gradients the streamlines are horizontal and the equipotential lines are vertical, then the onedimensional equation (in dimension x) that describes the groundwater flow through time, t (i.e., in transient conditions: $\partial h/\partial t \neq 0$) can be written as follows:

$$k\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + R = S_y\frac{\partial h}{\partial t} \tag{2}$$

where k is the saturated hydraulic conductivity, h is the hydraulic head from the horizontal impermeable layer of the unconfined aquifer, R is the recharge (e.g., effective precipitation or deep infiltration), and S_y is the specific yield. Eq. (2) is known as the Boussinesq equation [19]. In steady-state conditions there is no change of head in time (i.e., $\partial h/\partial t = 0$), and Eq. (2) reduces to:

$$k\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + R = 0 \tag{3}$$

If recharge is not present, equations (2) and (3) simplify, respectively to:

$$k\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = S_y\frac{\partial h}{\partial t} \text{ (Transient conditions)} \tag{4}$$

$$k\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = 0 \text{ (Steady-state conditions)} \tag{5}$$

Equations (4) and (5) describe the groundwater flow in the laboratory installation presented in this paper in transient and steady-state conditions, respectively. Usually, the set of hypotheses required to obtain an analytical solution limit their application, almost exclusively, to steady-state conditions. The numerical solutions are more flexible as transient conditions considering variations in the properties of the medium, distribution of the recharge, and initial and boundary conditions in space and/or time can more easily be taken into account. One analytical solution of Eq. (5) (for steady-state condi-

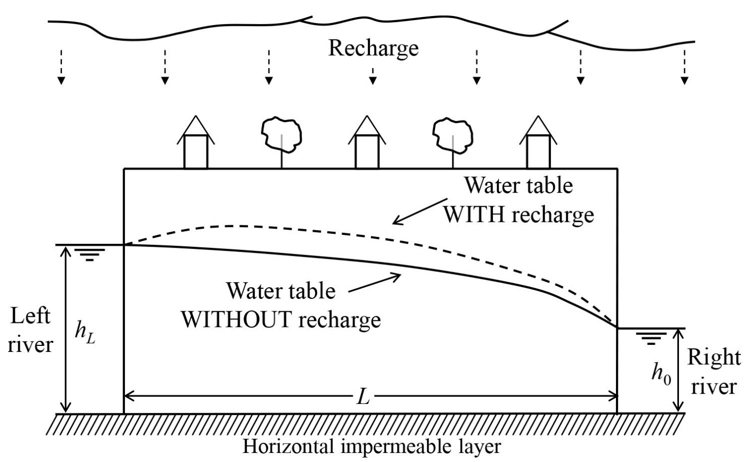


Fig. 1. Schematic representation of the water table in an unconfined aquifer between two rivers with and without recharge (h_L and h_0—hydraulic head from the horizontal impermeable layer in the left and right rivers, respectively; and L—distance between rivers or length of the porous medium).

tions), and two numerical solutions of Eq. (4) (for transient conditions) are detailed next.

2.1 Steady-state conditions

Equation (5) can be solved by direct integration to determine the groundwater flow per unit width, q, under steady-state conditions as follows [1]:

$$q = k\frac{\left(h_L^2 - h_0^2\right)}{2L} \tag{6}$$

where h_L and h_0 represent constant hydraulic head from the horizontal impermeable base layer of the aquifer in two parallel channels located a distance L apart (see Fig. 1).

2.2 Transient conditions

Eq. (4) is a nonlinear second order partial differential equation that can be linearized considering that the depth of the water table above the horizontal impermeable layer of the aquifer varies only slightly, and then one constant value for saturated thickness (D) can therefore be adopted. This assumption allows us to rewrite equation (4) and describe the groundwater flow in transient conditions as follows:

$$k\frac{\partial}{\partial x}\left(D\frac{\partial h}{\partial x}\right) = S_y\frac{\partial h}{\partial t} \quad \text{or} \quad kD\frac{\partial^2 h}{\partial x^2} = S_y\frac{\partial h}{\partial t} \tag{7}$$

and equation (7) can then be solved by implicit or explicit finite difference numerical methods as detailed next.

