A Model Employing the Overlapping Distribution Method to Predict the Success of Engineering Students in Supplementary Examinations*

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A model is proposed based on the statistical concept of the overlapping distribution method (ODM). The model is then applied to estimate the number of students that can be expected to qualify for and to pass supplementary examinations. The ODM prediction model was validated using 19 sets of examination events involving 858 civil engineering students in two different universities. It was found that lowering the mark cut-off criteria from 45 to 40%, increases the number of students that qualify for supplementary examinations, by up to 75%. In turn, the number of students that successfully sit for supplementary examinations, increases by about 40%. The proposed model promises to be useful in informing policy on conflicting issues that arise from balancing the work load under large class sizes on one hand, and offering wider opportunity to as many students as have the potential to pass.

Keywords: overlapping distribution method; supplementary examinations policy; engineering assessments; throughput

1. Introduction

In an earlier article [1], a statistical technique referred to as the overlapping distribution method (ODM) was discussed as a potential concept for estimating the number of students that have the potential to pass supplementary examinations. The present study further develops the concept into a model then employs it to examine matters that relate to policy and promotion of throughput.

Further to various researches on academic performance of students [1, 2], advanced learning techniques are also being explored. These techniques include computerized simulation and tracking of cognitive abilities [3–5], Halpern critical thinking analysis [6], use of machine learning to design tests and examinations [7], prediction of course selection by students [8]. Also, attempts have been made to determine indicators or predictors of students' performance [9–12]. In [13], it is shown that a student's performance in formative assessment is strongly indicative of his/her summative assessment results. Personality is reported to be related to beliefs about intelligence. A study by [9] found that personality traits (and not cognitive abilities) gave the most significant prediction of academic performance in essay-type questions and summative examinations. They concluded that conscientious and introverted students were likely to perform better at universities.

In this paper, a model is developed based on the ODM concept. The model is then validated as a statistical technique that can be meaningfully used to predict the number of students that are likely to qualify for and pass supplementary examinations, having failed their original summative assessment. Actual 10 year assessment data of civil engineering modules, was used in the validation. Among South African universities, the admission criteria for supplementary examinations varies from a threshold cut-off mark of 35 to 45%, depending on the institution's policy. In most cases, there is no clear scientific basis justifying the adoption of a particular cut-off mark over the other. In the present study, the ODM model was applied to examine this policy criteria.

2. Reliability and the ODM concept

2.1 The overlapping distribution method

In most natural phenomena, variability is characterised by normal distribution. This is also true of class marks of students (Fig. 3). By definition, failure occurs when loading (S) exceeds resistance (R). Since both quantities R and S are random variables, the difference between their normal distributions forms an overlapping area which itself is a normal distribution, as shown in Fig. 1. The overlap area (A) gives the probability of failure, $P(f)$ expressed in Equation (1) [14].

$$P(f) = P(R - S < 0) \qquad (1)$$

Practically, the overlap area represents a proportion that would experience failure during a given event. For a normal distribution, the overlap area consists of A/2 = A(S) = A(R) = the probability of failure, P_f [9]. In applying the ODM, *S-distribution*
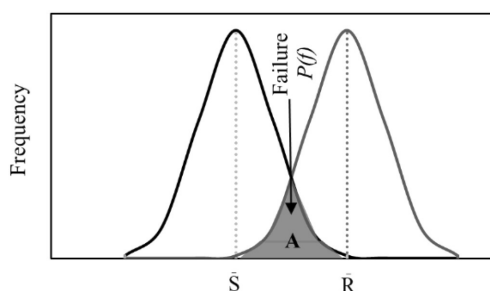
Fig. 1. Overlapping normal distribution curves.

represents the group of students that fail a summative assessment while *R-distribution* represents the group of students that pass it. The area *A(S)*, gives the percentage of students that fail summative examinations but could pass a supplementary examination, given the opportunity to write it.

The scoring of scripts is an end stage of a test or an examination event. Typically, a minimum mark of 50% is required for a pass to be awarded. During scoring, marks from the class will exhibit a normal distribution with some marks falling at the borderline of a pass and a fail, such as marks between say, 40 and 49%. The practice at universities is to give supplementary examinations to students of this category (S-group). Failure of a student to pass the second examination opportunity is interpreted as an indication of cognitive incapability.

