International Journal of Engineering Education Vol. 40, No. 1, pp. 69–74, 2024
Printed in Great Britain

0949-149X/91 $3.00+0.00

Inverted Pendulum Projects in Controls Education: A Five-Year Journey*

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This paper presents the iterative design of a novel cart/pendulum system for feedback control education. Because the system is inexpensive, students can be allowed to take it home, allowing them to interact with it at any time. In order for the system to enhance student learning, multiple hardware, software, and modeling issues had to be overcome. The final version of the project seems to enhance student learning and help students appreciate control theory.

Keywords: feedback control; inverted pendulum; pedagogy; project-based learning

1. Introduction

Dynamic systems and feedback control courses can be intimidating, mathematically intensive, and abstract. As such, these courses could particularly benefit from engaging pedagogies such as hands-on experiments and projects [1, 2]. In light of flipped courses and the impact of the pandemic on education, it can also be beneficial to have low-cost experiments that students can take home [3–6]. Take home or student-owned experiments have a number of potential benefits: students can engage with them at all hours, on campus lab space is not required, etc.

This paper tells the story of the iterative development of a low-cost, engaging project in a first course in dynamic systems and feedback control. Designing these projects/experiments poses several challenges. Affordability is a big constraint. Additionally, the projects should not be too hard or too easy.

Another challenge in designing feedback control experiments is that the control logic needs to be executed at hard real-time intervals on the order of 1–2 milliseconds. There are also many sensor and actuator mechatronic details that need to be handled (analog-to-digital conversion, reading encoders, creating PWM signals, etc.). It is difficult to ensure real-time execution and perform the other signal processing needs using a standard computer operating system. This has lead to multiple commercial systems for real-time feedback control, but most of these are too expensive for student owned or take home experiments. A low-cost alternative is to use a micro-controller, but writing micro-controller C code to implement real-time feedback control can impede student learning.

In feedback controls courses, it is challenging to create projects that do not accidentally devolve into PID tuning with little connection to control theory or modeling. PID tuning projects can still teach some important feedback control concepts such as the ability of feedback to alter the dynamics of a system. However, PID tuning projects do not increase students' depth of understanding of many other important control design concepts. Because the inverted pendulum problem is open-loop unstable, PID tuning is difficult and frustrating. Accurate modeling reveals that the cart/pendulum plant powered by a DC motor has a zero at the origin, so that it cannot be stabilized without an integrator term. This is fairly counter-intuitive and further complicates trying to solve this problem using only PID tuning. What really brings this project together in terms of student learning and engagement is that a decent model combined with root locus control design leads the students to a good starting point for their PID control gains. Feedback control theory and dynamic system modeling allow them to solve a problem that is very difficult to solve using just trial and error. This provides a solid and convincing answer to the age-old question, "Why do we have to learn this stuff?".

The first several versions of this project were poorly received by students and did not significantly contribute to student learning. The project was ultimately successful when hardware problems were fixed, a useful model of the dynamic system was developed, and software was created that facilitated student success. The final version of the project helped students make connections between root locus theory and successfully balancing an inverted pendulum.

2. Background and Introduction

In the fall of 2017, a colleague challenged me to do a project involving self-balancing robots in my dynamic systems and control course. This colleague does not work in the area of dynamic systems or

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* Accepted 21 November 2023.

