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Impact of Prior Knowledge, Learning Style, and Problem Nature on Students Performance in a Flipped Engineering Mathematics Class*

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Although many studies have demonstrated the effectiveness of flipped learning in terms of performance enhancements, there is a lack of research investigating the factors that can affect students' performance in introductory engineering mathematics courses using flipped learning. This study investigated how different factors, including prior knowledge, learning styles, and types of problems, can affect the flipped classroom students' performance in engineering mathematics. Before and after participating in flipped learning covering the concept of ordinary differential equations, 139 engineering students' testing and survey data were collected. The results showed that, first, two learning styles including converging and assimilating played a major role in problem-solving, and significantly predicted their final test score. Second, when engaging in real-life and non-routine problems individually or in collaboration with group members, students' scores in the post-test were increased. This study concluded that instructors could enhance students' performance in engineering mathematics by integrating flipped learning with their current curriculum, helping students apply the abstract concepts of mathematics to authentic situations, and considering students' learning styles as a factor in successful flipped learning.

Keywords: flipped learning; prior knowledge; learning style; problem types; engineering mathematics

1. Introduction

In modern society, it is necessary to develop resources that bring about outstanding scientific and technological abilities in order to secure competitiveness and innovation and ensure the country's development and overall prosperity [1]. In the past, a country's national competitiveness was determined by the quantitative and qualitative superiority of production factors such as natural resources, land, and capital. However, in the 21st century, skilled personnel in the STEM fields is becoming increasingly important [1]. Therefore, the quality of engineering personnel afforded by higher education is vital and impacts the future of the nation [2]. All over the world, closer attention has been paid to the importance of providing engineering education that can cultivate students' in-depth understanding, along with creativity, logical thinking, problem-solving skills, and integrated thinking skills [3]. Nevertheless, for nearly a century, accredited engineering programs have been faced with high attrition rates due to students' withdrawing from core courses during their first year. One of those courses is "Engineering Mathematics" (e.g., Calculus, Differential Equations, and Algebra).

The main focus of many of today's mathematics classes is transferring the professor's knowledge to the learners in a manner in which the students receive the lesson's contents passively. Making matters worse, teachers compound the problem when asking students with diverse learning experiences and varying learning styles to solve the same standardized problems. As a result of this flawed learning methodology for teaching mathematics – a core course in engineering – an increasing number of engineering students are dropping out of their majors, leaving engineering courses behind. What could have been a chance for these students to learn key concepts in their field, and potentially procced in engineering-related careers, becomes a reality filled with students leaving engineering colleges, accumulating learning deficits, and undermining the number of qualified engineering personnel and resources in the field.

Given the limitations brought about by traditional classes, flipped learning is emerging as an

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effective instructional model that can overcome these challenges [4]. Flipped learning is a type of blended learning that combines online and offline classes [4] in a way that reverses – or flips – the processes and activities involved in traditional lecture-style classes. Students are asked to go over the teacher's instructional content (usually via short videos and lecture slides) before class. Then, during the class, they participate in learner-centered interactions, including problem-solving or discussion depending on the different types of tasks [5]. Classroom time is used for Q&A, discussion, and practice, in whole-class format or small groups.

Many studies have demonstrated the effectiveness of flipped learning in terms of performance enhancements [6–8]. Nevertheless, there is a lack of research conducted in the introductory mathematics courses in engineering considering students' prior knowledge, different learning styles, and the different kinds of problems, which have a tremendous impact on students' learning. Due to this gap, instructors and researchers utilizing flipped learning in their engineering mathematics courses need to consider ways to implement more effective techniques to optimize this learning methodology. With this in mind, this study addressed how different factors, including students' prior knowledge, learning styles, and type of problems, can explain the performance in the flipped classroom for Engineering Mathematics.

2. Background

2.1 Flipped Learning

With the recent surge of interest in flipped learning, this instructional model is being applied in various settings from elementary schools to higher education. Flipped learning is an educational technique consisting of interactive group learning activities in the classroom and direct individual computer-based lessons outside of the classroom [4]. In the traditional lecture-style classes, students receive and understand knowledge and concepts in an authoritative and one-way manner led by the instructors. In other words, one instructor delivers knowledge to a large number of students collectively, and students identify and acquire this formalized knowledge through repetitive exercises. On the other hand, flipped learning is a learner-centered model based on learning activities that occur in three-steps: before, during, and after classes.

Because pre-class activities in flipped learning replace traditional classroom teacher-led instruction with individual learning, there is extra time secured for meaningful learning activities and peer interaction in the classroom. In general, students learn the concepts necessary for the in-class activities using learning materials (e.g., readings, pre-recorded video lectures, and other multimedia resources) before class begins. After learning the basic concepts, students take lecture notes to study or take quizzes to enhance their understanding.

During the in-class activities, students may ask the teacher questions about areas they did not understand in the pre-class preparation. The teacher/instructor will then provide the students with additional explanations and address students' misconceptions. As students familiarize themselves with this process of sharing basic learning content through the Q&A session, they become better equipped to participate in more enhanced student-centered collaborative activities in the future. What follows the Q&A session is small group learning, in which the groups collaborate, discuss, or solve the provided tasks. When students engage in discussion or inquiry activities in groups, they form common knowledge through a communication and interaction process that involves exchanging thoughts and opinions on the given task.

At the end of the group activity, each group presents a summary of the discussion or inquiry. All the students attend every other group's presentation, compare the outcomes, and go through a process of questioning and discussion. In this process, the knowledge shared by the whole class is restructured. After the group presentation and discussion are complete, the teacher briefly summarizes and organizes the core of the learning contents.