The classical Saul'Yev explicit numerical method is found to be unconditionally stable solving equation (7) as follows:

$$S_y\frac{h_i^{t+1} + h_i^t}{\Delta t} = kD\frac{h_{i+1}^t - h_i^t - h_i^{t+1} + h_{i-1}^{t+1}}{(\Delta x)^2} \tag{8}$$

If equation (8) is solved for h_i^{t+1}, it can be rewritten as follows:

$$h_i^{t+1} = \frac{cc \times h_{i+1}^t + (1 - cc) \times h_i^t + cc \times h_{i-1}^{t+1}}{(1 + cc)} \tag{9}$$

where $cc = \frac{kD}{S_y}\frac{\Delta t}{(\Delta x)^2}$.

Another approach to solve equation (7) is to apply the CrankNicolson implicit numerical method which is also unconditionally stable:

$$S_y\frac{h_i^{t+1} + h_i^t}{\Delta t} = kD\frac{1}{2}\frac{h_{i+1}^{t+1} - 2 \times h_i^{t+1} + h_{i-1}^{t+1} + h_{i+1}^t - 2 \times h_i^t + h_{i-1}^t}{(\Delta x)^2} \tag{10}$$

Equation (10) can be rewritten with cc as follows:

$$-cc \times h_{i-1}^{t+1} + 2 \times (1 + cc) \times h_i^{t+1} - cc \times h_{i+1}^{t+1} = cc \times h_{i-1}^t + 2 \times (1 - cc) \times h_i^t + cc \times h_{i+1}^t \tag{11}$$

where h_{i-1}^{t+1}, h_i^{t+1} and h_{i+1}^{t+1} are calculated simultaneously by solving a system of linear equations.

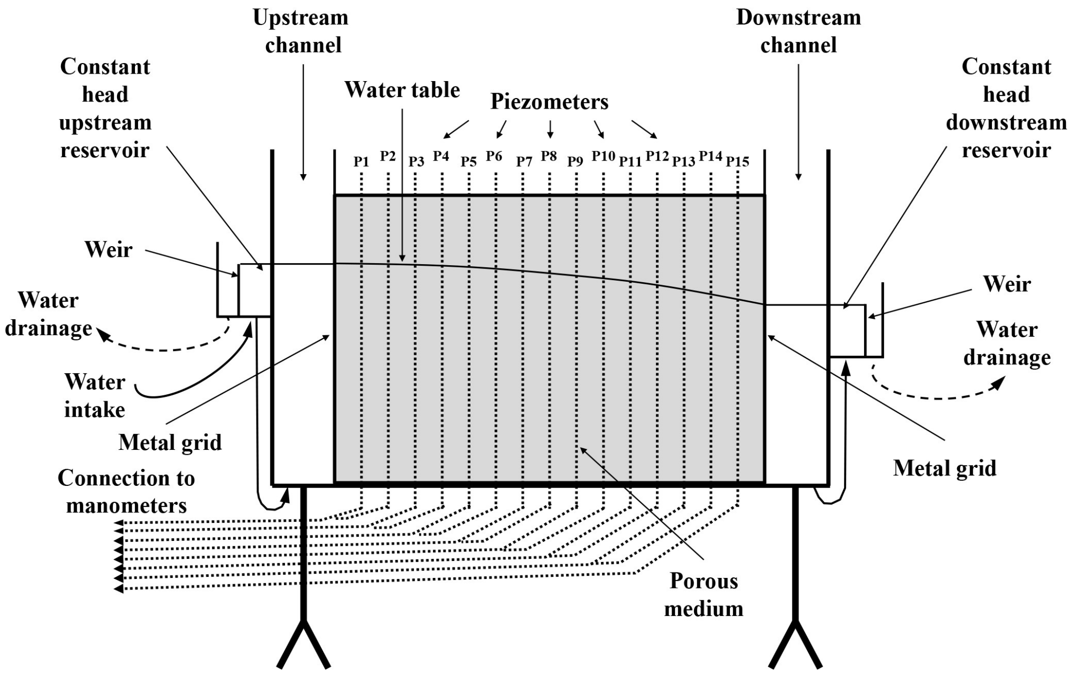


Fig. 2. Laboratory installation (see also photographs shown in Figs. 3 and 4).

3. Laboratory installation

The laboratory installation idealized to simulate the groundwater flow in a homogeneous and isotropic unconfined aquifer between two parallel channels is presented schematically in Fig. 2. The porous medium is in the central part of the laboratory installation (grey filled). At each end there is a channel separated from the porous medium by a metal grid and connected to a small constant head reservoir. The water levels in the upstream channel and downstream channel are kept constant by the weir placed on each constant head reservoir and the excess water is drained from the system. Piezometers regularly spaced in the porous medium and connected to a set of manometers facilitate following the variation of the water table between the two channels.