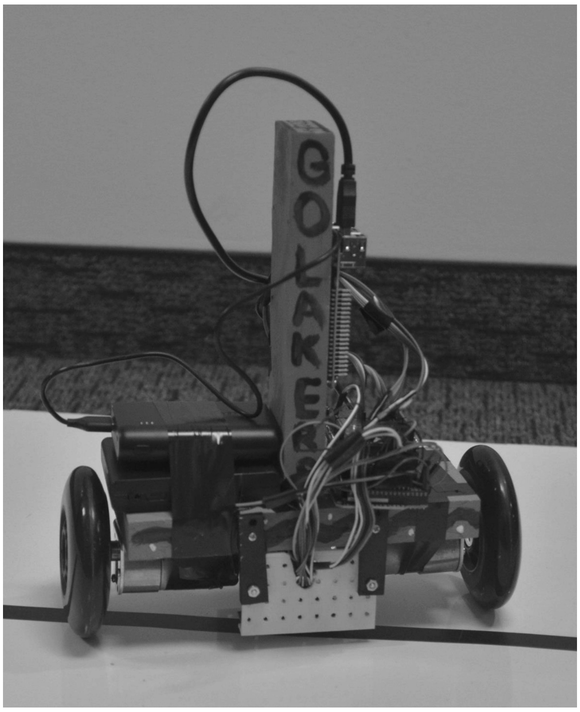

Fig. 1. A self-balancing robot built by students in the first year that this project was assigned.

feedback controls. The project is basically an implementation of the classical inverted pendulum problem. At the time there were many “makers” posting videos on the internet of successfully creating these robots. Since I assumed that I had a deeper knowledge of dynamic systems and feedback control than the makers posting the videos, I was sure that the project would be relatively easy and could be a great vehicle for student learning. But I fell on my face (metaphorically) as I watched my students’ robots fall over (literally). And so, I began a five-year journey toward teaching my students to solve an inverted pendulum project and to give them the tools they would need to succeed.

The first two years of this project involved two-wheeled, self-balancing robots as shown in Fig. 1. The robot uses a gyroscope/accelerometer to estimate its angle of orientation/tilt and then the robot should make the necessary adjustments to keep itself vertical. A Raspberry Pi was used as a small desktop computer to allow wireless remote login to the robot. An Arduino was used to implement the real-time feedback control logic. The Arduino and the Raspberry Pi communicated via a USB serial connection.

=Watching students struggle with this project the first two times it was assigned led to several questions:

- Can the tilt angle estimate based on the gyroscope/accelerometer be trusted?
- Are we able to correctly zero the angle and build well balanced robots so that when the robot is measured to be vertical, balancing it should be reasonably easy?
- How do we develop a model of these robots that captures the essential dynamics and facilitates control design while using only modeling techniques that are appropriate to a first course in system dynamics?

3. Course and Project Description

This project is part of a first course in dynamic systems and control taught at the junior level. The course is three credits of lecture and one credit of lab. This project is completed during the last 5–6 weeks of lab (rougly one third of the toal lab sections). This is the only project in the course.

The project consists of three separate events: – vibration suppression of the pendulum – line following while also suppressing vibrations – balancing the pendulum in the unstable position (vertically upward).

Students are expected to use root locus to design there feedback controllers.

3.1 Learning Outcomes

Through this project, students will learn:

- how to use root locus to design a control system and implement it on a physical system;
- how the pole locations predicted by the root locus relate to the response of the system;
- that feedback control can alter the response of a dynamic system;
- that feedback control can stabilize an unstable system.

4. The “Running Lean” Problem

One common problem that occurred in the first several offerings of this project is what the author has called the “running lean”. PD control mildly improves the stability of the system in that the robot no longer immediately falls over. But what happens is that the robot starts to accelerate in the direction that it is leaning, but the acceleration is not enough to balance the robot. The robot essentially maintains a constant lean until it runs into a wall or another obstacle. Correcting this problem was quite perplexing for some time.

5. Robot Chassis Revision: a Cart/ Pendulum System

The second revision of the robot for this project is shown in Fig. 2. The revised robot consists of a pendulum attached to a cart that is driven by two DC motors. Starting in the third offering of the