Many scholars have claimed that flipped learning, in accordance with the learning processes described above, has the following characteristics:

First, flipped learning increases learners' self-directedness. Students who were passive learners in traditional lecture-like classes now actively participate in their learning because lipped learning transforms them into active subjects that become more self-directed [9]. Self-directed learning is particularly promoted because pre-class learning happens in the students' homes and at their own pace.

Second, interaction during class increases because flipped learning is a method in which teacher-student and student-student interactions are at the forefront. As the "lectures" in the classroom become pre-class activities, students communicate via different learning activities with *the right to speak* [5].

Third, individualized learning becomes possible and takes place in accordance with the speed and the ability of individual learners. In flipped learning, students will learn individually and following their different learning styles before class [4], at their own pace, and repeat [10].

Finally, collaborative learning is enabled. Instead of lecture-style classes, in flipped learning, the instructor becomes a facilitator so that teaching and learning can occur among peers. The emphasis on flipped learning is given to the classroom being a space for learning through students' interaction and cooperation.

Due to these characteristics, there have been numerous studies that demonstrated the effectiveness of flipped learning. For example, Bates and Galloway [11] applied flipped learning to physics classes at British Universities, resulting in students' better engagement, high satisfaction, and better performance in the class. Enfield's [12] research reported that flipped-learning classes in a Cinema and Television Arts department in the U.S. had a positive effect on students' learning immersion and self-efficacy, and helped them better understand the content. In addition, several studies reported the positive effect of flipped learning on students' enhanced achievement [13], higher-order thinking skills [14], and perception of time management skills [15] in Engineering Mathematics for higher education. The recent meta-analysis also showed an overall significant effect (Hedges' $g = 0.298$) of the flipped learning over the traditional classroom regarding students' achievement in mathematics [16].

2.2 Students' Learning Styles

The context in which we are using here the term *learning styles* refers to *Experiential Learning Theory (ELT)*, aiming to take into account the differences in individuals' learning practices [17]. Kolb & Kolb [18] define what they call the *experiential learning cycle* that relates and corresponds to the operations of different regions of the cerebral cortex, namely the following: 1. Sensory and post-sensory, 2. Frontal integrative, 3. Premotor and motor, and 4. Temporal integrative [19]. Kolb & Kolb [20, p. 194] termed four learning modes that correspond to the four different cerebral cortex regions as 1. *Concrete Experience (CE)*, 2. *Abstract Conceptualization (AC)*, 3. *Active Experimentation (AE)*, and 4. *Reflective Observation (RO)*.

The ELT model entails two dialectically related modes of grasping experience, (CE) and (AC), and two dialectically related modes of transforming experience, (RO) and (AE). Kolb describes ELT as a "dynamic view of learning based on a learning cycle driven by the resolution of the dual dialectics of action/reflection and experience/abstraction." [21, pp. 50–51]. Therefore, learning arises from resolving the creative tension between the four learning modes [18, 20]. The four learning styles can be described [22, 23] as follows:

Convergent. Learners with a *convergent* learning style perceive information through abstract conceptualization (AC) and process it through active experimentation (AE). They like to learn by doing, and they do not like lectures or reading for a long time. In Kolb's terms, they "converge" fast and make decisions, cutting through to the essentials. They prefer the lab setting and the experiment. They tend to work faster individually, and they would like their instructor to be a coach and a facilitator so that they can take a more active role.

Divergent. Students that possess a *divergent* learning style perceive information through concrete experience (CE) and process it through reflective observation (RO). They seek personal meaning and interaction with the instructor and their peers. They learn well through discussion and collaboration, and they are interested in the humanistic aspects of the learned material. Kolb calls this kind of learner "divergent" because they can see things from different perspectives. As a consequence, they excel in brainstorming. For them, the ideal instructor acts as a motivator and shows them how the taught material fits the big picture.

Assimilative. Scholars with an *assimilative* learning style perceive information through abstract conceptualization (AC) and process it through reflective observation (RO). They prefer lectures and seek a conceptual understanding of what they are learning. Kolb calls this learning style assimilative because learners with this learning style take separate pieces of information, analyze them, organize them, and "assimilate" them into a whole. They like order, tend to be detail-oriented, and thrive on procedures and following directions. In their approach, they are careful, methodical, and cautious, trying to avoid errors. They like to see the teacher as an expert and a leader.

Accommodative. Learners with an *accommodative* learning style perceive information through concrete experience (CE) and process it through active experimentation (AE). They are enthusiastic and prefer self-discovery as a learning method. They like to follow their own pace, and they want interaction with others and group discussion. They are problem-solvers and risk-takers, and they tend to learn from their mistakes. Kolb calls this learning style accommodative because learners in this category take what they have learned and adapt, change, and improve it. The instructor's role for accommodators is that of an evaluator and a remediator. These learners need to be encouraged for self-discovery and self-teaching.

ETL's *learning style model* is the most widely accepted and has received extensive empirical support [24]. Many applications have been implemented, and much research has been conducted relative to ELT in college-level educational contexts. Orhun [25] concludes that applying teaching methods based on students' learning styles will simplify the selection process for the most effective teaching strategy, enhancing engineering students' performance in calculus courses. Also, Konak, Clark, and Nasereddin, [26], although they focus on the importance of collaborative learning strategies to promote learning in virtual computer laboratories, they highlight the potential benefits of identifying the students' learning styles. Konak, Clark, and Nasereddin [26] state that studying the interactions between the Kolb Experiential Learning Cycle stages and the students' learning styles could lead to more knowledgeable guidelines to design hands-on activities that are most appropriate for them. There have been several studies to identify engineering students' learning styles. The vast majority of the learners were nearly equally split between the convergent and the assimilative learning styles [27]. Another study [28] had the same results, with most engineering students being convergers or assimilators. On the other hand, the results produced by two studies [29, 30] about the engineering students' learning styles showed that they were mostly assimilators.

