International Journal of Engineering Education Vol. 38, No. 4, pp. 879–891, 2022
Printed in Great Britain

0949-149X/91 $3.00+0.00

Trends in Using Student-Centred Approaches in Mathematics and its Connection with Science, Technology, and Engineering*

DROBNIČ VIDIC
Andreja. Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, Ljubljana, Slovenia.
E-mail: andreja.drobnic@fmf.uni-lj.si

Problem-based, project-based and inquiry-based learning (PBL, PjBL, IBL) are student-centred approaches that emphasise interdisciplinary connection between subjects in all levels of education. After presenting the theoretical similarities and differences among these three approaches, we review the use of these approaches in mathematics education and its connection with science, technology, engineering (STEM). We analyse all articles from Web of Science database that examine one of the target approaches at various educational levels in mathematics and with its connection with other subjects. The distribution of the selected articles in 5-year periods allows us to delineate trends in the connection of math with science and other STEM subjects through student-centred approaches, and in the methodology used in the studies. Detailed analysis of selected articles with experimental design, where effect size is or can be measured, gives us an insight in common characteristics of science and engineering incorporations in mathematical education sphere. Our aim is to find out differences between students-centred approaches in mathematics as a sole subject, in mathematics connected with science, and in mathematics connected with all the STEM subjects from practitioners' point of view. Among other things our analysis reveals that integration of all the STEM subjects is a current trend in math education and it is mostly used with PjBL approach. The benefits of such STEM connection are naturally expected for mathematics as a knowledge base as well as for engineering as a field with wide range of knowledge applications. Finally, we encourage teachers and practitioners to implement such a school practice that conceives mathematics as making sense and to be applicable in a real life and in other fields of education – especially engineering.

Keywords: inquiry-based learning; interdisciplinary mathematics education; problem-based learning; project-based learning; STEM connection (science-technology-engineering and mathematics connection)

1. Introduction

Mathematics is a base for other academic disciplines such as engineering or science. Therefore, a connection between math and other disciplines at higher levels of education seems to be natural and effective. Conceiving mathematics as making sense should help promote conceptual changes in mathematical practice to value idea generation and design activity; connections generated from such a shift will support teaching and learning not only in individual science, technology, engineering, and mathematics (STEM) disciplines, but also in integrated STEM education [1].

In order to make mathematics valuable to future engineers, we incorporated problem-based learning (PBL) into statistics as a part of mathematical content for students at a tertiary level program in Slovenia [2]. This student-centred instruction encourages active participation of students that usually learn in small groups; learning is triggered by meaningful real-world engineering problems. Moreover, combination of PBL and project-based learning (PjBL) was implemented at the secondary level of education where STEM subjects were connected with a year-long "Energy as a value" projects followed by shorter problems about energy at each secondary school STEM subject [3]. The acronym PBL is often used also for project-based learning that can be misinterpreted as problem-based learning [4]. Both learning approaches foster active participation of students in small groups' learning and demand a longer period of time for the problem-solving process. However, there are some differences in the context for learning and in some other aspects that are further exposed in the theoretical background (Section 1.1).

In recent years, a tendency to foster inquiry-based learning (IBL) in math and science education has been observed by the Ministry of Education in Slovenia, treated as an East European country in the mathematical education sphere. IBL has been presented as a "new" approach to the elementary and secondary math teachers that enables students to explore a particular situation in a similar way as researchers and scientists would do it [5]. In this student-centred approach, students actively participate in a question-driven learning process, supported by meaningful contexts [6]. They are faced with an unknown situation or problem that needs to be solved; they work individually as well as in groups as scientists usually do and they take responsibility for their own learning [7].

* Accepted 2 January 2022.

1.1 Theoretical Background of Student-centred Constructivist Interdisciplinary Approaches

A set of reforms in science education from 1990 until 2011 promoted less-structured problem-, project- and inquiry-based learning approaches where teachers facilitate rather than direct learners' actions [8]. Brief theoretical overview of these approaches is given in alphabetical order.

Inquiry-based learning. The term IBL is not in widespread use throughout the educational literature; some other terms such as enquiry-based learning, inquiry learning, research-based learning etc. are also in use [9]. Additionally, definitions for inquiry-based education vary; inquiry itself can be considered as a way of engaging in science and more recently, engineering-related practice [8]. In the IBL practice questions or problems provide a context for active learning [10]. Students pose questions, make observations, plan and make investigations, analyse and communicate the results ([6, 11]). Science teachers usually centre students' activities around 5E steps: Engagement, Exploration, Explanation, Elaboration, and Evaluation [12]. This means that students create their own scientifically oriented questions; give priority to evidence in responding to questions; formulate explanations based on evidence; connect explanations to scientific knowledge; and finally communicate and justify explanations [13, p. 27]. Cooperative work is very useful for the mentioned processes. There is also a new role for a teacher to encourage students for inquiry, to pose questions and to integrate previous knowledge into the process of learning new things. The new role includes: orienting students toward constructive use of prior knowledge; supporting and guiding when necessary their autonomous work; managing small group as well as whole class discussions; encouraging the discussion of alternative viewpoints; and helping students to make connections between their ideas [14].

Problem-based learning. PBL is also a student-centred approach where problems trigger learning of a particular content. Problems take a central role in the learning process and constitute the motivation for the student's activities [15]. Problems should be professionally relevant and as close as possible to real-life situations. That means that they omit borders between various subjects. They are usually not structured, not routine and they are open, which means that problems have more than one correct solution. Such problems from real world are authentic and cannot be found in regular students' workbooks [16].

Students are confronted with a problem prior to the acquisition of new knowledge. In order to be able to solve a problem, students must activate their prior knowledge and integrate new knowledge in their own cognitive structures in order to establish a connection with the prior information [17]. Students' active involvement in problem solving and the exchange of opinions in group work within PBL enable early detection of misconceptions in knowledge. A seven-step model has become established in PBL in medicine: clarification of unclear terminology and concepts; definition of the problem; brainstorming; list of possible explanations; formulation of learning aims and key tasks; independent search for additional information outside the group; report, synthesis and testing of the new information [18]. In various PBL implementations problems may vary significantly in scope: from short single discipline problems to longer multidisciplinary problems [10]. Self-regulated learning in small groups enables students to acquire new skills for effective cooperative work, skills for searching various sources and acquiring new information as well as skills to efficiently present new knowledge to peers and others. The so-called transferable skills can be easily transferred into other disciplines. In addition, students become gradually more familiar with problem solving in a small group [18]. Getting more and more open problems, students practice problem-solving skills with the help of a teacher. The teacher's role is changed, too. He/she becomes a facilitator that poses questions, facilitates learning process, helps students to stay in the way that leads them to finish their problem solving process, and solve the problem. Therefore, students solve the problem by themselves in a small group, while facilitator guides the learning process.