The laboratory installation is shown in Fig. 3. A metallic structure with dimensions 2 m × 0.3 m × 1 m (length × width × height) holds a set of glass walls (12 mm thick) that allow visualizing the porous medium, piezometers, and water levels in channels. The porous medium is contained by a stainless metal grid in the inner central zone of the metallic structure (the length of the porous medium between the metal grids is 1.5 m). The central zone was filled with a mixture of sand and clay up to a height of approximately 0.65 m. Fifteen piezometers regularly spaced (at approximately 95 mm apart) were installed in the inner central zone and linked to manometers (Fig. 4A). Fig. 4B details the linkage of the piezometers beneath the metal structure to plastic tubes connected to the manometers. Fig. 4C is a close-up view of the reading of the water levels in the manometers. Fig. 4D shows how the weir keeps the water level constant in the upstream reservoir and the hydraulic equilibrium with the upstream channel. Plastic balls were inserted in the constant head upstream reservoir to minimize turbulence from water intake (also in Fig. 4D).

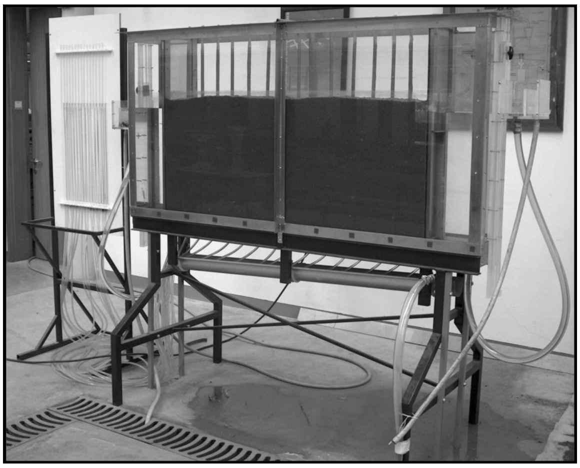
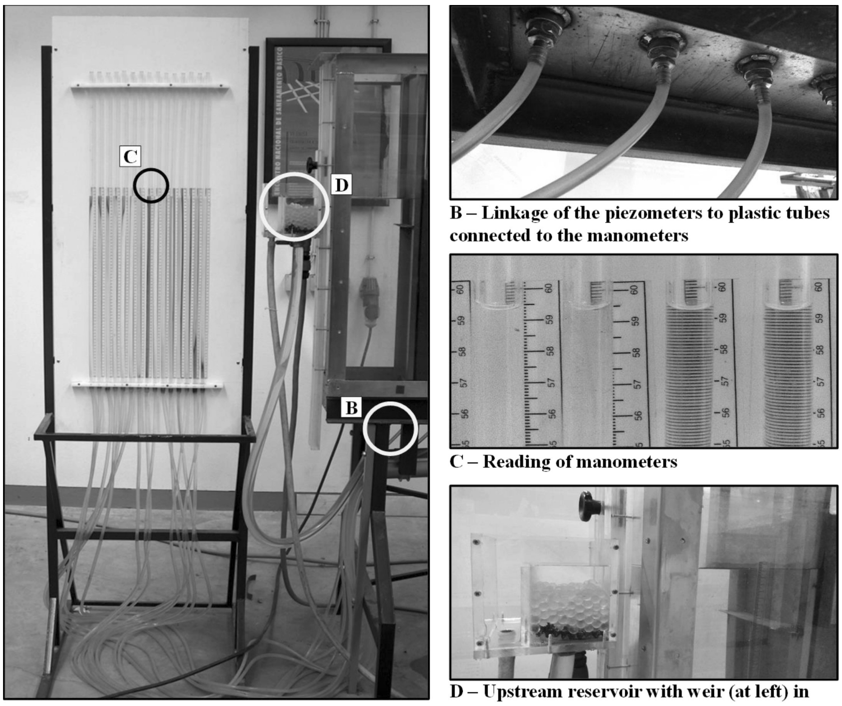


Fig. 3. Photograph of the laboratory installation.