However, flipped learning strategies applied under ETL learning styles in higher education, particularly in the engineering mathematics context, is a relatively new approach. Hence, more implementations and more research should be conducted to investigate the impact students' learning styles have on their academic accomplishments.

2.3 Types of Problems in Engineering Mathematics

Because engineering mathematics courses are designed for all students who choose engineering-related majors, they include a wide range of mathematics content (e.g., Calculus, Differential Equations, and Linear Algebra) taught in a short timeframe. Engineering mathematics courses are designed for students to acquire the least amount of mathematical knowledge needed to use mathematics as a tool for understanding the essential engineering concepts that will be the basis of their future engineering-related majors. Therefore, engineering mathematics education aims to foster students' mathematical skills and knowledge for practical purposes, and to the extent that students will not have any issues taking major-related engineering courses. To achieve this goal more effectively, various efforts are currently being made in engineering mathematics classes, and one of them is the provision of different types of problems. According to Guven., Aydin-Guc, and Ozmen [31], providing different kinds of problems not only determines the direction of mathematics learning activities but also affects the lesson's strategy and evaluation.

Mathematics problems can be divided into *verbal*, *non-routine*, and *real-life/applied* problems [32]. The verbal mathematics problems refer to formalized problems that can be addressed by recalling the general algorithm already presented as a known typical solution. Students can solve these types of problems through already known procedures; by applying mathematical knowledge such as concepts, principles, and laws learned in numbers and calculation, shapes and measurements, texts and equations, and probabilities and statistics. In short, these kinds of problems can be solved by the simple algorithm obtained from the standard instruction.

In non-routine problems, the clear procedures and proper algorithms needed for the solutions are not readily available. To solve non-routine problems, although students' basic mathematics knowledge is required, students need to develop their solutions through creative thinking and map out ways to find the answers instead of merely using their existing knowledge. Practice, which might be helpful to solve verbal problems, is not so useful when it comes to solving non-routine problems. These problems do not have any simple algorithm to follow and require devising a creative problem-solving strategy. Non-routine problems cannot be solved using a formal solution, and, instead, students need to draw graphs or use other means to discover the patterns. A non-routine problem can confirm students' mathematical cognitive thinking, creativity, and fluency. Faulkner, Earl, and Herman [33] consider engineering undergraduate students' ability to formulate and solve non-routine mathematics problems as one of the essential competencies, namely the *Problem Tackling Competency* as defined by Niss and Højgaard [34].

The last type is the real-life/applied mathematics problem. The topics and the materials used in constructing these kinds of problems are taken from students' real life, and the solutions include both verbal and non-routine explanations. Real-life problems are the most appropriate for demonstrating the applicability of mathematics. Solving real-life problems enable students to acquire mathematical knowledge, add to their problem-solving techniques, enhance their understanding of mathematics applications, and arouse their interest in learning mathematics. Engineering students testify to that by recognizing as a strength of a course's strategies the use of real-life applications because, as they said, it gave them a "taste" of what the engineering world is today [35]. Furthermore, adapting mathematical models to represent real-life or simulated engineering problems [36] and relating the mathematical knowledge, techniques, skills, and ideas acquired previously to the engineering context [37] are considered by scholars of grave importance. The problem-solving techniques that engineering students are taught were not proved useful when learned in isolation. Graduate engineering students could not develop their critical thinking and creativity to use problem-solving strategies to deal with real-life problems. When the context in which the students acquired the knowledge is removed from the actual real-life applications, that knowledge is not available to the students [36]. Radzi et al. [36] advocate using real-life mathematics problems to avoid the rote learning and rote memorization attitude that students transfer from school to university.

Many scholars have assumed that verbal problem-solving learning outcomes can easily be transferred to non-routine or real-life problem-solving. Nevertheless, recent studies have shown a clear difference in the problem-solving process between verbal, non-routine, and real-life problems [38–40]. Dunkle, Schraw, and Bendixen [41] concluded that the skills required to solve verbal and non-routine/real-world problems were independent. Shin, Jonassen, and McGee [42] suggested that solving real-life problems requires not only cognitive skills but also non-cognitive abilities, such as evidence-based argumentation skills and metacognition. That does not necessarily mean that real-life problems must be part of the curriculum for students in mathematics education. It means that the appropriate kinds of problems should be developed considering students' level of knowledge, learning content, and learning/teaching model.

However, few studies have investigated which types of problems in engineering mathematics courses adopting flipped learning can enhance engineering students' mathematical knowledge and understanding of the concepts. Thus, it is essential to identify the kinds of problems that should be considered when designing effective flipped learning for engineering mathematics education.

2.4 Prior Knowledge

Bloom [43] defined prior knowledge as knowledge, skills, and abilities necessary to successfully carry out a given series of new learning tasks and Gagne [44] defined it similarly as a lower learning task linked to higher learning tasks in various hierarchical learning tasks. Prior knowledge is the right understanding of the characteristics of learning tasks, which is necessary to achieve the learning goals. As a learner, knowledge of oneself means knowing the personal academic strengths and weaknesses. Also, knowledge of the learning process can be understood as the knowledge that sets the correct learning goals and carries out the learning activities to achieve them.