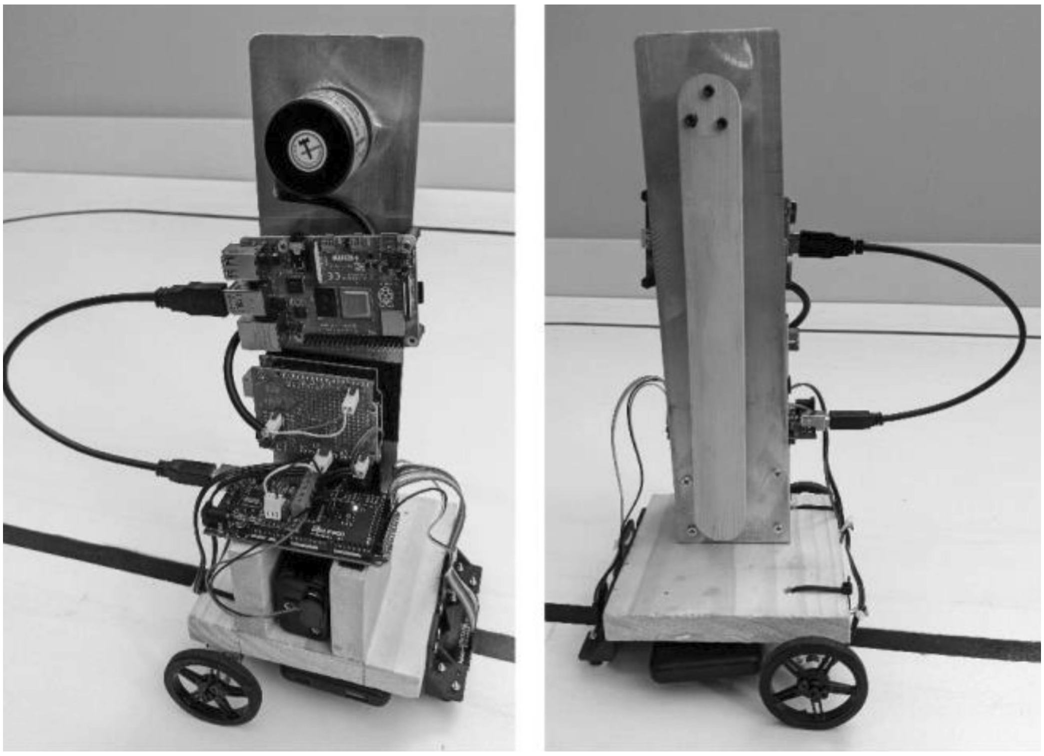

Fig. 2. The revised robot chassis – a cart/pendulum system used in the last two years of the project.

project, the students were given a robot chassis and no longer asked to design and build their own. This version addresses several of the concerns from the initial self-balancing robot. The pendulum angle is measured directly using an encoder. This means that the gyroscope/accelerometer has been eliminated along with any filtering techniques. It should be much easier to build a well balanced pendulum. System identification can easily be conducted in the pendulum down position, which will hopefully lead to insights into the development of a pendulum up model. The robot can also be used for an initial vibration suppression project with the pendulum in the down position that should help students feel successful. For more details on the design of the cart/pendulum system, see [7].

The chassis is a mix of off-the-shelf parts, 3D printed parts, CNC-ed plastic, along with a small number of custom made parts. The are three custom PCBs that are made from prototyping shields purchased on Amazon. Any reader that is interested in building one of these robot systems is encouraged to contact the author.

6. Development of an Accurate Transfer Function System Model

The inverted pendulum problem has been studied since at least the 1960s [8]. Hundreds of papers have been written about the inverted pendulum and it is considered a benchmark for nonlinear control approaches. See [9, 10] for fairly recent surveys. See [11] for an example of a landmark paper on the subject.

Many of the models in the literature assume a force or torque input and the models are typically fairly complicated. Practically speaking, the input to this system is a PWM voltage to the DC motors. At steady state, the PWM duty cycle is proportional to motor speed. However there are dynamics involved when the PWM duty cycle changes. It is often assumed that pendulum motion could back drive the cart. The gear ratio of the DC motor prevents any back driving from happening with this system.