B – Linkage of the piezometers to plastic tubes connected to the manometers

C – Reading of manometers

D – Upstream reservoir with weir (at left) in hydraulic equilibrium with channel (at right)

A – Manometers and stand

Fig. 4. Photographs of (A) the manometers and stand for registration of the piezometric levels in time, and (B, C and D) details of the laboratory installation.

4. Laboratory test

4.1 Materials

The material required to execute the experimental runs on the laboratory installation (Fig. 3) include manometers and stand (Fig. 4), measuring tape, 1 L container, electronic weighing scale for discharge estimation by the volumetric method (accuracy: 0.1%—if the readout of the electronic scale is 0.1 g, the water volume collected should be above 100 mL), and paper and pen/pencil to register manually the readings from the manometers, or a video recorder.

4.2 Preparation of the experiment

The preparation of an experimental run involves the saturation of the porous medium, and the establishment of a horizontal water level in the laboratory installation as follows:

1. Set the two weirs of the constant head reservoirs at the same height. The porous medium should not be entirely submerged.
2. Open the valve of the water intake to the laboratory installation to saturate the porous medium up to the weir's level. The soil is considered to be fully saturated when the water level read in the manometers is the same in all piezometers. In this step, it may be required to flush the air bubbles from the tubes that connect the piezometers to the manometers.
3. Register the dimensions (height $\times$ length $\times$ width) of the porous medium, the distance between the piezometers, and the hydraulic head in the laboratory installation.

4.3 Experimental run

During an experimental run, enough water must be supplied to the constant head upstream reservoir of the laboratory installation to guarantee a constant head. The measurements for the determination of the saturated hydraulic conductivity and the comparison of the experimental results with analytical and numerical solutions of the equation describing the groundwater flow in saturated porous media should be made as follows:

1. Quickly drop the water level in the downstream reservoir to create a difference in the hydraulic head between the two channels, and record the new level in the downstream channel. It is also possible to raise the upstream reservoir.
2. Record the evolution of the water levels in all piezometers through reading of the manometers. The time interval of the readings has to be adapted to the characteristics of the porous medium of the laboratory installation. Normally, the first five readings should be made at 30 seconds, 1, 2, 4 and 6 minutes after the start of the experimental run, and the following readings are performed at 3 minute intervals thereafter.
3. Make three measurements of the discharge by the volumetric method using the 1L container when the steady-state condition is established. This state can be assumed when the differences between 3 consecutive measurements are smaller than 2%.
4. Close the water supply to the laboratory installation.

4.4 Report questions

The students should report on the following three questions:

1. Estimation of the saturated hydraulic conductivity of the porous medium, k, with Eq. (6), using the difference of hydraulic head (Δh) between the two channels created during the experimental run and the average of the three measurements of the discharge by the volumetric method to determine the groundwater flow per unit width, q ($q = \bar{Q}/B$, where $\bar{Q}$ is the average of the 3 measurements of discharge and B is the width of the porous medium);
2. Show in graph form the evolution of the groundwater level between the two channels, in time, and comment on it;
3. Comparison of the observed evolution of the groundwater level with numerical simulation in transient conditions using the saturated hydraulic conductivity, k, estimated in step 2; an average saturated thickness of the porous medium, $D \approx (h_L + h_0)/2$; and a characteristic value of specific yield, S_y, for the geologic material of the porous medium, with application of the classical Saul'Yev explicit method, Eq. (9), and the CrankNicolson implicit finite difference numerical method, Eq. (11).

Questions 1 and 2 are suitable for BSc level using a homogenous porous medium, and question 3 is for MSc level.

4.5 Example

In a laboratory experiment, the porous medium was saturated to a height of 0.584 m following the steps described in Section 4.2. The experimental run (Section 4.3) began by dropping the water level in the downstream reservoir to the height of 0.369 m. Fig. 5 shows the evolution of the water levels in all piezometers through the experiment. This figure shows an increase of the gradient of the water table on the direction of the downstream reservoir.