The importance of prior knowledge is emphasized by Ausubel's advanced organizer and schema theory. According to Ausubel [45], acquiring knowledge means that new knowledge is added to the existing cognitive framework (i.e., schema). Meaningful learning, which helps learners expand and reorganize the schema through the interaction with new information, is an important process that uses prior knowledge as meaningful information and derives meaning in relation to the new knowledge. Ausubel referred to advanced organizers as a way to promote meaningful learning. Advanced organizers are resources that come before presenting a new learning task, which is a means of strengthening learner's cognitive structure and promoting the absorption of new information. In other words, the advanced organizers are a technique that presents new learning content to students before the beginning of instruction to help them relate them to prior knowledge. Advanced organizers facilitate bridging the gap between what learners already know and the new knowledge to be acquired, and scaffolding, which is defined as a support to help learners participate in tasks beyond their current abilities, can also play a similar role [46–48].

Pre-class activities, which are part of the flipped learning processes, provide students with learning materials related to the upcoming lesson's learning contents and can play the role of an advanced organizer [49]. Based on this fact, one can assume that prior knowledge built by these pre-class activities or already exist as schema can affect students' performance in flipped learning. However, few studies investigated the role of prior knowledge in affecting the outcomes in flipped learning through empirical research.

2.5 Research Question

To what extent can students' learning styles, the type of problem, and their prior knowledge influence their understanding of Engineering Mathematics using Flipped Learning?

3. Methods

3.1 Participants and Group Composition

Most of the 139 students who participated were majoring in engineering and various natural science departments. Their ages ranged between twenty and twenty-four, while most were male students (92%). At the beginning of each semester, students took two tests. The first test was a set of conceptual problems that measured the students' prior knowledge of the course material. The second test was Kolb's learning style inventory (LSI), by which the students were categorized into four different groups: convergent, divergent, assimilative, and accommodative. The students were assigned to four-member groups so that their average score on the conceptual test were similar ($p > 0.05$). The assigned groups remained unchanged throughout the semester.

3.2 The Course and Course Design

This study involved four sections of "Engineering Mathematics I," a course on ordinary differential equations, in one large-scale private university located in South Korea. One of the study researchers was the instructor of all sections, on the same days of the week, at different times. The course materials, assignments, and in-class activities were identical for all sections. The instructor had been teaching the course for seven years and could manage four sections without much variance. The only difference was the number of students in each section (39, 14, 48, and 38).

For this study, students learned one concept (i.e., Spring-mass system, equivalent system, linear system, and SIR model) every week for a total of four weeks. Students submitted their pre-class homework after watching two or three fifteen-minute-long video clips, which were about the topics of the following class. This pre-class homework was comprised of three conceptual questions related to the watched video clips. During class, students participated in "voting" activities based on the framework of *Peer Instruction* [50] or *Hypothesis-Experiment-Instruction* [51]. First, each student solved the given problem individually, and then, the students joined their groups. The group members shared their thoughts, rationale, and opinions on the right answer and voted their correct answers. Finally, the instructor examined the voting results, clarified possible misconceptions, and pointed out any existing pitfalls. The group discussion usually was taking between fifteen to twenty minutes. The instructor determined the discussion's duration based on the problem's difficulty and the students' engagement level.

After finishing four weeks' learning, students took the exam to check their understanding of four concepts related to ordinary differential equations covered by this flipped classroom.

3.3 Data Collection

Prior knowledge test. This test measured students' basic knowledge of functions, continuity, differential equations, and function graphing, which is necessary to learn ordinary differential equations covered in the class. This test, which consists of 12 items, was validated by two experts in the engineering mathematics field and was highly reliable (Cronbach's alpha = 0.92). The maximum score students could get from this test was 100.

Learning Style (see Appendix A). Kolb's Learning Style Inventory (LSI) Version 3.1, consisting of 12 items, was used to identify the participants' learning styles by measuring how much each student's learning style corresponded to the four learning styles as defined by Kolb [52]. The score the participants could get from this survey ranged from 12 to 48, and, based on their results, each participant was placed into one of four learning style quadrants: Convergent, Divergent, Assimilative, and Accommodative. The reliability for each of the four-dimensional constructs (based on N = 139) was very good (Cronbach's alpha ranging from 0.84 to 0.89).

Performance in different kinds of problems. Four problems (two verbal, one non-routine, and one real-life) and provided to each group for individual and collaborative learning in the classroom to assess the influence of different kinds of problems on students' understanding of contents related to ordinary differential equations. Fig. 1. shows examples of different kinds of problems.

The first example, the speedometer, is a real-life/applied type of problem because it is about an instrument most students use almost daily or are all familiar with it. This problem demonstrates the applicability of mathematics by finding a mathematical model of the speedometer. For the second example with the equivalent system, students cannot merely apply a known algorithmic solution. Instead, they need to implement their creative skills to form a unique problem-solving strategy. Hence, the second example belongs in the category of the non-routine type of problems. The last two examples, the linear system and the SIR model, are verbal type mathematical problems because they can be solved through known procedures. They require familiar to students algorithmic procedures for solving ordinary differential equations. The type of each problem and its content were validated by one expert in assessment and an instructor who has expertise in Engineering Mathematics. Students' performance on each problem was coded as a dichotomous variable (i.e., right or wrong).

Post-Test (see Appendix B). The measure of individual student understating in the course was the score of one test. The test consisted of 14 open-ended problems about four concepts (*Mass-spring system*, *Equivalent system*, *linear system*, and *SIR Model*) covered by Flipped learning. The score for the post-test ranged from 0 to 100. An instructor of this course and one of the researchers – who had expertise in engineering mathematics – worked independently to score students' responses in the test. Then they met to discuss score discrepancies and came to a consensus. The post-test interrater

Speedometer

How do you model a speedometer?
Which regime do you need to choose?
(Explain that spring-mass system with a damper is used to make a speedometer and the system needs to be tuned to be practically usable).