Project-based learning. PjBL allows students to learn by doing and by applying ideas; in PjBL students engage in real world activities that are similar to the activities that adult professionals engage in [19]. Krajcik and Blumenfeld describe project-based environments with five key features: learning starts with a driving question or problem to be solved; students learn and apply important ideas in the discipline; they engage in collaborative activities to find solutions; while engaged in the inquiry process, students are scaffolded with learning technologies that help them participate in activities; they create tangible products (artefacts) that address the driving question. Such authentic real-world problems that students use to produce a tangible product over extended periods of time are often called projects [19]. Projects are usually interdisciplinary [4]. Students have to use knowledge of various disciplines as well as different skills: to find appropriate information, to work as a team, to present results in the way that becomes valuable to potential users, etc. However, students do not only apply knowledge; they build new knowledge from a professional domain, while developing transferable skills important for new projects [4].

PjBL cannot be seen as synonymous with project work that follows traditional instruction in such a way that the project serves to provide illustrations, examples or additional practical applications for material taught. Projects are constantly used in engineering education, but these "application" projects are not considered to be instances of PjBL neither are projects in which students learn things that are outside the curriculum. The central activities of the PjBL must involve the transformation and construction of knowledge and activities should be student driven [20]. The teacher's role is to facilitate, advise, guide, monitor, and mentor learners, not just to conduct lectures and laboratory work. Sometimes, the teachers act as instructors to provide direct instruction or give explanatory knowledge or research skills, and sometimes as facilitators, helping learners find resources or resolve problems [12].

1.2 Comparison of IBL, PBL and PjBL

A brief overview of IBL, PBL and PjBL may give an impression that they are quite similar. All the approaches are student-centred, they are all based on constructivist principles and relevant problems, they all emphasize interdisciplinarity, develop self-directed learning skills, demand active involvement of students in the learning process and searching various sources to find appropriate information, they extend over a longer period of time and require the teacher to step out of the traditional role [9]. All of these approaches seem to be more efficient if students work actively in small groups.

A comparison of PBL and PjBL at tertiary level was made by Perrenet, Bouhuijs and Smits [15]. They noted that both are based on self-direction, collaboration, and multidisciplinary orientation. However, they also identified some differences between the two approaches. Projects used in PjBL are closer to professional reality and therefore take a longer period of time than PBL problems. PjBL is more directed to the application of knowledge, whereas PBL is more directed to the acquisition of knowledge. Management of time and resources as well as task and role differentiation is very important in PjBL, while skills for effective search and problem solving activities are important in PBL. Hmelo-Silver, Dunkan and Chinn [21] argue that PBL and IBL are guided approaches, organized around relevant, authentic problems or questions, place heavy emphasis on collaborative learning, but they differ in their origin. Moreover, students in IBL are engaged in investigations and develop specific-reasoning skills, while in PBL students are engaged in inquiry, learn strategies and develop problem solving skills. Although, all three approaches are promotors for inductive teaching and learning where instruction begins with a specific situation, case or problem to observe and after a certain learning procedure with new rules and facts to be generated at the end of the process, they differ in the product at the end as well as on its own history, research base, proponents, as argued by Prince and Felder in [10]. IBL, PBL and PjBL differ in historical aspects, in the main principle, in the instructional procedures and in the outcomes [12]. We summarise the main differences identified by the above-mentioned authors in the Table 1.

Oguz-Unver and Arabacioglu [12] argue that each of the three approaches can be used in all the environments, but the reason why we prefer one approach is determined by particular characteristics of individual approaches. Some authors claim that IBL is an overarching approach that involves other student-centred approaches [8, 9, 10]. However, it is also important to know that these approaches can be adapted for various disciplines and the described differences between them can become less visible.

Table 1. Differences between IBL, PBL, PjBL and their main characteristics

Characteristics	**IBL**	**PBL**	**PjBL**
Origin	Practices of scientific inquiry	Medical education	Engineering education
Context for learning	Question or problem or situation	Ill-structured open real-world problem	Real major project
Emphasis on. . .	Conceptual understanding	Acquiring knowledge	Applying or integrating knowledge
Philosophical aim	Raising questions	Problem solving	Producing a project
Specific learning outcomes	Comprehension of nature of scientific inquiry, critical thinking	Effective problem-solving skills, intrinsic motivation, lifelong learning	Process skills, ability to produce a tangible project/artefact
Key elements	Exploration, raising questions, invention	Prior knowledge activation, elaboration of knowledge	Learning by producing/creating artefacts
Student learning	Individual, group	Small group	Small group
Teacher's role	Leader, facilitator	Facilitator, coach	Facilitator, advisor

All three mentioned approaches can bring forward interdisciplinarity. PjBL projects are usually built around an intersection of topics from two or more disciplines [20]. PBL is a pathway towards interdisciplinary learning that is possible when the identified problems are ill defined and not necessarily situated within a specific scientific paradigm [22]. This can be true also for IBL settled in mathematics because math applications in many different domains are sources of significant questions at various levels of education [14]. Students are expected to develop teamwork skills, ICT skills as well as other transferable skills applicable across disciplines through active involvement in such approaches. Interdisciplinarity is also a trigger for teachers to implement one of these three types of instruction. To be able to talk about interdisciplinary connection through a student-centred approach, mathematics and at least one another school discipline need to be exposed by problems and learned by participants.

1.3 Research Aims

In mathematics education, the traditional approach can be seen to be dominated by theory and not to address the needs of most students [23]. However, there have been calls for reforming mathematics instruction by considering more innovative pedagogical approaches that can bring forward critical thinking, math applicability and interdisciplinary connection with various disciplines [23]. Student-centred approaches such as PBL, PjBL and IBL seem to be a good choice. However, it remains unclear which of these target approaches are implemented in mathematics with interdisciplinary connections and what trends in such learning can be seen from high-quality incorporations. It is therefore very important to review the use of student-centred approaches in the field of mathematics in the twenty-first century.

Trends of empirical evidence on culturally relevant and inquiry-based science education settings were identified [8] and diversity of PBL/PjBL implementations in engineering was posed in the literature review [24]. We have also found a systematic analysis about trends in STEM education of publicly funded projects [25], and trends about innovative approaches in mathematics at higher education in the first decade of the 21st century [23]. In the last article [23] authors conclude, that already published papers about innovative approaches such as PBL or IBL could be a motivating indication for further research investment to contribute to this emerging shift. Hopefully, the present research can fill this gap. In order to find out how these target approaches are implemented worldwide in the field of mathematics and its connection with other subjects such as engineering or science we pose the following research questions (RQ):

RQ1. What trends are revealed in the examined studies of IBL, PBL and PjBL in mathematics and its connections with other subjects?

RQ2. What are the differences in experimental implementations of IBL, PBL and PjBL in mathematics and its connections with other subjects?

For the purposes of incorporation of an approach it is important to take into account studies of various research designs, such as observation design with description of concrete implementation, survey with statements of teachers' beliefs about realisation, case study with details of students' reactions in one realisation, quasi-experimental design with results of students' improvement. Following Brown [8], we therefore decided to search for articles about various research designs in RQ1. These articles need to be published in educational journals with (Social) Science Citation Index-Expended (SSCI or SCI-E) to ensure quality, evidentiary basis and peer-review status.