Table 1 gives the threshold cut-off mark criteria for admission to supplementary examinations at some universities in South Africa. It can be seen that the requirements vary from 35 to 45% depending on the institution. In most cases, the rationale used to decide on the threshold cut-off mark does not seem to be clear but appears to involve factors such as large class sizes, work load of academic instructors, ability of the students to pass given a second chance etc. This paper attempts to contribute towards a scientific and mathematical basis for deciding this threshold criteria.

2.2 Conceptualization of ODM for supplementary examinations

After conducting script marking of summative assessments, two groups of students typically emerge, namely the "Fail" group (F-) and the "Pass" group (P-). Due to errors which inevitably occur in any marking of scripts and in the conduct of examinations, students that obtain the borderline pass /fail mark, may receive an incorrect mark. So,

some students in the F-group should ideally belong to the P-group and vice versa. As such, students that fail marginally with a mark below 50% are usually given a second opportunity to pass, by way of writing supplementary examinations. These students form the "Supplementary" group (S-). The relationship between the P-group, F-group and S-group can be represented statistically as overlapping normal distributions [1]. However, the size of the S-group depends on the threshold cut-off mark, which may be as low as 35%.

2.3 Theoretical success rates in supplementary examinations

During the process of developing the proposed model, the 50% mark was set as a fixed point against which the cut-off threshold of the F-group was adjusted at sliding mark levels of 45, 40, 35%, to determine the S-group. The probability that students admitted to the S-group at each of these thresholds, would pass supplementary examinations, was then calculated [1].

Table 2 summarizes the theoretical probabilities and likely pass rates for supplementary examinations. It can be seen that students who score very low marks during summative assessment, particularly $\leq$ 30% are unlikely to pass a second examination opportunity, i.e. if they were given to write a supplementary assessment. In fact, it takes twenty (20) students (of S-group) with a mark of 30% in summative assessment, for only one of them to pass a supplementary examination. It then becomes relevant to ask whether at all it is worth allowing students of that category to take supplementary examinations. From results in Table 2, it becomes apparent that students with a great possibility of passing, when given a second assessment opportunity, are those that obtain a mark $\geq$ 40% in summative assessment. It may also be noted that if a threshold cut-off mark of 45% is used, it would deny an opportunity to students that obtain a mark of 40 to 44%, of whom 30% would pass a supplementary assessment.

Fig. 2 gives the theoretical prediction of expected distribution of students that fail summative examinations. It can be seen that no cognitively capable student is expected to obtain marks below 25% in summative examinations, under normal circumstances. Should it happen, it may be considered that special circumstances may have played a role in that student's academic performance. Similarly,

Table 1. Eligibility criteria for supplementary assessment in some South African universities

Institution	J	W	P	Kz	Un	Rh	Ct	Uf
Minimum summative exam mark (%)	45	40	45	40	40	35	45	40

Table 2. Prediction of pass rates for supplementary examinations

Threshold cut-off mark (%)	Probability (%)	S-students that are likely to pass	Is it worth giving supplementary exam opportunity?
30	4.6	0.5/10	Not
35	13.2	1/10	Not, perhaps!
40	31.8	3/10	Yes
45	61.8	6/10	Yes

students that obtain an examination mark between 30 to 35% should be relatively a small number. The largest number of students that fail summative assessment fall in the range of $\pm$ 40% mark, followed closely by those with a mark of $\pm$ 45%. This observation justifies the argument that using 45% mark as a cut-off threshold criteria for students qualifying to do supplementary examinations, would be inappropriate as it excludes a significant number of students in the 40 to 44% range, that also have the potential to pass supplementary examinations. A discussion whether to give supplementary examination to students that obtained 30 to 35% in summative examination, should depend on the number of F-group of students that fall into this category. It was shown in the foregoing that this category of students has a very low likelihood of passing supplementary assessments, which is a policy issue to be considered alongside implications on the work load of academic instructors.

3. Validation of the concept

3.1 Assessment data

Validation of the ODM concept described in the foregone, was done using real data sets of examination results taken from South African universities. The data used were drawn from four modules

taught over a period of 10 years by the same instructor.