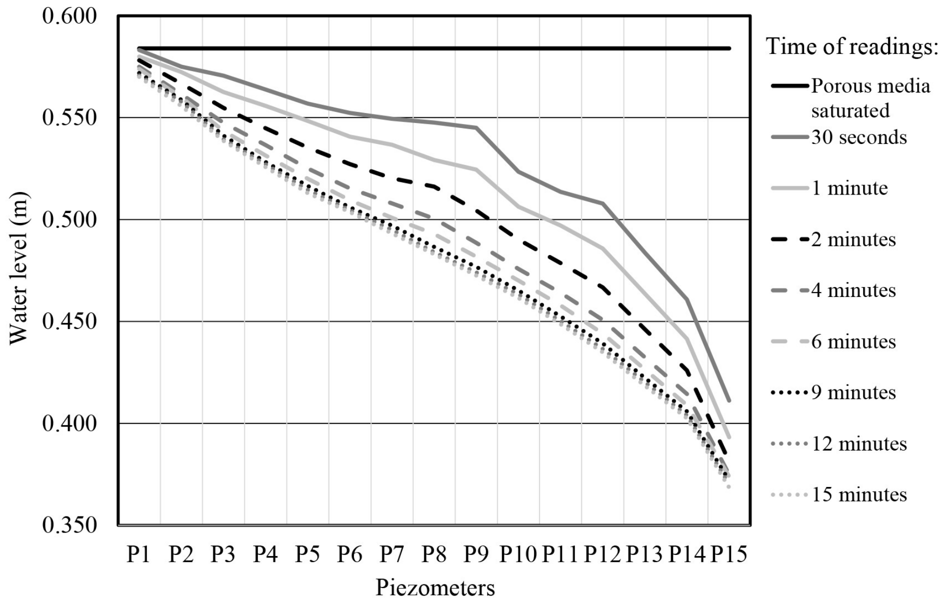


Fig. 5. Evolution of the water levels in all piezometers registered by two students in a laboratory experiment. Readings for saturated porous medium (horizontal line) and after 30 seconds, 1, 2, 4, 6, 9, 12 and 15 minutes from the beginning of the laboratory test.