If at the beginning of educational innovation publications are in form of descriptions of developmental work and case studies, later on experimental studies show that the innovation has become well established and recognized. To identify the differences in experimental implementation of the target approaches in mathematics and its connection with science or STEM we analyse such types of studies in RQ2. Despite good theoretical perspectives it is unclear whether the target approaches are well recognized and effective in the field of mathematics.

Effectiveness of educational approaches is usually measured in meta-analyses. In such studies only quasi-experimental designs with experimental and control groups are analysed and effect sizes can be calculated in order to describe the effectiveness of an approach. Detailed examination of experimental designs with measures of effectiveness of a target approach enables us to form a holistic picture about the differences of IBL, PBL and PjBL in math implementations and current trends. Some existing meta-analyses have analysed PBL [26], computer-based problem-centred approaches in STEM [27], and IBL in mathematics and in science [28] showing that these approaches are more used in science than in math. The impact of learning mathematics through PBL in secondary schools was investigated in [29]. Among 28 articles, connection between math and other disciplines was only detected in one article. In year 2021, there was a meta-analysis published about PBL in mathematics at primary level of education in Indonesia and several other countries [30]. There were 16 studies included in the analyses, but only 4 of them were published in the two articles that are indexed in Scopus database. However, we have not found any systematic review or meta-analysis about trends in all target student-centred approaches in the field of mathematics at all levels of education.

We begin by a brief overview of these approaches and theoretical comparisons between them, after that we describe the methodology of articles' selection in our study and continue by the presentation of the measured characteristics. We analyse trends in IBL, PBL or PjBL implementations in math and its connection with science, and /or engineering and trends in characteristics of such educational implementations in 5-year periods. Moreover, using the subset of experimental studies, we provide a more detailed analysis of differences in interdisciplinary connection of the target approaches.

2. Method

2.1 Search Criteria

We made a selection of high-quality studies to determine trends in using IBL, PBL or PjBL in mathematics and its connection with other subjects and to identify some differences between these approaches in experimental educational research. We started with Web of Science (WoS) journals' database, because this database is freely accessible, and it is the only one that is relevant for research distribution in our institution. Despite one selected database, our main sample was not too small. We searched through all WoS journals' databases for articles with SSCI or SCI-E in the category of Education – Scientific disciplines or Education – Educational research through three 5-year periods: from the start of the year 2003 to the beginning of year 2018, when our study began. The 1st period starts with year 2003, the 2nd period starts with year 2008, the 3rd period starts with year 2013. Year 2003 was a beginning of periods for two reasons: to make possible comparisons of another research with the same period [31], and because STEM acronym was established in 2001, Institute of Education Sciences was established by the Education Sciences Reform Act of 2002 and STEM education research has been mainly administered and managed by this institute since 2003 [25].

The methodology of the systematic search is presented on Fig. 1. The first round of results with words for the target approaches in mentioned period yielded 26508 results. However, before the last round exactly 100 articles with the phrase word "inquiry-based", 44 with "problem-based" and 62 with "project-based" remained for full text reading. Even though the target word "mathematics" was entered, almost half of the articles dealt with the science. In most of the articles that do not deal with math, IBL was used. After full text reading, we excluded 34% of articles that do not deal with the field of mathematics. The reason for this result could be found in an extra keyword "mathematics" in some journals, whenever a statistical analysis was involved. Moreover, 17.6% of the reminding articles do not examine deeply one of the target approaches. In the selected articles, the target approaches are named as problem/project/inquiry based and are thoroughly defined in the studies. The authors promised to involve active participation of self-directed learners in problem-solving processes and the teacher's different role over a longer period of time. However, active learning processes in small groups, the context of learning as well as possible interdisciplinary connections of subjects are not always described in detail to readers. A selection of n = 112 articles was used for further analysis.

2.2 Data Characteristics

We included articles reporting on studies with experimental and non-experimental design. Our coding scheme categorises the characteristics of the target approaches. We used the following coding scheme with 5 categories: Learning subject, Educational level, Research method, Research design, Participants. The category Learning subject is the most important for the study about various subjects' connection implementations through the target approaches. We are also interested in the educational level of implementations and participants in them. Having teachers as participants means that an approach is still in the process of development because they still learn how to implement an approach into practice. The research method and design are needed for creating a subsample of experimental studies for RQ2.

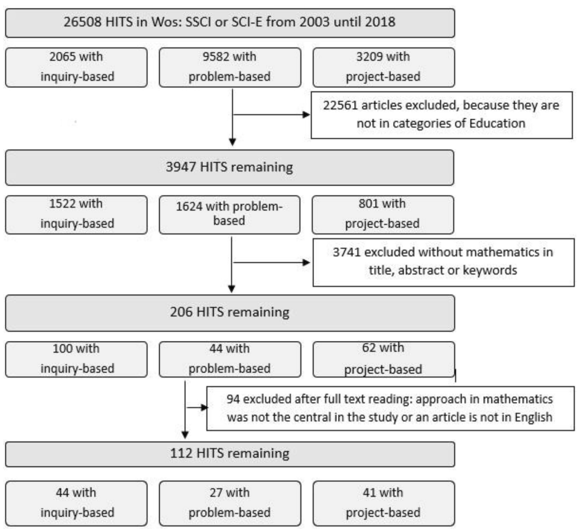

Fig. 1. Literature screening flowchart.

Learning subject. We distinguished between Mathematics (thought independently from other subjects), the connection of Math and science and STEM as a combination of all educational fields: science, technology, engineering and mathematics. If mathematics was connected with any other subject we categorised the article under the category Other.

Educational level. In our categorisation, we refer to the lowest level of education, at which a particular approach is examined, e.g., Elementary level (usually Year 5–11, labelled often as K 6); Middle level (usually Year 11–15, labelled often as K 7–9), Secondary level (usually Year 15–18, labelled often as K 10–12); University level as a tertiary level (usually Year 18 and up), Not clear. Some researchers use an approach at various educational levels. In such cases the article was categorised according to the lowest educational level. For instance, if a connection between mathematics and science was examined at an elementary and a middle level together, we categorised the article into the category Elementary level. If participants were teachers, the level corresponded to the level of the target contexts.

Research method. In our analysis we classified the research methods using three common subcategories: Quantitative, Qualitative and Mixed method. In some rare cases it was not clear which method was used.

Research design. There are many ways to classify research designs. Cohen, Manion and Morrison [32] identify eight main styles of educational design: experiments; quasi-experiments; single-case research and meta-analysis; ex post facto designs; action research, case studies, internet-based research and computer usage; surveys, longitudinal, cross-sectional and trend studies; historical and documentary research; naturalistic and ethnographic research. These research designs can take in quantitative as well as qualitative research, together with small- and large-scale approaches. In our analysis, some of the above-mentioned research designs were not observed in the selected articles. Therefore, the final categorisation included the following design-related subcategories: Experiment with comparative groups (for various (quasi) experimental designs); Experimental one group pre-post design (for an experiment with one group using pre and post-test); Case study, that includes also action research; Survey, that includes also longitudinal studies, internet based research; Descriptive observation, that includes also documentary research; Reviewing other research with various literature reviews and meta-analysis; Not clear, if the design is not clearly presented.