Altogether, a total of 19 data sets involving 858 students were used in the study. Class sizes of the modules varied from 17 to 86 students, with an average of 47 students. Accordingly, the class sizes were of small to medium size categories. The pass rate in summative examinations ranged from 81 to 100%, except one module that had a pass rate of 56.5% in one academic year. Fig. 3 shows the general statistical characteristics of the students' examination results, for some of the modules. The other examination events, whose histograms are not presented in Fig. 3, invariably gave similar characteristics [13]. It can be seen that the examination results exhibit the normal distribution characteristics, without exceptions. The overall average mark for each examination event, was generally between 50 and 60%.

The number of students that sit for supplementary examinations is always lower than those that failed the summative assessment. This is partly attributed to policy situations in which some students are not allowed to sit supplementary examinations for having failed a number of modules during summative assessment. In other cases, students may terminate studies or fail to present themselves for examinations, due to exceptional circumstances such as sickness etc. It can be seen in Fig. 4 that the proportion of students that failed summative examinations and also presented themselves for supplementary assessment opportunity, was between 60 and 100%.

3.2 Prediction of students that qualify for supplementary examinations

The ODM concept described in Section 2.0, was applied to compare the actual versus predicted
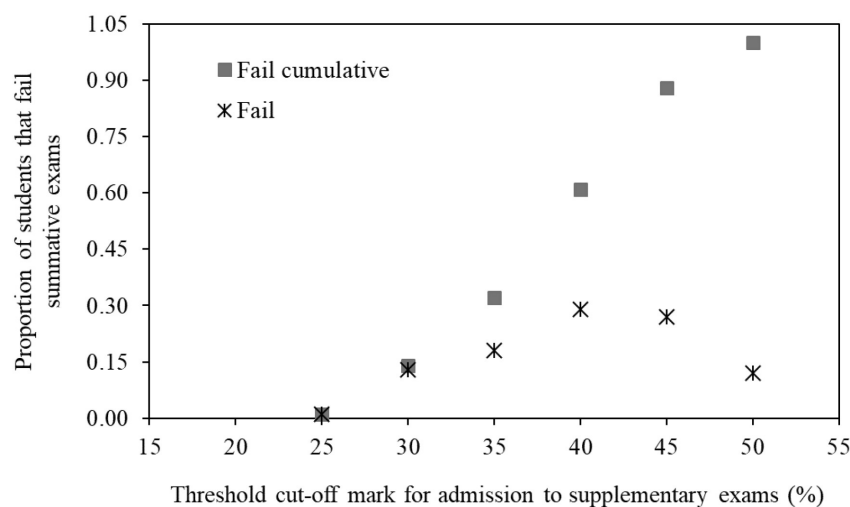
Fig. 2. Distribution of students failing summative examinations.
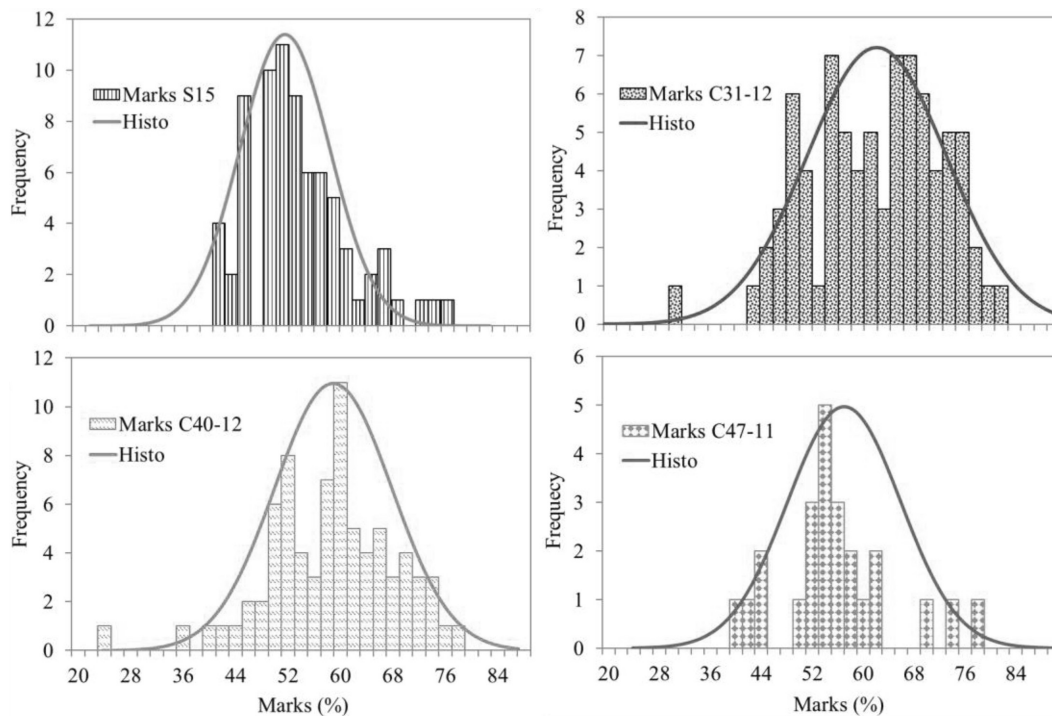
Fig. 3. Normal distributions exhibited in examination events of the various modules.
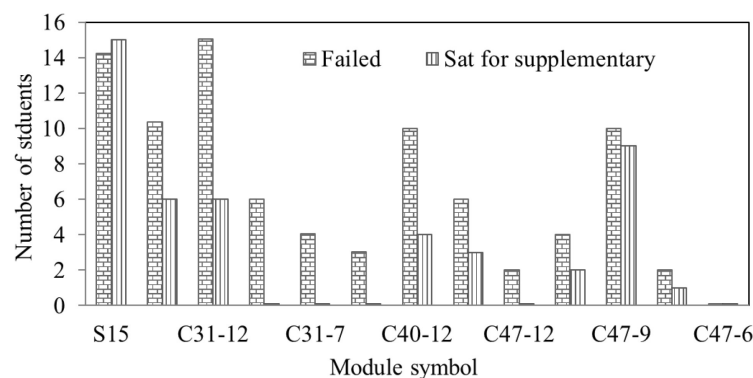