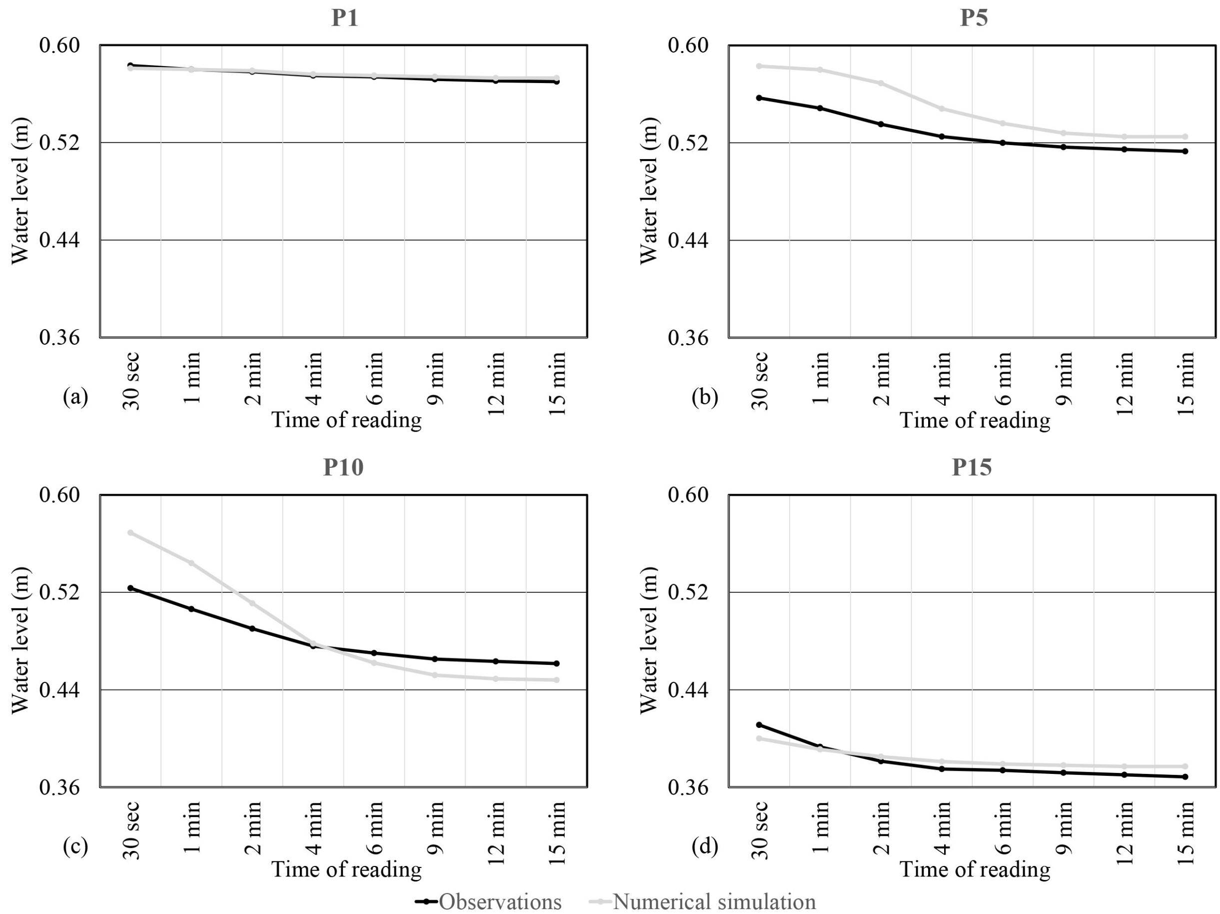


Fig. 6. Comparison between observations and numerical simulation with the Saul'Yey explicit method about the evolution of the water levels in piezometers (a) P1, (b) P5, (c) P10, and (d) P15.

From the Darcy's law, as the crosssectional area is smaller on the direction of the direction of the downstream reservoir, the hydraulic gradient must be greater so that the quantity of water flowing though the porous medium is nearly constant [4].

The discharge by the volumetric method was determined after the last reading of the water levels. The average of the total discharge from the three measurements ($\bar{Q}$) was 1.42×10^{-5} m^3/s. Then, by applying Eq. (6) (with $B = 0.3$ m and $L = 1.5$ m – see Section 3), the saturated hydraulic conductivity (k) was estimated in 6.93×10^{-4} m/s (typical range of well sorted sands in [4]: 10^{-5}–10^{-3} m/s).

Fig. 6 shows the comparison of the observed water levels in four piezometers (P1, P5, P10 and P15) with results from the numerical simulation by the Saul'Yey explicit method (Eq. 9), with $D = (0.584 + 0.396)/2$ m (see Section 4.4), $\Delta t = 1$ s, $\Delta x = 0.05$ m, and $S_y = 0.25$ (average value for sands in [4]). In general, Fig. 6 shows a good agreement between the observations and the numerical simulation in all piezometers through time. The differences between observations and the numerical simulation are related with some heterogeneity of the porous medium, and reading errors (accuracy and timing of the records). On the contrary, the differences cannot be justified with uncertainty in the value of S_y as minimal differences were verified with 0.20 and 0.35 (minimum and maximum values for sands in [4]).

4.6 Assessment of laboratory installation

The laboratory installation has been used in many one-semester courses on hydrology to improve students' knowledge in hydrogeology. A conceptual understanding beforehand has proven to increase engagement of students and to contribute to an interactive laboratorial activity (see Introduction). Therefore, the hands-on experience is typically introduced in the second half of the semester after lectures with exposition of theoretical concepts and principles on groundwater flow and resolution of simple practical exercises. Notwithstanding, during the laboratory test some undergraduate students are corrected about some common misunderstandings, such as considering in the Darcy equation the longitudinal section instead of the cross-section, incorrect reading of the water levels or making confusion between the total discharge and the discharge per unit length. The knowledge acquired by the students with the laboratorial activities is assessed both through a written report prepared in group and oral examination. Additionally, in final exams, some students refer explicitly the experience gained in the laboratory when answering to questions dealing with groundwater flow and subsurface drainage. Also, in annual institutional questionaries' for the evaluation of courses, often students stand out the importance of the hands-on experience on their learning process, and the course punctuation has increased in approximately 10%.

As future developments, the complexity of the laboratory activity can be increased by including the simulation of recharge by rainfall to test and verify analytical and numerical solutions that describe the groundwater flow in such conditions. Furthermore, it would be interesting to visualize and explain the behaviour of the phreatic surface with heterogeneous or stratified porous media, or non-horizontal impermeable base layer.

5. Conclusions

A laboratory installation was built for demonstration and studying the groundwater flow in saturated porous media. The installation available at the University of Coimbra (Portugal) has been used uninterruptedly since the academic year 2004/2005 (i.e., for 13 years in 2017/2018) by more than 2000 undergraduate, graduate and PhD hydrology students. The installation is simple to use and has been proven to be robust and useful in improving students' knowledge on modelling principles of groundwater flow in saturated porous media.

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