Over vs Critical vs Underdamping?

Equivalent System

Find the equivalent system to the following one.

$$y'' + ay' + by = \delta(t), \qquad y(0) = 0, y'(0) = 0$$

1. $y'' + ay' + by = 0, \quad y(0) = 0, \; y'(0) = 1$
2. $y'' + ay' + by = 1, \quad y(0) = 0, \; y'(0) = 0$
3. $y'' + ay' + by = 0, \quad y(0) = 0, \; y'(0) = 0$
4. $y'' + ay' + by = 0, \quad y(0) = 0, \; y'(0) = 1$

Linear System

Find the linear systems.

A $\begin{array}{c} x' = 2x + 3y \\ y' = x^2 - y \end{array}$

B $\begin{array}{c} x' = 2x + 3y + \sin t \\ y' = 3x - y \end{array}$

C $\begin{array}{c} x' = x + y + z \\ y' = z \\ z' = x \end{array}$

D $\begin{array}{c} x' = -\sigma x + \sigma y \\ y' = -xz + \gamma x - y \\ z' = xy - bz \end{array}$

SIR Model

R: Recovery Model

1. $R'(t) = C_1 days I(t)$
2. $R'(t) = -\frac{1}{C_1 days} I(t)$
3. $R'(t) = \frac{1}{C_1 days} R(t)$
4. $R'(t) = -\frac{1}{C_1 days} R(t)$

Fig. 1. (a) Speedometer: real-life problem, (b) Equivalent: non-routine problem, (c) and (d) Linear system and SIR model: verbal problems.

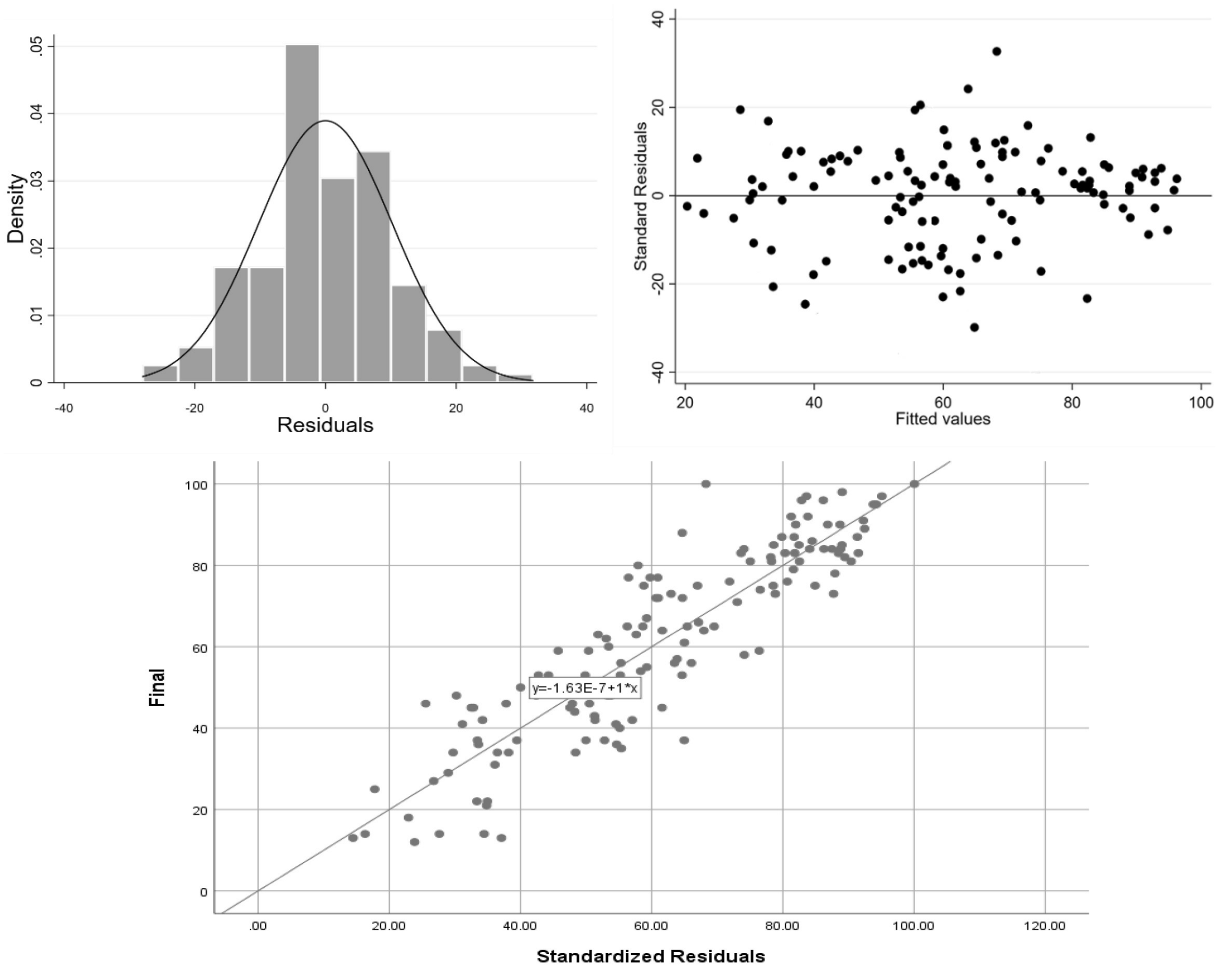

Fig. 2. Histogram of residuals and two scatter plots of Linearity of Residuals and Equal Variance of Residuals.