Fig. 4. Comparison of students that failed summative assessment with those that presented themselves for supplementary examinations.

number of students that presented themselves for supplementary examinations, as given in Fig. 5. In the analysis, actual pass rates determined from data were used in the prediction calculations. The predictions shown in Fig. 5 have been done for the 45 and 40% cut-off thresholds. It can be seen that when the 40% mark cut-off criteria was used, the predicted number of students qualifying for supplementary examinations is slightly higher than actual, while the prediction tends to be lower when 45% mark cut-off criteria is used. The ODM model does not account for the proportion of students that failed summative examinations yet never undertook supplementary assessment, as discussed in Section 3.1. This discrepancy led to over-prediction for the 40% cut-off threshold.

It is also evident in Fig. 5 that as the number of students who sat for supplementary examinations increased, the prediction became less accurate. The tendency of these data to "fan" out is evident in the plot of residuals given in Fig. 6. The cause of the observed heteroscedasticity is not clear. However, these tendencies typically occur with model predictions involving various natural phenomena, as reported in the literatures [15–18].

In practice, the actual examination pass rate for a future examination would be unknown. In such cases, the typical pass rate may be estimated from historical data as an average value. In this study, the influence of using an average historical value of pass rate in the model, was investigated. Accordingly, an average pass rate of 83% obtained from the data was used in the model to predict the likely number of students that would qualify and write supplemen-
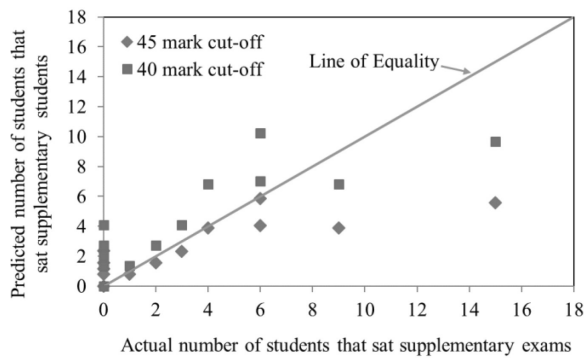
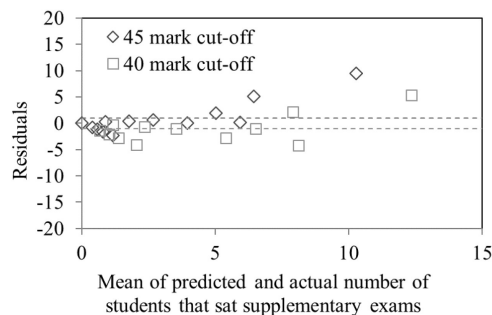**Fig. 5.** Predicted versus actual number of students that sat supplementary exams.