Even though some studies used a combination of method designs, we did not use the term Mixed design. If a study included a (quasi-) experiment with experimental and control groups where each group is analysed with a pre-test and a post-test, we categorised such an article as Experiment with comparative groups and not Experimental one group pre-post design. Instead of treating such articles as two or more separate studies, we categorised such articles in the umbrella category. In some articles, methodology could not be clearly determined, because all paper details were not publicly available.

Participants. Typical participants are: Students, Teachers, Prospective teachers, Students and teachers or Other/not clear, if some other type of participants was analysed, e.g., staff or principals.

We used these five categories for comparison in the results section. The studies were categorised by two independent researchers who classified the articles. The agreement in categories for all articles in the sample was 89.3%. After more detailed reading and discussion about unequal categorisation the consensus was made between both researchers. We analysed trends in 5-year periods: the 1st period from year 2003, the 2nd period from year 2008 and the 3rd period from year 2013 and performed statistical χ^2 tests to verify if the described category and the examined periods are dependent variables. We used SPSS for Windows and rejected all the hypotheses at significance level of $\alpha = 0.05^*$ or $\alpha = 0.01^{**}$.

Categories Research method and Research design were used further on for creation of a subsample with experimental design studies where effect size is or can be measured. We analysed these studies in more detail to check trends in interdisciplinary connections and differences in such interdisciplinary connections. The results are summarized in the next paragraphs.

3. Results

Looking through the WoS database, 112 articles were found to examine one of the target approaches in the field of mathematics or its connection with science or engineering in most cases. These articles were published in 50 various educational journals. The most often journal outlets are listed in Fig. 2. *Eurasia journal of mathematics, science and technology education* features 10 articles, while *International journal of science and mathematics education* features 7 articles. The next tree journals in Fig. 2 are non-mathematical, among them is *International journal of engineering education*. Five journals published 4 such articles, among them are *Educational studies in mathematics* and *Mathematical thinking and learning*. One journal published 3 articles,

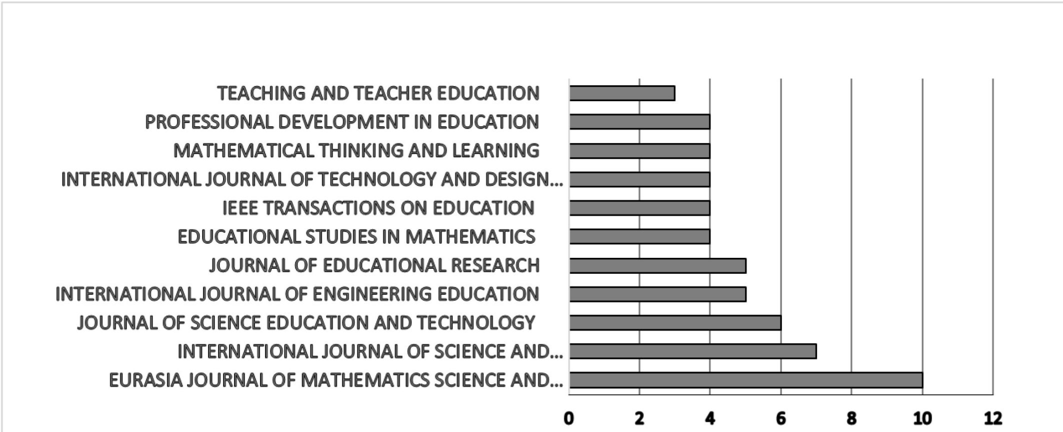

Fig. 2. WoS journals with at least three articles with IBL, PBL or PjBL in mathematics.

followed by twelve journals with 2 articles and the remaining journals with only one such article.

3.1 Trends of the Target Approaches in the Field of Mathematics and its Connection with Other Subjects

The aim of this subsection is to identify answers to RQ1. In the field of mathematics, IBL is used in 44 articles, PBL in 27 articles and PjBL is used in 41 articles. There are 33 of the target approaches that examine mathematics only, 27 of them deal with math and science, 48 approaches examine STEM, and 4 of them either combine math with social science or with computer science, or the combination is not clear. We divided all 112 articles in three 5-year periods. As shown on Fig. 3, there are considerable differences in the frequency of articles that examine one of the target approaches in math across 5-year periods: 4 (3.6%), 26 (23.2%) and 82 (73.2%) respectively. Based on the raising frequencies it seems that these target approaches' realisations in school mathematics increased over the measured 5-year periods worldwide.

There are not many differences in frequencies between IBL and PjBL implementations through periods (Fig. 3). However, PBL dominate only in the first period. It is very interesting to note that mathematics (without other subjects) is the most frequently examined in the first period, math in combination with science is mostly studied in the second period, and STEM is the most popular in the third period. These frequencies indicate that the number of connected subjects in these approaches increased over the target 5-year periods. There are minor differences in the educational levels, at which the approaches were used. We can observe in Fig. 3 that average educational level slowly raises through the periods. In the first and the last 5-year period, quantitative and qualitative methods are used equivalently, while qualitative methods slightly prevailed in the second period. Case study is the main research design in all 5-year periods and students are the main type of participants in all three 5-year periods.

In each measured category, the frequency was found to increase over the periods except for prospective teachers as participants that are used most often in the second period as shown in Fig. 3; this is a consequence of growing frequency of articles in each period. This led us to study the changes in percentages within the individual subcategories. Taking these percentages into account (see the shares in the shaded parts of columns in Fig. 3), IBL and PjBL were found to increase over the time due to a decrease in the use of the PBL approach. Percentages in STEM integration increase over the time due to subcategory Mathematics. There are no deviations in category Educational level as well as in Research methods over the 5-year periods. In category Research design, experimental one group pre-post design and reviewing other research have increased in comparison to other mentioned designs. Finally, students and teachers together as participants become increasingly prominent. However, if we merge first period with very low frequency and second period together and provide statistical tests of independency, only the category Learning subject depends on periods significantly ($n = 108$, $\chi^2 = 17.615$, $p = 0.000$**). Although the category Participants is also related to the chosen periods ($n = 98$, $\chi^2 = 8.941$, $p = 0.030$*), a quarter of theoretical frequencies do not reach an adequate level more than 5. Educational approach ($n = 112$, $\chi^2 = 1.936$, $p = 0.380$), Educational level ($n = 104$, $\chi^2 = 4.471$, $p = 0.215$), Research method ($n = 107$, $\chi^2 = 0.526$, $p = 0.769$) or Research design ($n = 105$, $\chi^2 = 7.375$, $p = 0.117$) used in the studies do not depend on periods significantly. Moreover, a quarter of theoretical frequencies for the statistical test about Research design do not reach an adequate