Table 1. Results of Four Different Assumption Tests

Assumption	Test	Results
Linearity of Residuals	Pearson Correlation (between Final and Standardized Residuals)	$r(137) = 0.89$, $p < 0.01$
Independency of Residuals	Durbin-Watson d-statistic	Durbin-Watson value = 1.61
Normality of Residuals	Shapiro-Wilk W test for normal data	$W(139) = 0.99434$, $p > 0.05$
Equal Variances of Residuals	Breusch-Pagan test for heteroskedasticity	X^2 (1) = 3.26, $p > 0.05$

Table 2. Learning Styles of Participants

Learning Styles	N	Percent (%)
Convergent	67	48.20
Divergent	19	13.67
Accommodative	13	9.35
Assimilative	37	26.62

reliabilities, which show the degree of concordance concerning grading, were 0.95 (Krippendorff's alpha. [53]). The overall Cronbach's alpha value of items' reliability in the test was 0.87.

3.4 Data Analysis

Multiple Linear Regression was conducted to predict engineering students' mathematical understanding of ordinary differential equations considering their learning styles, prior knowledge, and the performance on each of the different types of problems (i.e., Speedometer, Equivalent, Linear, and SIR). Before conducting multiple linear regression, we tested four assumptions of linear regression: Linearity of Residuals, Independence of Residuals, Normal Distribution of Residuals, and Equal Variance of Residuals.

A histogram and two scatter plots in Fig. 2 satisfied the four linear regression assumptions. The following statistical results in Table 1 also show no issues to conduct Multiple Linear Regression.

In addition, all variables' VIF values to detect multicollinearity were less than 10 [54]. All variables were included in this regression model.

4. Results

Descriptive statistics of Learning Styles in this study are shown in Table 2. Most participants had 'convergent' (48.20%) and 'assimilative' as their learning styles (26.62%).

Table 3 shows the means, standard deviations, and 95% confidence intervals of a prior knowledge test and post-test.

The multiple regression was conducted by the enter method so that all nine independent variables were included simultaneously in the model. According to the ANOVA table, the multiple regression model, including nine independent variables, was significant, and 62% of the data fit this regression model, $F(9, 129) = 23.77$, $p < 0.01$, with an R^2 of 0.62 (see Table 4).

Students' performance in real-life problem ($p < 0.01$), and non-routine ($p < 0.05$), and two learning styles, convergent ($p < 0.05$), and assimilative ($p < 0.01$) were statistically significant predictors of students' final test scores (see Table 5). However, there was no statistically significant linear dependence of the mean of the post-test score on students' prior knowledge, learning styles of divergent and

Table 3. Means, Standard Errors, and 95% Confidence Intervals of Prior Knowledge and Post Tests=

			95% Confidence Interval	
	Mean	Std. Error	Lower	Upper
Prior	47.14	1.23	44.70	49.58
Post	61.37	1.96	57.49	65.24

Table 4. ANOVA Table for Goodness of Fit of Regression Model

Model	Sum of Squares	df	Mean Squares	F	Sig.
1. Regression	45910.81	9	5101.20	2.377	0.00
Residual	27679.48	129	214.57		
Total	73590.29	138			

Note: $R^2 = 0.62$, Dependent Variable: Posttest.

Table 5. Regression Analysis of Predictor Variables Related to Students' performance in Engineering Mathematics Class

Model	Coefficients	Std. Error	t	Sig
(constant)	26.04	9.08	2.87	0.00**
Prior	0.13	0.09	1.45	0.15
Convergent	22.66	8.79	2.58	0.01*
Divergent	–0.32	9.24	–0.03	0.97
Accommodative	8.75	9.53	0.92	0.36
Assimilative	43.33	9.15	4.73	0.00**
Verbal 1 (Linear)	–1.28	2.74	–0.47	0.64
Verbal 2 (SIR)	0.15	2.83	0.05	0.96
Real-life (Spring-Mass)	10.50	2.76	3.80	0.00**
Non-routine (Equivalent)	6.31	2.71	2.33	0.02*

Note. ** < 0.01, * < 0.05.

accommodative, and verbal-type problem (p >0.05).

5. Discussion

To achieve academic objectives effectively, instructors need to implement various teaching methods for learners. With even more people noticing the problems with the existing lecture method in teaching engineering mathematics, the flipped classroom has been regarded as an effective alternative teaching method [4]. While the empirical research using flipped classroom in mathematics in engineering has been widely conducted, most of them are simple comparative studies of the traditional and flipped classroom in a short period in terms of students' performance. Due to this lack of design principles and the breadth of the topics needed to be covered, instructors in the field of engineering mathematics are often reluctant to implement flipped learning. Therefore, the study at hand was not about researching the effectiveness of flipped learning in engineering mathematics classes since this was already demonstrated by several empirical research [13] and meta-analyses [16]. Instead, this study investigated the extent to which students' learning styles, prior knowledge, and the types of problems provided predict students' understanding of ordinary differential equations at the end of class.

In the multiple linear regression, significant predictors of students' final test scores were the convergent and assimilative learning styles and the non-routine and real-world problem types. In the following sections, we unpack and discuss the predictors in light of the literature.

5.1 Predication of Post Score by Students' Learning Style

The finding that the learning style of most engineering students participating in this study was either convergent or assimilative is consistent with prior research [28, 30]. These two learning styles (convergent and assimilative) were also significant factors affecting students' post-test scores in flipped learning for the engineering mathematics class. This result can be explained by considering the characteristics of the two learning styles, which correspond to the requirements for successful flipped learning in studying and understanding the abstract concepts related to mathematics.