This project is being implemented in a first course in dynamic systems and feedback control and the students have not learned state-space modeling. So, the modeling challenge is to come up with a transfer function model that captures the essential dynamics of this system while taking all of these practical details into account. In order to arrive at this transfer function model, the instructor worked through a cart/pendulum system model based on a Lagrange approach inspired by a YouTube video [12]. This lead to two, coupled equations of motion: one for the cart and one for the pendulum. The cart equation of motion implied back driving by the pendulum and did not include the gear ratio of the motor. The cart equation was thrown out and replaced with a transfer function between PWM duty cycle and the displacement of the cart based on known DC motor dynamics, namely:

$$\frac{X}{PWM} = \frac{N}{s(s+p)} \tag{1}$$

The pendulum equation of motion from the Lagrange analysis was then linearized and rearranged into a transfer function with cart displacement as the input and pendulum rotation as the output:

$$\frac{\theta}{X} = \frac{(1/l)s^2}{(s+\omega_n)(d-\omega_n)} \tag{2}$$

Multiplying these two transfer functions together gives the inverted pendulum transfer function with PWM duty cycle as the input and pendulum rotation as the output:

$$\frac{\theta}{PWM} = \frac{\hat{N}s}{(s+p)(s+\omega_n)(s-\omega_n)} \tag{3}$$

Because the design of the cart/pendulum system allows experiments to be run with the pendulum in the up or down positions, this modeling approach can be verified by linearizing about the pendulum down position and running various tests to validate the model (swept sine, pulse input, . . .).

It is important to note the zero in the transfer function. The combination of the unstable pole and the zero at the origin makes control design very challenging. A pure integrator is essential to cancel the zero. This is somewhat counter-intuitive: integral terms are not usually thought of as increasing the stability of a system. This also means that PD control alone cannot stabilize this system. Given the model and the knowledge that an integral term is essential, students were able to use the root locus approach to design a controller that stabilizes the pendulum in the upright position.

7. What makes a Project "Good" for Student Learning?

One pedagogical issue related to project-based learning (PBL) that is debated in the literature is what level of challenge and practical detail makes a project appropriate for student learning. Projects often involve real-world details that are difficult to model theoretically (such as friction or the effects of digital time delay on control design). The evolution of this project has led to several observations related to creating course projects that promote student learning.

First, students prefer projects where they feel successful. It might seem obvious, but trying to convince students that they learned a lot even if their robot fell over was difficult. Student course evaluations for the first two years of this project were not very good.

Table 1. Students course evaluations over the span of this project. Number of students refers to the number completing the course evaulation. Completion percentage refers to what percentage of students enrolled in the course completed the survey. The mean summative rating refers to the mean value of the overall summative rating for the course which "represents the combined responses of students to the four global summative items and is presented to provide an overall index of the class's quality." Note that the project was not included in the 2019 offering of the course.

Year	Number of Students	Completion Percentage	Mean Summative Rating
2017	28	82%	3.1
2018	21	64%	3.4
2019*	17	85%	3.8
2020	16	70%	3.2
2021	9	50%	3.6
2022	10	48%	4.4

Second, making a project easier does not necessarily make students happier. In the third year of this project, students did vibration suppression control on the cart/pendulum system, but were not asked to keep the pendulum upright in the unstable position. Student course evaluations that year were lukewarm.

The ideal scenario seems to be a challenging project where theory and practice come together and students feel successful. In the fifth year of the project, students were able to balance the pendulum in the upright position by using a root locus to design their PID controller based on a transfer function model. Student evaluations that year were the highest they have ever been with some students commenting on getting to work on a project related to the author's research. This was the first time that the student course evaluations accurately reflected the instructor's efforts and what was learned in the course.

Student course evaluations over the course of the project are summarized in Table 1. The number of students in the table refers to the number who completed the evaluation. The completion percentage refers to the fraction of students completing the evaluation compared to total course enrollment. The summative rating is on a scale from 0-5. Note that the project was not conducted in 2019 (a slewing beam vibration suppression project was done instead).