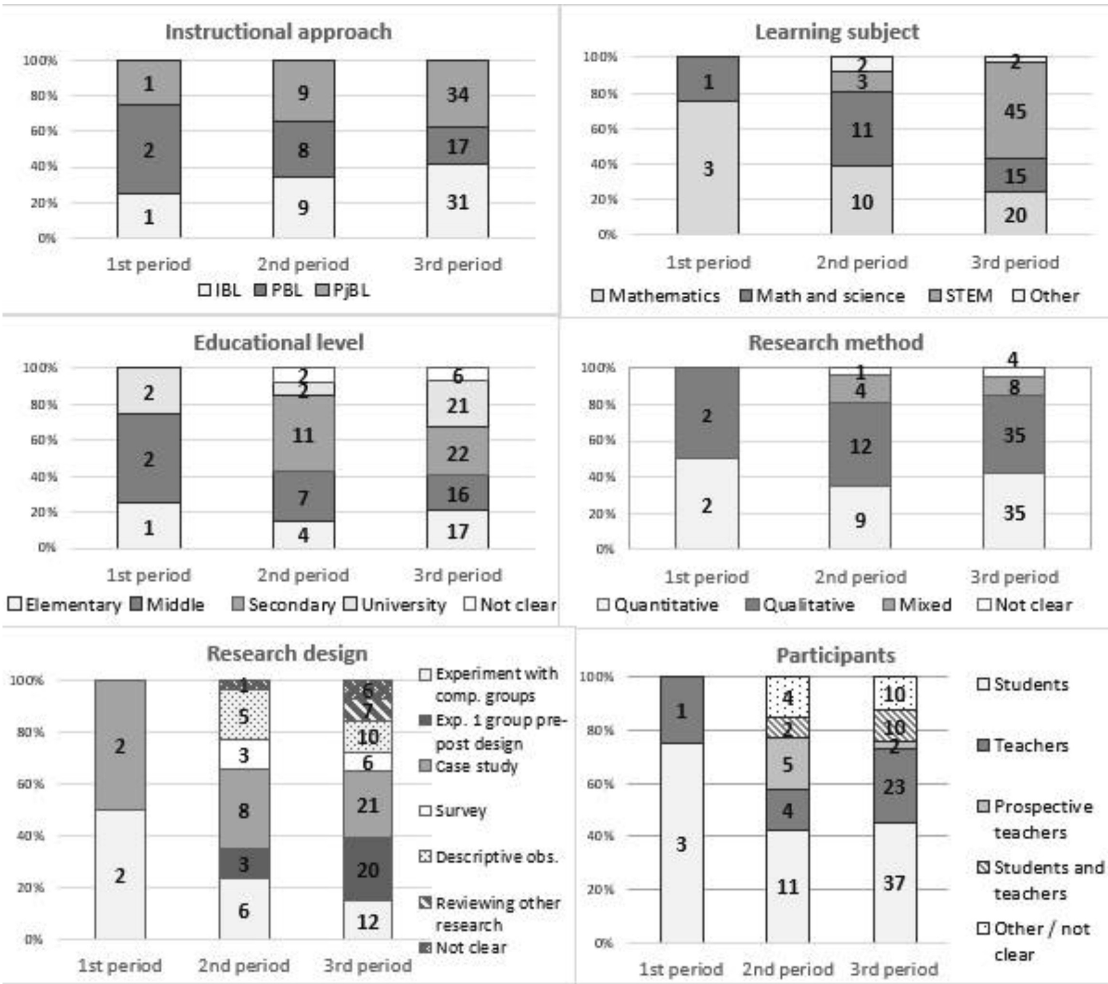

Fig. 3. Trends in measured categories of IBL, PBL and PjBL in the examined studies.

level more than 5. Therefore, we check only experimental designs (experimental one group pre-post design and experiment with comparative groups), and these designs depend on related periods ($n = 43$, $\chi^2 = 4.083$, $p = 0.043$*).

These results highlight the following changes: increase of PjBL and IBL approach due to decrease of the use of PBL approach; increase of STEM subjects' connection due to mathematics with no subject connection; increase of one group pre-post design and reviewing other research due to other mentioned designs; and increase of students and teachers together as participants due to involvement of prospective teachers. Among these perceived differences only the trend in subject connection has been confirmed with a statistical test, while the trends in design as well as in participants used in research have been verified with restricted statistical tests. Analysing a subsample of experimental designs in more detail, we can probably find relationship between the chosen experimental designs and examined 5-year periods.

3.2 Comparison of the Target Approaches with Experimental Design

In this subsection we focus on studies from our sample with experimental design, where effect size is or can be measured, thus trying to analyse the RQ2. There are 43 such articles shown in Fig. 3 with Experiment with comparable groups and Experimental one group pre-post design. However, in one article PBL and IBL are examined together, and in another article PBL and PjBL are used together. Therefore, a subsample with $n = 41$ articles (studies) is examined in detail.

The distribution of articles across 5-year periods is very similar as in the main sample: 2 (4.8%), 9 (22%), and 30 (73.2%) respectively. In the first period both studies examined PBL, while in the next period all approaches have almost the same share of occurrence. In the third period PBL has the lowest frequency. In the first and second period, mathematics is examined sole or with the science, while STEM subjects' connection "explode" in the third period. Most STEM connections are used with PjBL approach (Table 2). The amount of one group pre-post design is much bigger in the third period than in the first and second periods together which was detected also in the previous statistical test. In such studies only one group of participants is examined in detail to verify a development of participants' knowledge, skills or attitudes through one of the target student-centred approaches. Median for the number of participants in groups is 52 for the first two periods and 66 for the third one. Time of examination is 24 weeks for the first two periods and 9 weeks for the third period. Data in Table 2 show that the time shorter than a week is used for IBL examination (with one exception). Characteristics of experimental studies substantiate results in the previous Section 3.1. Moreover, in all studies discussing comparison of IBL/PBL/PjBL and a more traditional approach, authors claim that innovations are more or at least equally effective than the traditional approach. However, the effectiveness of the target approaches through meta-