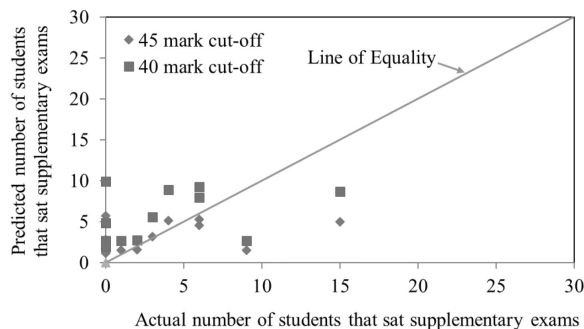**Fig. 6.** Residuals for predictions at 45 and 40% mark cut-off.

Fig. 7. Prediction of the number of students for supplementary examinations based on average exam pass rate.

tary examinations. The prediction results based on average pass rates, are given in Fig. 7. It is interesting to note that similar or better predictions were obtained by using an average pass rate, compared to using the individual module pass rates (Fig. 5).

3.3 Hypothetical analysis of success rates in supplementary exams, work load and policy

Application of the ODM model is herein demonstrated through hypothetical analysis on the implications of the cut-off mark criteria, class size, work load and the proactive policy to promote throughput. To examine these issues, a hypothetical case study was conducted for class sizes varied from 20 to 700 students, using two pass rates of 50% and 75%

which are deemed to be representative of poor and very good academic performance in typical engineering classes, respectively. For each class size and pass rate, calculations were done to determine the number of students that would be likely to (i) fail a summative examination, (ii) qualify for supplementary examination based on 45 and 40% mark cut-off criteria, and (iii) pass supplementary examination under the 45 and 40% mark cut-off criteria. In each case, results were analysed with respect to the policy on the threshold cut-off criteria, and its association with class size and work load.

Table 3 gives results of the study. It can be seen that for a class of 80 students with a pass rate of 75%, 20 students may be expected to fail their summative assessment of which eight (8) students qualify for supplementary examination under the 45% mark cut-off. The number qualifying for supplementary examination increases to 14, if a 40% mark cut-off is applied, i.e. six (6) more students would receive a second opportunity to pass. After writing the supplementary examination, five (5) and seven (7) students would pass under the 45 and 40% mark cut-off criteria, respectively. These results imply that by changing the eligibility criteria from 45 to 40% mark cut-off, the number of students passing supplementary examination increases by 40%. Similarly, for a class of 300 and 50% pass rate, 150 students would be expected to fail summative assessment. Of these, 59 and 102 students would qualify for supplementary examination under the respective eligibility criteria of 45 and 40% mark cut-off. Thus by lowering the eligibility cut-off mark from 45 to 40%, 44 more students would get a second opportunity to progress by writing supplementary examination. Of the students that would write supplementary examination, 36 would pass under the 45% mark cut-off while 50 students would pass under the 40% mark cut-off. Again by lowering the eligibility cut-off mark from 45% to 40%, the number of students that fail summative assessment but pass their supplementary examination increases by 39%. This ODM analysis assumes that 100% of the students who qualify for supplementary assessment would present themselves to undertake the examination, which is not true as discussed in Section 3.0. However, the validity of the analysis is not affected by the assumption.