8. The Raspberry Pi Plus Arduino Control Approach

This project uses the combination of a Raspberry Pi and an Arduino to form a powerful, low-cost, open-source feedback control system. The Arduino is a micro-controller that interfaces with the sensors and actuators. Arduino micro-controllers are very popular and inexpensive. There are many sensors and actuators designed to interface with them. There are also many software libraries available for using the sensors and actuators with an Arduino. The Raspberry Pi is a mini-computer with significantly more RAM and computing power than an Arduino. It also provides wifi and a remote desktop environment for the students to log into. Being able to combine the strengths of these two popular devices creates a powerful system. However, communication between the Raspberry Pi and the Arduinos becomes a key bottleneck for the system. In the fourth year of the project, students were allowed to program the Raspberry Pi in Python while the Arduinos ran standard code that was given to them. This approach reduced the students' pain related to syntax, but it restricted the controller to a 100 Hz update frequency because of USB serial latencies. This relatively slow control frequency prevented the students from successfully balancing the pendulum.

Switching to i^2c and essentially removing latency was one key part of student success in year five of the project. By using i^2c, the digital control frequency was increased to 500 Hz. For more details on combining Raspberry Pi and Arduino for real-time feedback control, see [13, 14].

9. Block Diagram Modeling and Auto-Generation of Raspberry Pi C++ Code

Another important aspect of this project was equipping students to program a real-time feedback control system while keeping them from being bogged down by low-level coding details. Programming a micro-controller to perform real-time feedback control is fairly complicated. Students in the first two years of this project were required to directly write C code for their Arduinos. This approach hindered student learning because it was difficult to untangle feedback control concepts from coding syntax. Additionally, if students are writing their own C code, they are not constrained to think in terms of transfer functions and feedback control block diagrams. If students successfully complete the project, but do not use feedback control properly, the learning objectives of the project will not be achieved.

To address these issues, the author created a software library that allows students to describe the control logic using a block diagram. The software then auto-generates Raspberry Pi C++ code based on the block diagram. This approach significantly reduced student programming frustration while constraining them to think in terms of transfer functions and controls block diagrams. In this approach, i^2c was used for the communication between the Raspberry Pi and Arduino, significantly reducing communication latencies and enabling 500 Hz digital control. This software was another key part of the students' success in year five.

All of the software developed to support this project is open-source. There are two Python libraries available on pypi. They can be installed using pip. The back-end library for describing a block diagram system, drawing the block diagram, and auto-generated C++ code for Arduino or Raspberry Pi is called py_block_diagram. There is a tkinter gui for block diagram creation called pybd_gui. pybd_gui depends on py_block_diagram. The C++ code that is auto-generated by Python depends on two C++ libraries, one for Arduino [15] and one for Raspberry Pi [16].

10. Comparison of Student Performance in 4th and 5th Year of the Project

In the fourth year of the project, students were writing their control logic in Python on the Raspberry Pi and communicating with the Arduino over USB serial. The latency of USB serial communication limited the digital control frequency to roughly 100 Hz. The students in year four were given the transfer function model, but were not explicitly instructed to place a pole at the origin. In year four, the longest time that any student was able to balance the pendulum was 3.5 seconds.

In the fifth year of the project, students successfully designed PID controllers that stabilized the pendulum in the upright position. The students in year five wrote their control logic using custom block diagram software written by the author. That software then auto-generated C++ code for the Raspberry Pi (the Arduinos were running standard code that was also provided). The Raspberry Pi and Arduinos communicated using i^2c, which essentially eliminated latency. This combination allowed the control logic to be executed at 500 Hz. The students in year five were also explicitly instructed to place a pole at the origin. Most teams used the root locus method to design PID control. In year five, a cap of seven minutes was placed on the pendulum balancing event. Five out of ten teams were able to balance the pendulum for the maximum seven minute time. The average time for balancing the pendulum was 300 seconds.

11. Conclusions

In conclusion, this paper has presented the iterative design of a novel and engaging feedback controls project. The experimental cart/pendulum system is relatively low cost, so that students can potentially be allowed to take the system home. In the final iteration of the project, students were able to successfully balance the pendulum in the vertical position based on root locus control design. In order to use the root locus method, students had to be given an accurate model of the system. Students gained an appreciation for feedback control theory when they saw that the root locus method is much more powerful than PID tuning, especially for unstable systems.

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