Table 2. Characteristics of experimental studies from the subsample

Period	Approach	Subject	Level	Design	Method	Time-weeks	Participants	Number
1	PBL	Math	Primary	Comparative groups	QUN	6	students	50
1	PBL	M+Sci	Uni	Comparative groups	QUN	112	students	?
2	IBL	Math	High	Comparative groups	QUN	0.01	students	120
2	PBL	Math	Primary	Comparative groups	QUN	56	students	179
2	PBL	Math	Uni	Comparative groups	QUN	24	prospective t.	40
2	PBL	Math	Primary	Comparative groups	QUN	4	students	28
2	PjBL	Math	High	Pre-Post	QUN	4	students	32
2	PjBL	M+Sci	Middle	Pre-Post	MIX	24	prospective t.	24
2	IBL	M+Sci	Middle	Comparative groups	MIX	56	students	67
2	PjBL	M+Sci	High	Pre-Post	MIX	24	prospective t.	65
2	PBL	M+Sci	High	Comparative groups	QUN	12	mix	36
3	IBL	Math	Uni	Comparative groups	QUN	112	mix	902
3	IBL	Math	Primary	Comparative groups	QUN	0.29	teachers	24
3	IBL	Math	Middle	Comparative groups	QUN	56	mix	60
3	PBL	Math	High	Comparative groups	QUN	4	students	167
3	PjBL	Math	High	Comparative groups	MIX	112	students	532
3	PBL	Math	Primary	Pre-Post	QUN	22	students	35
3	IBL	M+Sci	Primary	Pre-Post	QUN	112	mix	24
3	IBL	M+Sci	Middle	Pre-Post	MIX	2	teachers	49
3	IBL	M+Sci	Uni	Pre-Post	QUN	28	prospective t.	79
3	IBL	M+Sci	High	Pre-Post	QUN	16	mix	368
3	IBL	M+Sci	High	Pre-Post	QUN	9	teachers	25
3	IBL	M+Sci	Primary	Comparative groups	MIX	2	teachers	413
3	IBL	STEM	Primary	Pre-Post	QUN	0.86	teachers	36
3	PjBL	STEM	Uni	Pre-Post	MIX	5	students	30
3	IBL	STEM	Middle	Comparative groups	QUN	0.14	students	8
3	IBL	STEM	Middle	Comparative groups	QUN	0.14	students	278
3	IBL	STEM	Uni	Pre-Post	QUN	2	students	72
3	IBL	STEM	Uni	Pre-Post	QUN	3	students	85
3	PjBL	STEM	High	Comparative groups	QUN	168	students	1854
3	PjBL	STEM	Middle	Pre-Post	QUN	9	students	333
3	PBL	STEM	High	Pre-post	QUN	8	students	90
3	PjBL	STEM	High	Comparative groups	MIX	168	mix	120
3	PBL	STEM	Middle	Pre-Post	QUN	2	students	48
3	PjBL	STEM	High	Comparative groups	QUN	224	teachers	80
3	PjBL	STEM	High	Pre-Post	QUN	0.71	students	205
3	PjBL	STEM	Middle	Pre-Post	MIX	15	teachers	29
3	PjBL	STEM	High	Pre-Post	MIX	6	students	60
3	PBL	STEM	Uni	Pre-Post	QUN	24	students	20
3	PjBL	STEM	Uni	Pre-Post	QUN	24	prospective t.	60
3	PjBL	STEM	High	Pre-Post	QUN	12	students	39

analysis cannot be assessed due to incomparable methodologies of its calculation.

Detailed analysis of the 12 *mathematical* studies in the subsample with no connections with other subjects lead us to the conclusion that half of them favour examination *of PBL* (Table 2). In these studies, a classical *experiment with comparative groups* is mostly used; there are only two one group pre-post designs. Researchers that examine a student-centred approach in mathematics usually use *students* as participants. Median number of participants is 55, their educational level is between middle and secondary, and median time of examinations is 14 weeks. Median year of publication for mathematical studies is 2013.

Data of 11 studies about *math and science connection* in Table 2 show that *IBL* is mostly examined approach (7 studies), while the same share belong to PBL and PjBL. *Teachers* as participants dominate in these studies, there are only 2 studies with students as participants. *One group pre-post design* is used in 7 studies. Median number of participants is 57, educational level of math and science examinations is secondary level, and median time of examinations is 24 weeks. On the other hand, except for one, all student-centred approaches in the subsample with the time of implementation less than one week belong to IBL. Median year of publication is 2014.

Analysis of 18 *STEM* studies in the subsample lead us to conclusion that 10 of them favour examination *of PjBL* (Table 2), 5 IBL and 3 PBL. Almost two thirds of the studies use *one group pre-post design*, and more than two thirds use *students* as participants. Median number of participants is 66, educational level of STEM examinations is secondary level, and median time is 7 weeks. Median year of publication is 2016. Perceived differences in high-quality experimental implementations in mathematics as one discipline, in math with science connection and in all STEM disciplines' connection cannot be verified with statistical tests because the number of high-quality experimental studies is not big enough.

4. Interpretation and Discussion

Our findings are based on literature review of high-quality educational journals. In a total of 112 studies about IBL, PBL or PjBL from 50 various WoS journals, PBL is represented a little bit less frequently than IBL and PjBL. In the target approaches, mathematics is most often examined together with STEM subjects; less frequently, it is examined as a sole subject or together with science. A connection of mathematics with another subject is rare.

The division of the selected articles in 5-year periods offers an outline of trends in the use of the target approaches. To sum up, the number of articles examining IBL, PBL or PjBL has increased significantly over the selected 5-year periods. Growth is visible in all the approaches and all categories (except in one subcategory), which is natural, because the number of publications has increased over the years. For that reason, our analysis is centred on the growth over 5-year periods, expressed in percentages and verified with statistical tests of independency. Trends such as involving all STEM subjects through the target approaches, increasing diversity in research design and increasing diversity in participants were verified in our analysis and this is in line with increasing diversity of PBL/PjBL in engineering [24]. Firstly, STEM has increased significantly over the observed periods. This means that math integration not only with science but also with engineering and technology penetrates from reforming theory into practical implementations through the target approaches. Given that STEM subjects' connection is mostly examined with PjBL approach, it is not surprising that the number of PjBL studies has been growing. PjBL seems to be the most suitable for interdisciplinary connection of all STEM disciplines. However, more data are needed for statistical confirmation. Secondly, in the category Research design, experimental one group pre-post design and reviewing other research have increased in comparison to other mentioned designs. It is not surprising that a subcategory Reviewing other research has increased in comparison to other designs, because a certain time is needed for innovations' implementation before a review process is possible. On the other hand, experimental one group pre-post design has become a current trend. Despite many criticisms, recent statistical methodologies have started to allow the measurement of effect size in such designs [33]. Moreover, implementation of such design in practice is easier than implementation of a classical experiment with comparative groups. Finally, using students and teachers together as participants instead of prospective teachers is observed trend that indicate transfer from learning how to implement these approaches to implementing them into practice.

More detail analysis of experimental designs gives us insight of student-centred implementations in mathematics, math with science and overall STEM connection. *Mathematics* as a sole subject is typically examined through PBL with an experiment of comparative groups using students as participants such as for instance in study from year 2009 [34]. Students' knowledge and/or problem solving skills are usually compared in such rigorous experiment with comparative groups that is often difficult to realise in school setting. Interdisciplinary connection can be hidden in a real engineering context of given problems that trigger learning of mathematical content [2]. *Math with science* is typically examined through IBL with experimental one group pre-post design using teachers as participants such as for instance the study from 2014 [35]. In this study teachers of a professional development program learn how to teach interdisciplinary IBL math and science contents. Overall *STEM* integration is typically examined through PjBL with experimental one group pre-post design using students as participants such as for instance in the study from 2017 [36]. Students' attitudes toward STEM subjects' connection are measured in this study that shows increased interest towards STEM subjects and career after STEM PjBL. Integration of all STEM subjects is mostly used at higher levels of education to verify if students have improved their knowledge, skills or have changed attitudes through such interdisciplinary way of learning. However, we need to point out that in this research a power of interdisciplinary connection of subjects is not analysed. In some examined STEM connection mathematics has a minor role (e.g., [37]), while in some other studies mathematics has an equal role in learning as other mentioned subjects (e.g., [3]). To find out about this kind of integration of subjects, more research on students' math knowledge in STEM connection is needed in the future.