In flipped learning for engineering mathematics, the in-class given task should begin with the students' basic understanding of the pre-class abstract concept activity. Then it should progress to the challenging problems addressed by their advanced problem-solving skills, the practical application of concepts, and reasoning skills [13]. According to Kolb [52], the assimilators abstractly conceptualize the information and process it with perceptual and reflective observations. The assimilators are excellent in logic and precision and are familiar with inductive reasoning. They can integrate a wide range of ideas and understand them in various ways, so their ability to create theoretical models is excellent. The convergers, on the other hand, have the ability to conceptualize abstractly, perceptually, and actively experiment with the provided information. Because they can apply ideas [46], converges have excellent decision-making and problem-solving skills. They can also focus on particular problems through hypotheses-linked causes and approach tasks systematically and scientifically.

For this reason, both the assimilative and the convergent learning styles showed significantly positive correlations with performance in the engineering mathematics class using flipped learning. From this result, one of the effective ways to achieve successful outcomes in flipped learning is to obtain information about students' learning styles before class and provide the individualized learning process and tasks considering learners' different learning styles.

However, to realize the flipped learning class, the workload and the time needed from the instructor to design and develop all the necessary learning materials are immense. So, it might not be feasible to create individualized learning, taking into account the different learning styles. Instead, it may be more realistic to train students to adapt to the optimal learning style by acquiring various learning styles, based on the fact that learning style preferences can change according to environmental needs and circumstances.

5.2 The Influence of Different Types of Problems on Students' Understanding of Concepts

Results showed that students' performance in real-life ($p < 0.01$) and non-routine problems ($p < 0.05$) were statistically significant predictors of students' post-test scores in flipped learning for the engineering mathematics class. This result can be explained by the skills necessary to tackle non-routine and real-life problems, which are also necessary for a successful flipped learning implementation in understanding the abstract mathematical concepts in the engineering mathematics class. In non-routine problems, the solutions are not achieved through the taught procedures or known algorithms [41]. Students not only need their existing knowledge to solve these kinds of problems but also need to implement their creative thinking and construct a problem-solving strategy. In real-life/applied problems, the solutions include both verbal and non-routine explanations [39]. They are about the applicability of mathematics, and they "demand" the students' knowledge and problem-solving strategies in mathematics.

Similarly, for a successful flipped learning implementation in engineering mathematics, the problems become more challenging after the pre-class abstract concept activity, asking for the students' advanced problem-solving, practical application of concepts, reasoning skills, and collaborative argumentation skills. In this sense, non-routine and real-life problems enhance and ask for independent thinkers, individualized learning, self-directedness, and collaborative learning skills in flipped learning.

5.3 Insignificant Prediction of Post Score by Student's Prior Knowledge

Based on previous research, students' prior knowledge is one of the significant factors that predict the overall cognitive outcomes [55]. Nevertheless, in this research, the students' prior knowledge of functions, continuity, differential equations, and function graphing (which are the necessary basics to learn ordinary differential equations covered in the class) was not a significant predictor of students' post-test measurement of their understanding of ordinary differential equations. This result is somewhat perplexing but can be explained by three reasons. One plausible reason is that pre-class activities in flipped learning did not play a meaningful role as advanced organizers, as Ausubel claimed [45], due to lack of provision of scaffolding. The pre-class activities should be conducted by the students themselves without any support from instructors or peers, which could hinder their knowledge building, affecting students' final outcomes in flipped learning. Another possible reason is that the participants' level of prior knowledge ($M = 47.14$) was not high. According to several scholars [56], students with a low level of prior knowledge can achieve better performance in the class when they get direct and explicit support and feedback from instructors. The flipped classroom's learning process is such that students are responsible for their learning; all activities are conducted by individualized learning at their level and by collaboration with peers, rather than direct instruction from teachers [4]. This means that even students with low prior knowledge do not necessarily show low performance in flipped learning. On the contrary, high performance is not guaranteed for students with a high level of prior knowledge [57, 58]. This led us to conclude that students' prior knowledge was not a significant factor in predicting the post-test scores in this research. The last reason is the insignificance of verbal problems in predicting students' performance in flipped learning. Based on this study's results, the effects of verbal problems, mostly requiring students' prior mathematical knowledge for problem-solving [32], on students' performance were not significant. On the other hand, two types of problems (i.e., non-routine and real-life) that can be solved by creative thinking and advanced problem-solving skills rather than relying on prior knowledge were significant predictors in flipped learning [38, 39]. This fact can be the reason why students' prior knowledge was not a significant predictor that affects students' performance in flipped learning.

5.4 Limitations and Suggestions for Future Research

This research revealed the extent to which students' learning style and different types of problems can affect their understanding of engineering mathematics concepts through quantitative analysis. The limitations that emerged in this research are as follows.

First, this research did not directly verify flipped learning effectiveness in a college engineering mathematics class. However, several significant factors (i.e., learning styles and types of problems) to predict students' performance in flipped learning were identified. Therefore, further empirical research, including the variables analyzed in this study, needs to be carried out to investigate the effects of flipped learning.

Second, additional qualitative research can significantly contribute to answering the 'how' and 'why' questions, increasing the likelihood of a reasonable interpretation of quantitative results through the qualitative data describing students' perception and experience in flipped learning in detail and more in-depth.

Third, this research applied flipped learning to an engineering mathematics class over a relatively short learning period of four weeks. Therefore, there is a limit to generalizing the results obtained by applying a new instructional model for four weeks. Therefore, future research is required to verify this study results repeatedly for more than one semester.