From the foregone hypothetical analysis conducted, it is evident that lowering the eligibility criteria for supplementary examination from 45 to 40% cut-off mark, has a significant impact of increasing the number of students that would pass supplementary assessment towards their program completion or progress to the next level of study. For small classes of under 50 students, the number of students writing supplementary examination (S-

Table 3. Hypothetical case study of success rates in supplementary examinations for 75% and 50% pass rates under summative assessments

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Pass rate in summative exam (%)	Class size	Students failing (No.)	Students qualifying for *suppl. exam under eligibility of 45 cut-off (No.)	Students qualifying for suppl. exam under eligibility of 40 cut-off (No.)	Students passing suppl. under eligibility of 45 cut-off (No.)	Students passing suppl. under eligibility of 40 cut-off (No.)
75	20	5	2	3		
	50	13	5	9	3	4
	80	20	8	14	5	7
	100	25	10	17	6	8
	150	38	15	26	9	13
	200	50	20	34	12	17
	300	75	29	51	18	25
	500	125	49	85	30	42
	700	175	68	119	42	59
50	20	10	4	7		
	50	25	10	17	6	8
	80	40	16	27	10	13
	100	50	20	34	12	17
	150	75	29	51	18	25
	200	100	39	68	24	33
	300	150	59	102	36	50
	500	250	98	170	60	84
	700	350	137	238	85	117

*suppl.—supplementary examinations.

group of students) would typically be low and not of much concern. For medium and large class sizes, however, the S-group of students can become quite sizeable depending on the pass rate and cut-off mark. For example, in a class size of 100, the S-group for 40% mark cut-off criteria can range from 17 students at a high pass rate of 75% to 34 students at 50% summative examination pass rate. These numbers become quite excessive for larger class sizes. For a class size of 300 students and 50% examination pass rate, a 45% mark cut-off gives 59 students of S-group while a 40% mark cut-off gives 102 students of S-group. This high number of students in the S-group for large class sizes, would exert substantial additional work load on the academic instructor. As a result, instructors of large classes may prefer a more stringent mark cut-off criteria such as 45%, so as to control the number of students that would qualify for supplementary examination. Unfortunately, such stringency tends to conflict with the broader policy of promoting education access opportunities and of improving throughput. Accordingly, these factors actively playout within the academic system and should be considered crucial when determining policies on supplementary assessments. The complexity of this issue and robust debates that surround it, in addition to lack of a scientific basis, may explain the observed use of different mark cut-off criteria implemented by various universities, as earlier discussed (Table 1).

3.4 ODM model

Equation (2) gives the model derived in the foregone sections. To apply the model, it is necessary to first establish the values of three parameters comprising the class size (C) of the module, its typical pass rate for summative assessment (Pr), and the threshold cut-off policy for admission to write supplementary examination. The number of students that would pass supplementary examination (N_{ps}), is given by Equation (2).

$$N_{ps} = k.P_f(1 - Pr)C \qquad (2)$$

where k and P_f are the coefficients given in Table 4.

4. Conclusions

A prediction model is proposed based on the statistical concept of the overlapping distribution method (ODM). For a given module, the ODM model can be applied to predict the number of students that are likely to qualify for and pass supplementary examination. Investigations were conducted on the implications of using the eligibility criteria of 40% and 45% mark cut-off thresholds.

Table 4. Coefficients for the ODM model

Threshold cut-off mark (%)	45	40	35	30
Attendance coefficient, k	0.39	0.68	0.86	0.99
Probability factor, P_f	0.618	0.495	0.415	0.365

The effects of pass rates and class size on work load and policy issues, were also studied.

A reasonably good correlation was obtained between the model's predictions and the actual number of students that qualified for supplementary examinations. Lowering the eligibility criteria for supplementary examinations from 45 to 40% mark cut-off, has a significant impact of increasing the number of students that qualify for a second opportunity, by up to 75%. In turn, the corresponding number of students that eventually pass supplementary examinations increases significantly by about 40%.

The effects of class size, summative assessment pass rate, and eligibility criteria for admission to supplementary examinations, pose conflicts between balancing the work load of instructors for large class sizes and offering a wider opportunity to as many students as have the potential to pass supplementary examinations. The proposed prediction model promises to generate meaningful results that may inform such debates and policy considerations.

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