In WoS journals, STEM PjBL studies with comparative groups are rare; in our selection only 3 such studies are detected (visible in Table 2). One study pointed to difficulties in teachers' cooperation through PBL STEM interdisciplinary teaching without attending professional development program [3], in another two studies STEM PBL benefitted low performing students to a greater extent than middle and high performing students [38] and showed better math and science improvement of students with greatest fidelity of STEM PBL implementations with those of the lowest fidelity [39]. These two studies were carried out in the three Texas schools where teachers attended three years of sustained professional development program. These studies indicate that STEM PjBL interdisciplinary teaching is effective if teachers have a proper support of pedagogy training.

However, interdisciplinary connection of math, engineering, technology and science is a challenge for teachers: to prepare a good context that triggers learning of various disciplines, to synchronise different requirements of teachers, to incorporate activities into the curriculum at the right time and, of course, to adequately assess the knowledge of all included subjects in such interdisciplinary teaching [3]. Despite positive attitudes and desire to use such instructional strategies in math, the actual implementation often turns out to be more difficult and time consuming for many math teachers teaching at the different levels of education [40]. Math teachers need careful preparation for interdisciplinary teaching, (institutional or collegial) support in designing problems, enough time for interdisciplinary activities and teamwork skills for productive cooperation with engineering or science teachers. Although the review of PBL/PjBL in engineering revealed diversity in levels of implementation, a lack of pedagogy training is a common challenge [24]. IBL seems to be more flexible than PBL/PjBL with the time of implementation and the way of providing problem solving activities. That's why IBL might be a good choice for some novice teachers in using student-centred approaches. This might encourage them to continue student-based practice and gradually incorporate more rigorous approach with longer period of realisation. Good practice of STEM PBL in Texas schools can stimulate teachers to use PjBL in STEM interdisciplinary learning.

It is important to emphasize that it is not fair to compare the effectiveness of PBL approach on mathematical knowledge (compared to the more traditional approach), effectiveness of interdisciplinary math and science IBL teaching on teachers' development and effectiveness of STEM PjBL approach on students' skills/attitudes/knowledge development. However, perceivable differences of student-centred approaches in math, math with science connection and all the STEM subjects' connections can give us directions for future research.

4.1 Limitations

Our study has some limitations that need to be exposed. Our analysis was based on various articles from WoS database that examine the target approaches in the field of mathematics and its connection with other subjects. Choosing more databases would enlarge our sample but research would be even more time consuming, and the level of articles' quality could become lower. Secondly, using our selection of non-experimental and experimental studies, we could observe the target approaches' popularity in mathematics, while their effectiveness has not been measured. Thirdly, our research was limited only to the articles, retrieved through the search phrases "inquiry-based", "problem-based", or "project-based" that are used in acronyms. By adding some other search word phrases, we could enlarge our sample of studies. However, excluding these search phrases did not affect the comparison between IBL, PBL or PjBL approaches. We do not believe that a bigger sample would result in significant changes in results.

Finally, having started our research at the beginning of 2018, we included six articles published in this year although a 5-year period would elapse at the end with previous year (which is the year of article's acceptation).

5. Conclusion

This research shows that student-centred approaches are used worldwide in the field of mathematics to offer interdisciplinary connection between mathematics and engineering or science. Diversity of good practices is identified through WoS publications of IBL/PBL/PjBL approaches in various educational levels and in participants which is a sign that these approaches are recognized and well established in the field of mathematics education. Integration of all STEM subjects through these approaches is a current trend. No negative consequences on students have been detected in such implementations published in WoS journals. However, the number of such publications in mathematics education is lower than in science education. It seems that pedagogical innovations penetrate into math discipline more slowly than into other STEM disciplines.

STEM rapid development can help introduce ideas for exploring how mathematics can be taught and learned. Integration of all STEM subjects is mostly used with PjBL approach at higher levels of education when students are old enough to work in teams in interdisciplinary projects. In WoS journals PjBL STEM practice is mostly analysed with experimental one group pre-post design and enable students to improve their knowledge, skills or change their attitudes toward engineering career. These students' improvements are visible in all the analysed studies where teachers are prepared for such interdisciplinary teaching activities. Such STEM connection is considered as the best potential for interdisciplinary teaching and learning in student-centred mathematics education. The benefits of such STEM connection are naturally expected for mathematics as a knowledge base as well as for engineering as a field with wide range of knowledge applications. However, the impact of such approach in comparison to other learning approaches stays unclear because proper experimental realisations with comparable groups are rare. More high-quality experimental studies about STEM integration are needed in the future.

References

1. Y. Li and A.H. Schoenfeld, Problematizing teaching and learning mathematics as "given" in STEM education, *International Journal of STEM Education*, **6**(44), 2019.
2. A. Drobnič Vidic, Impact of problem-based statistics course in engineering on students' problem-solving skills, *International Journal of Engineering Education*, **27**(4), pp. 885–896, 2011.
3. A. Drobnič Vidic, Teachers' beliefs about STEM education based on realisation of the "Energy as a Value" project in the Slovenian school system, *International Journal of Engineering Education*, **33**(1B), pp. 408–419, 2017.
4. E. De Graaff and A. Kolmos, Characteristics of problem based learning, *International Journal of Engineering Education*, **19**(5), pp. 657–662, 2003.
5. B. Jessen, M. Doorman and R. Bos, *Priročnik MERIA za poučevanje matematike s preiskovanjem/MERIA (Practical Guide to Inquiry Based Mathematics Teaching*), Zavod Republike Slovenije za šolstvo: Ljubljana, 2017.
6. D. C. Edelson, D. N. Gordin and R. D. Pea, Addressing the challenges of inquiry-based learning through technology and curriculum design. *Journal of the Learning Sciences*, **8**(3–4), pp. 391–450, 1999.
7. K. Engeln, M. Euler and K. Maass, Inquiry-based learning in mathematics and science: a comparative baseline study of teachers' beliefs and practices across 12 European countries, *ZDM Mathematics Education*, **45**, pp. 823–836, 2013.
8. J. C. Brown, A metasynthesis of the complementarity of culturally responsive and inquiry-based science education in K-12 settings: Implications for advancing equitable science teaching and learning, *Journal of Research in Science Teaching and Learning*, **54**(9), pp. 1143–1173, 2017.
9. R. Spronken-Smith, Experiencing the process of knowledge creation: The nature and use of inquiry-based learning in higher education, *The Journal of Geography in Higher Education*, **2**, pp. 183–201, 2012.
10. M. Prince and R. Felder, Inductive teaching and learning methods: Definitions, comparisons, and research bases, *Journal of Engineering Education*, **95**(2), pp. 123–138, 2006.
11. K. Maass and M. Artigue, Implementation of inquiry-based learning in day-to-day teaching: a synthesis, *ZDM Mathematics Education*, **45**, pp. 779–795, 2013.
12. A. Oguz-Unver and S. Arabacioglu, A comparison of inquiry-based learning (IBL), problem-based learning (PBL) and project-based learning (PjBL) in science education, *Academia Journal of Educational Research*, **2**(7), pp. 120–128, 2014.
13. National Research Council (NRC), *Inquiry and the National Science Education Standards: A Guide for Teaching and Learning*, Washington, DC: National Research Press, 2000.
14. M. Artigue and M. Blomhøj, Conceptualizing inquiry-based education in mathematics, *ZDM Mathematics Education*, **45**, pp. 797–810, 2013.
15. J. C. Perrenet, P. A. J. Bouhuijs and J. G. M. M. Smits, The suitability of problem-based learning for engineering education: theory and practice. *Teaching in Higher Education*, **5**(3), pp. 345–358, 2000.
16. D. R. Woods, *Problem-based Learning: How to Gain the Most from PBL*, Hamilton: McMaster University, USA, 1994.
17. G. R. Norman and H. G. Schmidt, The psychological basis of problem-based learning: a review of the evidence, *Academic Medicine*, **67**(9), pp. 57–565, 1992.