6. Conclusion

This study helped further understand the prediction of students' performance in engineering mathematics by their prior knowledge, learning styles, and the types of problems in a flipped classroom. The results showed that, first, two learning styles (i.e., convergent and assimilative) in flipped learning for engineering mathematics played a major role in problem-solving and significantly predicted their final test score. Second, engineering students' performance on ordinary differential equations varied significantly depending on the different types of problems. Students' scores in the post-test were significantly increased when engaging in real-life and non-routine problems individually or in collaboration with group members. Implications from this study include that instructors can enhance students' performance in engineering mathematics by (a) integrating the learning process of flipped learning with the current curriculum, (b) helping students apply the abstract concepts of mathematics to an authentic situation, and (c) considering students' learning styles as one of the factors for successful flipped learning.

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Appendix

Appendix A. Kolb's Learning Style Inventory Survey

LEARNING-STYLE INVENTORY

The Learning-Style Inventory describes the way you learn and how you deal with ideas and day-to-day situations in your life. Below are 12 sentences with a choice of endings. Rank the endings for each sentence according to how well you think each one fits with how you would go about learning something. Try to recall some recent situations where you had to learn something new, perhaps in your job or at school. Then, using the spaces provided, rank a "4" for the sentence ending that describes how you learn *best*, down to a "1" for the sentence ending that seems least like the way you learn. Be sure to rank all the endings for each sentence unit. Please do not make ties.

Example of completed sentence set:

When I learn: _2_ I am happy. _1_ I am fast. _4_ I am logical. _3_ I am careful.

Remember: **4** = *most* like you **3** = *second most* like you **2** = *third most* like you **1** = *least* like you

	A		B		C		D	
1. When I learn:	___	I like to deal with my feelings.	___	I like to think about ideas.	___	I like to be doing things.	___	I like to watch and listen.
2. I learn best when:	___	I listen and watch carefully.	___	I rely on logical thinking.	___	I trust my hunches and feelings.	___	I work hard to get things done.
3. When I am learning:	___	I tend to reason things out.	___	I am responsible about things.	___	I am quiet and reserved.	___	I have strong feelings and reactions.
4. I learn by:	___	feeling.	___	doing.	___	watching.	___	thinking.
5. When I learn:	___	I am open to new experiences.	___	I look at all sides of issues.	___	I like to analyze things, break them down into their parts.	___	I like to try things out.
6. When I am learning:	___	I am an observing person.	___	I am an active person.	___	I am an intuitive person.	___	I am a logical person.
7. I learn best from:	___	observation.	___	personal relationships.	___	rational theories.	___	a chance to try out and practice.
8. When I learn:	___	I like to see results from my work.	___	I like ideas and theories.	___	I take my time before acting.	___	I feel personally involved in things.
9. I learn best when:	___	I rely on my observations.	___	I rely on my feelings.	___	I can try things out for myself.	___	I rely on my ideas.
10. When I am learning:	___	I am a reserved person.	___	I am an accepting person.	___	I am a responsible person.	___	I am a rational person.
11. When I learn:	___	I get involved.	___	I like to observe.	___	I evaluate things.	___	I like to be active.
12. I learn best when:	___	I analyze ideas.	___	I am receptive and open-minded.	___	I am careful.	___	I am practical.

Appendix B. Exam

Engineering Mathematics

Exam

Please remember that your solutions should be clear, concise, precise, and legible. Be sure to check you write your name and your class number on the exam paper before you hand in.

Name & ID No.: ______________________________

Class No.: ______________________________

1. (10) Solve the following ODE:

$$(xy^2 + 3x^2y)dx + (x + y)x^2dy = 0$$

2. (10) Solve the following ODE:

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$$

[Problems 3–5] Consider the following initial value problem (IVP),

$$\frac{dy}{dx} = x^2 - y^2, \qquad y(-2) = 0$$

3. (10) Sketch the isoclines for slopes –2, 0, 2, and sketch the slope field using this information.
4. (5) On the same graph, draw the solution curve of the above IVP.
5. (5) Estimate y (–100).

[Problems 6–7] Consider the following autonomous ODE:

$$y' = 2y - 3y^2 + y^3.$$

6. (5) Find all equilibrium solutions and state the stability of them.
7. (5) Sketch the graph of the solutions in the t y-plane. Be sure to include at least one solution with values in each interval above, below, and between the equilibrium solutions. You also need to specify the value at the points of inflection.

[Problems 8–10] Consider the following spring-mass system:

$$y'' + 2\zeta\omega_0 y' + \omega_0^2 = 0, y(0) = 0, y'(0) = 0.$$

8. (10) Using the Laplace Transforms, find the above spring-mass system's impulse response and sketch it.
9. (5) Describe the change of the mass' velocity near t = 0 when the impulse is applied at t = 0.
10. (5) If an arbitrary force f(t) is applied to the above spring-mass system, write the solution using the convolution integration. (You need to write down the integration explicitly).

[Problems 11–12]

11. (5) Find the Laplace Transform of the following function:

$$f(t) = t^2e^t - \int_0^t \tau \sin(t - \tau)\, d\tau + \delta(t - 2) .$$

12. (5) Find the non-negative real numbers A, ω, and θ for the following equation:

$$Re\left(\frac{ie^{2it}}{1+i}\right) = Acos(\omega t - \theta).$$

[Problems 13–14] If the fuel in a car engine is shut off, the car will eventually come to a rolling stop due to wind resistance and rolling friction. Assume that the rolling friction is given by $F_f = k_1 V$ and the wind resistance is given by $F_w = K_2 V$ where V is the speed of the car.

13. (5) Write down the equation for the velocity of the car. Assume that the car has mass M.

14. (5) Solve the resulting equation if $V(0) = V_0$.

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