18. D. Boud and G. Feletti. *The Challenge of Problem-based Learning*, Kogan Page: London, 1998.
19. J. S. Krajcik and P. Blumenfeld, Project-based learning, In Sawer, K. (ed.), *The Cambridge Handbook of the Learning Sciences*, Cambridge University Press: New York, 2006.
20. J. W. Thomas, *A Review of Research on Project-based Learning*, San Rafael, CA: Autodesk Foundation, 2000.
21. C. Hmelo-Silver, R. G. Duncan and C. Chinn, Scaffolding and achievement in problem-based and inquiry learning: a response to Kirschner, Sweller, and Clar (2006), *Educational Psychologist*, **42**(2), pp. 99–107, 2007.
22. A. A. Jensen, O. Ravn and D. Stentoft, Problem-based projects, learning and interdisciplinarity in higher education, In: Jensen A., Stentoft D., Ravn O. (eds): *Interdisciplinarity and Problem-based Learning in Higher Education, Innovation and Change in Professional Education*, Vol. 18. Springer: Cham, 2019.
23. M. Abdulwahed, B. Jaworski and A. R. Crawford, Innovative approaches to teaching mathematics in higher education: a review and critique, *Nordic Studies in Mathematics Education*, **17**(2), pp. 49–68, 2012.
24. J. Chen, A. Kolmos and X. Du, Forms of implementation and challenges of PBL in engineering education: a review of literature, *European Journal of Engineering Education*, **46**(1), pp. 90–115, 2021.
25. Y. Li, K. Wang, Y. Xiao, J. E. Froyd and S. B. Nite, Research and trends in STEM education: a systematic analysis of publicly funded projects, *International Journal of STEM Education*, **7**(17), 2020.
26. M. Demirel and M. Dağyar, Effects of problem-based learning on attitude: A meta-analysis study, *Eurasia Journal of Mathematics, Science and Technology Education*, **12**(8), pp. 2115–2137, 2016.
27. B. Belland, A. Walker and N. J. Kim, A Bayesian network meta-analysis to synthesize the influence of contexts of scaffolding use on cognitive outcomes in STEM education, *Review of Educational Research*, **87**(6), pp. 1042–1081, 2017.
28. A. W. Lazonder and R. Harmsen, Meta-analysis of inquiry-based learning: Effects of guidance, *Review of Educational Research*, **86**(3), pp. 681–718, 2016.
29. N. Mustaffa, Z. Ismail, Z. Tasir and M. N. H. M. Said, The impacts of implementing problem-based learning (PBL) in mathematics: A review of literature, *International Journal of Academic Research in Business and Social Sciences*, **6**(12), pp. 490–503, 2016.
30. S. Suparman, M. Tamur, Y. Yunita, T. T. Wijaya and S. Syaharuddin, Using problem-based learning to enhance mathematical abilities of primary school students: A systematic review and meta-analysis, *Jurnal Teori dan Aplikasi Matematika*, **5**(1), pp. 144–161, 2021.
31. A. Drobnič Vidic, Comparison of problem-based and inquiry-based learning in math education, *Proceedings of the 2nd International Conference of Engineering, Science and Mathematics Education*, pp. 125–145, The second International Conference on Mathematics, Science and Engineering Education, 8–10. November 2019, North Cyprus.
32. L. Cohen, L. Manion and K. Morrison, *Research Methods in Education (6th ed.)*, Routledge Publishers: Oxford, 2007.
33. A. Bakker, A. Cai, L. English, G. Kaiser, V. Mesa and W. Dooren, Beyond small, medium, or large: points of consideration when interpreting effect sizes, *Educational Studies in Mathematics*, **102**, pp. 1–8, 2019.
34. M. Cotič and M. Valenčič Zuljan, Problem-based instruction in mathematics and its impact on the cognitive results of the students and on affective-motivational aspects, *Educational Studies*, **35**(3), pp. 297–310, 2009.
35. C. Lotter, J. A. Yow and T. T. Peters, Building a community of practice around inquiry instruction through a professional development program, *International Journal of Science and Mathematics Education*, **12**, pp. 1–23, 2014.
36. E. H. Mohd Shahali, L. Halim, M. S. Rasul, K. Osman and M. A. Zulkifeli, STEM learning through engineering design: Impact on middle secondary students' interest towards STEM, *Eurasia Journal of Mathematics, Science and Technology Education*, **13**(5), pp. 1189–1211, 2017.
37. L. Hirsch, S. Berliner-Heyman and J. L. Cusack, Introducing middle school students to engineering principles and the engineering design process through an academic summer program, *International Journal of Engineering Education*, **33**(1), pp. 398–407, 2017.
38. S. Han, R. Capraro and M. M. Capraro, How science, technology, engineering, and mathematics (STEM) project-based learning (PBL) affects high, middle, and low achievers differently, *International Journal of Science and Mathematics Education*, **13**, pp. 1089–1113, 2015.
39. R. Capraro, M. M. Capraro, J. J. Scheurich, M. Jones, J. Morgan, K. S. Huggins, S. Corlu, R. Younes and S. Han, Impact of sustained professional development in STEM on outcome measures in a diverse urban district, *The Journal of Educational Research*, **109**(2), pp. 181–196, 2016.
40. R. S. Al Said, X. Du, H. Abdelkarim, H. M. ALKhatib, M. H. Romanowski 1, A. I. I. Barham, Math teachers' beliefs, practices, and belief change in implementing problem based learning in Qatari primary governmental school, *Eurasia Journal of Mathematics, Science and Technology Education*, **15**(5), 2019.

Andreja Drobnič Vidic is an associate professor at Faculty of Mathematics and Physics, University of Ljubljana (UL), Slovenia. She received her master's degree in Mathematics at Faculty of Mathematics and Physics and her Doctorate degree in Pedagogy at the Faculty of Arts from UL with dissertation about problem-based learning in engineering statistics. Her interests are new trends in mathematics education and engineering education, teaching and learning engineering statistics and mathematics, and applicable statistics in safety engineering. She received an IASE Excellence Award for the best paper by an early career author at the eight International Conference on Teaching Statistics (ICOTS8), introducing PBL in teaching statistics for future engineers. ICOTS conferences, held by International Association for Statistical Education (IASE) every 4 years, are the most important events on the international statistics education calendar. She is a member of the curriculum-development group of mathematics for schools in Slovenia, whose goal is, among other things, to include innovative teaching methods in the curriculum and connect mathematics with other school subjects.